TESLA TURBINE THEORY

2nd Edition

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paraphrase: Performance Characteristics of a Friction Disc Turbine by E.W. Beans, Penn. State U. Ph.D. thesis, June 1961. paraphrase of pages 1-3 of introduction.

The Tesla Turbine has two parts: the nozzle and the rotor. The nozzle accelerates the fluid , and the rotor creates torque by changing the fluid's momentum. The Tesla Turbine's rotor speeds up (its momentum changed) because of friction between the fluid and rotor. Most turbine rotors are bladed, high-tech fans.

Imagine a fluid flowing evenly and parallel across a solid disc. The difference of speeds between the fluid and the disc creates a shear stress that slows the fluid down. The layer of fluid next to and parallel to the disc is slowed to the same speed as that of the disc, a relative velocity of zero. This layer slows down the layer of fluid next to it, to nearly zero relative velocity. Each succeeding layer of fluid is slowed down less the farther away they get from the disc where the relative velocity between fluid and disc is zero. This pattern of adjacent layers of fluid moving at different speeds is a "velocity profile."

By slowing the fluid down, the shear stress (or drag force) reduces the fluid's momentum and kinetic energy and raises its temperature. If the disc is fixed to an axis perpendicular to it (a shaft and bearings), the drag force can create torque equal to the moment of the drag (force x force arm). If the disc is free to turn on the shaft, the torque will accelerate the disc, passing kinetic energy of motion from the fluid to the disc. As the disc speeds up, the relative velocity between it and the fluid decreases. This reduces the shear stress between disc and fluid while also reducing torque. The disc will keep speeding up until the torque produced by the relative velocity between disc and fluid equals the torque externally applied to the axis of rotation. The rotating disc is the rotor of the Tesla Turbine.

The amount of torque depends on the amount of shear stress between disc and fluid. Increasing shear stress increases torque. So there should be more torque when there is a higher relative velocity between disc and fluid, and when flow is turbulent. Torque is made up of the shear stress and the area of the rotor disc; it can be increased by using more discs. Nikola Tesla's friction disc turbine has a series of smooth flat discs fixed parallel to each other on a shaft and housed in a cylinder. Fig. 1 shows the fluid path through a typical Tesla Turbine. The fluid accelerates through a nozzle or series of nozzles and enters the turbine tangentially at its outside radius. It flows between the discs, where its momentum changes, and then the fluid discharges at the inside radius as shown in the drawing by the dotted line. Also shown is the spiral path between the discs. As the angular* velocity of the discs

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*angular: pertaining to quantities related to a revolving body that are measured in reference to its axis of revolution

increases, the shear stress across the discs decreases and and the spiral path becomes longer, the coils of the spiral being closer to each other.

As in all radial inflow turbines, a pressure drop must exist between the outside and inside radii (coils) to get a flow. The main purpose of this pressure drop is to overcome the centrifugal force of the fluid, not to overcome the frictional resistance to flow. Thus the Tesla Turbine might be classified as a reaction turbine. But, as shown later, this pressure drop is large compared to other turbines and results in low efficiencies.

(Editor's note: the thesis is 186 pages long, and is only one of several degree papers written about the Tesla Turbine in the 50's and 60's.)

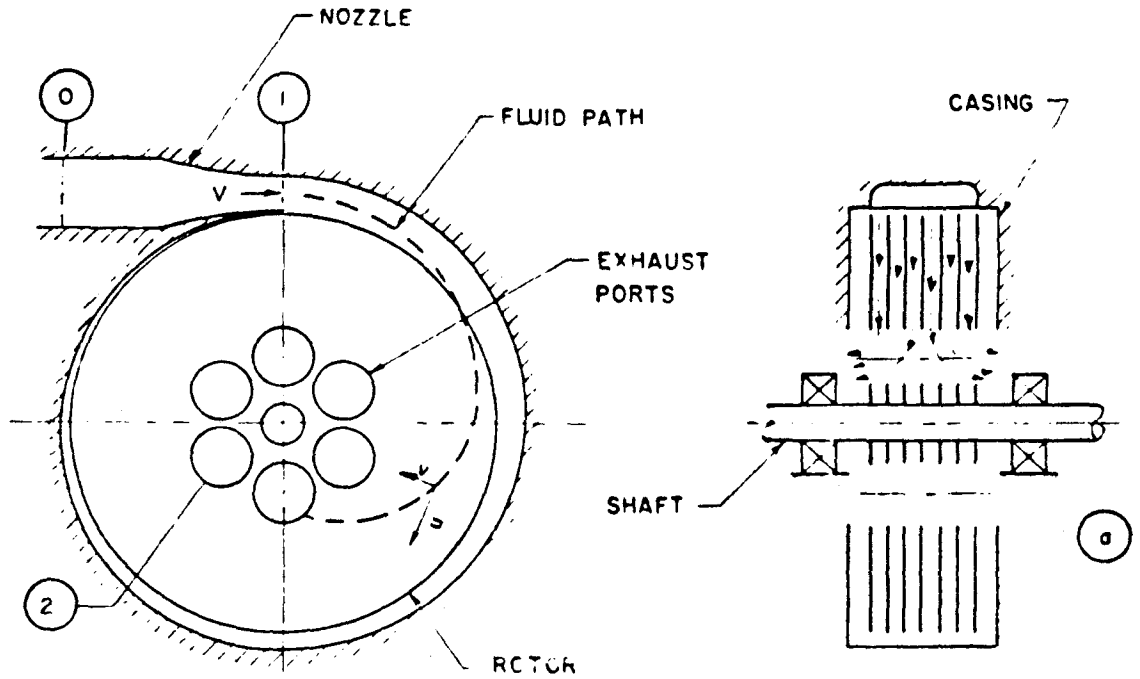

FIG. 1 SCHEMATIC OF TESLA TURBINE

Tesla Turbine Design Criteria
taken from Tesla and Possell

Possell: ...nozzles based on Coanda Effect.
...stamp discs--cheapest way to fabricate--spacers between discs; use hardened steel.
...(Possell): Bernoulli's equation: total pressure = velocity squared plus static pressure ($P_{total} = v^2 + P_s{}^2$)(?).
When fluid comes off periphery of pump rotor, the total pressure is all velocity. Then into divergent diffuser.
Turbine: "series of nozzles'" is opposite of diffuser, converting pressure and temperature into velocity.
...automatic overspeed protection--as load is removed & rotor gains speed, centrifugal force increases at a rate that is the square of the speed; centrifugal force soon equals the pressure ratio across the rotor from nozzle to exhaust, causing turbine speed to level off.
...spacers between discs are of appropriate thickness to keep the flow of the effluent between discs in a laminar range for highest efficiency.
...discs welded together on the shaft. Shaft mounted in two outboard bearings, continuously lubricated (steam turbine).
...from pictures: discs are bolted together in one or two rows of concentric circles, depending on size of rotor--holed in disc are full (no struts)--the two outside discs are thicker than the rest. Spacers must be where bolts go through that hold discs together.
...9" rotor weighing less than 20 lbs. can develop 200 bhp.
9 3/4" rotor x 2" wide, 110 hp with free exhaust, 9000 rpm.

Tesla: ...(pump): fluid subjected to two forces: one acting tangentially in direction of rotation, the other radially outward. Combined effect of two forces is to accelerate the fluid to the outlet in spiral path. The path of the fluid can be estimated by determining the # of revolutions required to renew the fluid passing through the chamber and multiplying it by the ratio between the mean speed of the fluid and that of the discs. The flow is approximately proportional to the surface area of the rotor & its effective speed. Therefore, performance increases quickly with increase of size & speed. Disc gap should be greater the larger the diameter, the longer the spiral path, & the greater the viscosity. Spacing should be such that the entire mass of the fluid before leaving the rotor is accelerated to a nearly uniform velocity, not much below that of the periphery of rotor under normal working conditions; almost equal to it when the outlet is closed & the particles move in concentric circles.
...(turbine): with rotor turning freely, rim will attain speed close to the max of that the fluid in the intake jet, & the path of the fluidwill be relatively long, consisting of many almost circular turns. When loaded, rotor is slowed, motion of fluid is slowed, the # of turns are reduced, and

the path is shortened. In general, the torque is indirectly proportional to the square of the velocity of the fluid relative to the rotor and to the effective area of the discs, and inversely to the distance separating them. The machine will generally perfprm its maximum work when the effective speed of the rotor is ½ that of the fluid. But for highest economy, the relative speed or slip for any given performance should be as little as possible. This condition can be approximated by increasing the active area and reducing the disc gap. With fluid admitted through a convergent-divergent nozzle, pressure is converted wholly or in part into velocity. Rotor absorbs kinetic momentum as fluid whirls, with decreasing speed, to the exhaust. When expansion is complete in the nozzle, pressure in peripheral clearance between rotor and case is small.
...In one respect, turbine and pump are essentially different. In the pump, the radial or static pressure due to centrifugal force is added to the tangential or dynamic force, thus increasing the effective head. But in the turbine, the radial or static pressure due to centrifugal force reduces the effective head & the velocity of radial flow toward the center. In turbine, desirability of a great torque calls for increased # of discs & smaller gap, but in pump, rotary effort should be the smallest & the speed the greatest practicable.
Scientific American, 9-30-11...(re: turbine)...
...hardened & tempered steel discs
...in 18" turbine, fluid covers a path 12-16 ft. long
...resistance to passage of fluid between discs is approx. proportional to the square of the relative speed which is at a maximum toward the center of the discs & is equal to the tangential velocity of the steam. Therefore the resistance to radial escape is very great & further enhanced by the centrifugal force acting outwardly.
Engineering News Record, 10-12-11...(pump): fluid follows short or long path depending on quantity allowed to escape from outlet. With unrestricted exhaust, there's little resistance to radial flow (that is from the backing up & development of pressure) & the tangential slip between disc & fluid is large. Pressure in casing depends on velocity of fluid leaving periphery of disc. Max pressure with exhaust throttled is proportional to disc speed. Velocity head converted to pressure head in diverging discharge nozzle. Power absorbed is proportional to square of slip..
...whole surface of each disc is effective in impelling fluid.
...quantity of fluid discharged is proportional to disc area: capacity increases nearly directly with length along shaft (rotor width) and with square of diameter.
...disc spacing increases with viscosity & diameter & decreases with allowable slip.
..fluid should be delivered from discs at not much below peripheral speed under normal load.
...(turbine): if shaft were blocked, fluid would take short

path from inlet to exhaust. With rotor turning, centrifugal force makes fluid take a longer spiral path.
...torque developed increases with difference in peripheral velocity of disc and of fluid in contact. Torque rises with the square of the slip. When shaft runs free without load, speed rises & the centrifugal counter-force on the fluid (traveling in long spiral, almost concentric paths) causes casing pressure to ruse nearly to supply pressure, difference being only that required to do the work of supplying losses.
...max work done at about 50% average slip, but max efficiency comes with small slip.
1906...Tesla's first model--compressed air--35,000 rpm--rotor 6" diameter--discs 1/32" thick--mounted on center of shaft 6" long, 1" diameter in middle & stepped down to ½" diameter on ends. Set in casing made of four parts bolted together. Eight discs; rotor width ½". Clearance between walls of case & faces of rotor 1/64". 30 hp.
1910 Model...12" discs, 10,000 rpm, 100 hp
...larger rotor diameter = slower rpm @ given output.
...18" diameter, 25 discs 1/32" thick, total rotor thickness 3½"; path of fluid 12-16 ft., 200 hp on steam.

N. TESLA.

TURBINE.

APPLICATION FILED JAN. 17, 1911.

1,061,206. Patented May 6, 1913.

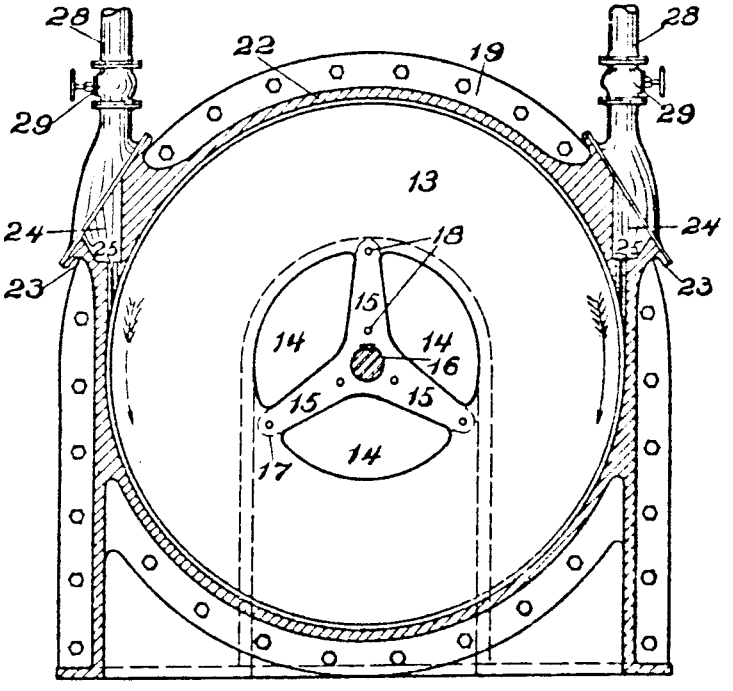

Fig. 1.

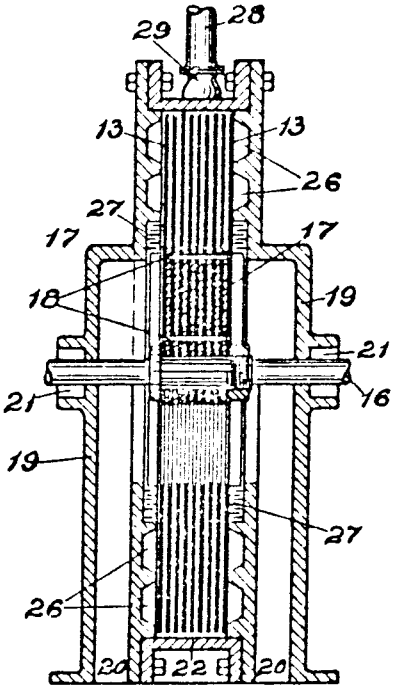

Fig. 2.

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By his Attorneys
Howson Page Cooper & Hayward

UNITED STATES PATENT OFFICE.

NIKOLA TESLA, OF NEW YORK, N. Y.

TURBINE.

1,061,206. Specification of Letters Patent. Patented May 6, 1913.

Original application filed October 21, 1909, Serial No. 523,832. Divided and this application filed January 17, 1911. Serial No. 603,049.

To all whom it may concern:

Be it known that I, NIKOLA TESLA, a citizen of the United States, residing at New York, in the county and State of New York, have invented certain new and useful Improvements in Rotary Engines and Turbines, of which the following is a full, clear, and exact description.

In the practical application of mechanical power, based on the use of fluid as the vehicle of energy, it has been demonstrated that, in order to attain the highest economy, the changes in the velocity and direction of movement of the fluid should be as gradual as possible. In the forms of apparatus heretofore devised or proposed, more or less sudden changes, shocks and vibrations are unavoidable. Besides, the employment of the usual devices for imparting to, or deriving energy from a fluid, such as pistons, paddles, vanes and blades, necessarily introduces numerous defects and limitations and adds to the complication, cost of production and maintenance of the machines.

The object of my invention is to overcome these deficiencies and to effect the transmission and transformation of mechanical energy through the agency of fluids in a more perfect manner, and by means simpler and more economical than those heretofore employed. I accomplish this by causing the propelling fluid to move in natural paths or stream lines of least resistance, free from constraint and disturbance such as occasioned by vanes or kindred devices, and to change its velocity and direction of movement by imperceptible degrees, thus avoiding the losses due to sudden variations while the fluid is imparting energy.

It is well known that a fluid possesses, among others, two salient properties, adhesion and viscosity. Owing to these a solid body propelled through such a medium encounters a peculiar impediment known as "lateral" or "skin resistance," which is twofold, one arising from the shock of the fluid against the asperities of the solid substance, the other from internal forces opposing molecular separation. As an inevitable consequence a certain amount of the fluid is dragged along by the moving body. Conversely, if the body be placed in a fluid in motion, for the same reasons, it is impelled in the direction of movement. These effects, in themselves, are of daily observation, but I believe that I am the first to apply them in a practical and economical manner in the propulsion of fluids or in their use as motive agents.

In an application filed by me October 21st, 1909, Serial Number 523,832 of which this case is a division, I have illustrated the principles underlying my discovery as embodied in apparatus designed for the propulsion of fluids. The same principles, however, are capable of embodiment also in that field of mechanical engineering which is concerned in the use of fluids as motive agents, for while in certain respects the operations in the latter case are directly opposite to those met with in the propulsion of fluids, and the means employed may differ in some features, the fundamental laws applicable in the two cases are the same. In other words, the operation is reversible, for if water or air under pressure be admitted to the opening constituting the outlet of a pump or blower as described, the runner is set in rotation by reason of the peculiar properties of the fluid which, in its movement through the device, imparts its energy thereto.

The present application, which is a division of that referred to, is specially intended to describe and claim my discovery above set forth, so far as it bears on the use of fluids as motive agents, as distinguished from the applications of the same to the propulsion or compression of fluids.

In the drawings, therefore, I have illustrated only the form of apparatus designed for the thermo-dynamic conversion of energy, a field in which the applications of the principle have the greatest practical value.

Figure 1 is a partial end view, and Fig. 2 a vertical cross-section of a rotary engine or turbine, constructed and adapted to be operated in accordance with the principles of my invention.

The apparatus comprises a runner composed of a plurality of flat rigid disks 13 of suitable diameter, keyed to a shaft 16, and held in position thereon by a threaded nut 11, a shoulder 12, and intermediate washers 17. The disks have openings 14 adjacent to the shaft and spokes 15, which

may be substantially straight. For the sake of clearness, but a few disks, with comparatively wide intervening spaces, are illustrated.

The runner is mounted in a casing comprising two end castings 19, which contain the bearings for the shaft 16, indicated but not shown in detail; stuffing boxes 21 and outlets 20. The end castings are united by a central ring 22, which is bored out to a circle of a slightly larger diameter than that of the disks, and has flanged extensions 23, and inlets 24, into which finished ports or nozzles 25 are inserted. Circular grooves 26 and labyrinth packing 27 are provided on the sides of the runner. Supply pipes 28, with valves 29, are connected to the flanged extensions of the central ring, one of the valves being normally closed.

For a more ready and complete understanding of the principle of operation it is of advantage to consider first the actions that take place when the device is used for the propulsion of fluids for which purpose let it be assumed that power is applied to the shaft and the runner set in rotation say in a clockwise direction. Neglecting, for the moment, those features of construction that make for or against the efficiency of the device as a pump, as distinguished from a motor, a fluid, by reason of its properties of adherence and viscosity, upon entering through the inlets 20, and coming in contact with the disks 13, is taken hold of by the latter and subjected to two forces, one acting tangentially in the direction of rotation, and the other radially outward. The combined effect of these tangential and centrifugal forces is to propel the fluid with continuously increasing velocity in a spiral path until it reaches a suitable peripheral outlet from which it is ejected. This spiral movement, free and undisturbed and essentially dependent on the properties of the fluid, permitting it to adjust itself to natural paths or stream lines and to change its velocity and direction by insensible degrees, is a characteristic and essential feature of this principle of operation.

While traversing the chamber inclosing the runner, the particles of the fluid may complete one or more turns, or but a part of one turn, the path followed being capable of close calculation and graphic representation, but fairly accurate estimates of turns can be obtained simply by determining the number of revolutions required to renew the fluid passing through the chamber and multiplying it by the ratio between the mean speed of the fluid and that of the disks. I have found that the quantity of fluid propelled in this manner, is, other conditions being equal, approximately proportionate to the active surface of the runner and to its effective speed. For this reason, the performance of such machines augments at an exceedingly high rate with the increase of their size and speed of revolution.

The dimensions of the device as a whole, and the spacing of the disks in any given machine will be determined by the conditions and requirements of special cases. It may be stated that the intervening distance should should be the greater, the larger the diameter of the disks, the longer the spiral path of the fluid and the greater its viscosity. In general, the spacing should be such that the entire mass of the fluid, before leaving the runner, is accelerated to a nearly uniform velocity, not much below that of the periphery of the disks under normal working conditions, and almost equal to it when the outlet is closed and the particles move in concentric circles.

Considering now the converse of the above described operation and assuming that fluid under pressure be allowed to pass through the valve at the side of the solid arrow, the runner will be set in rotation in a clockwise direction, the fluid traveling in a spiral path and with continuously diminishing velocity until it reaches the orifices 14 and 20, through which it is discharged. If the runner be allowed to turn freely, in nearly frictionless bearings, its rim will attain a speed closely approximating the maximum of that of the adjacent fluid and the spiral path of the particles will be comparatively long, consisting of many almost circular turns. If load is put on and the runner slowed down, the motion of the fluid is retarded, the turns are reduced, and the path is shortened.

Owing to a number of causes affecting the performance, it is difficult to frame a precise rule which would be generally applicable, but it may be stated that within certain limits, and other conditions being the same, the torque is directly proportionate to the square of the velocity of the fluid relatively to the runner and to the effective area of the disks and, inversely, to the distance separating them. The machine will, generally, perform its maximum work when the effective speed of the runner is one-half of that of the fluid; but to attain the highest economy, the relative speed or slip, for any given performance, should be as small as possible. This condition may be to any desired degree approximated by increasing the active area of and reducing the space between the disks.

When apparatus of the kind described is employed for the transmission of power certain departures from similarity between transmitter and receiver are necessary for securing the best results. It is evident that, when transmitting power from one shaft to another by such machines, any desired ratio between the speeds of rotation may be obtained by a proper selection of the diameters of the disks, or by suitably staging the

transmitter, the receiver or both. But it may be pointed out that in one respect, at least, the two machines are essentially different. In the pump, the radial or static pressure, due to centrifugal force, is added to the tangential or dynamic, thus increasing the effective head and assisting in the expulsion of the fluid. In the motor, on the contrary, the first named pressure, being opposed to that of supply, reduces the effective head and the velocity of radial flow toward the center. Again, in the propelled machine a great torque is always desirable, this calling for an increased number of disks and smaller distance of separation, while in the propelling machine, for numerous economic reasons, the rotary effort should be the smallest and the speed the greatest practicable. Many other considerations, which will naturally suggest themselves, may affect the design and construction, but the preceding is thought to contain all necessary information in this regard.

In order to bring out a distinctive feature, assume, in the first place, that the motive medium is admitted to the disk chamber through a port, that is a channel which it traverses with nearly uniform velocity. In this case, the machine will operate as a rotary engine, the fluid continuously expanding on its tortuous path to the central outlet. The expansion takes place chiefly along the spiral path, for the spread inward is opposed by the centrifugal force due to the velocity of whirl and by the great resistance to radial exhaust. It is to be observed that the resistance to the passage of the fluid between the plates is, approximately, proportionate to the square of the relative speed, which is maximum in the direction toward the center and equal to the full tangential velocity of the fluid. The path of least resistance, necessarily taken in obedience to a universal law of motion is, virtually, also that of least relative velocity. Next, assume that the fluid is admitted to the disk chamber not through a port, but a diverging nozzle, a device converting wholly or in part, the expansive into velocity-energy. The machine will then work rather like a turbine, absorbing the energy of kinetic momentum of the particles as they whirl, with continuously decreasing speed, to the exhaust.

The above description of the operation, I may add, is suggested by experience and observation, and is advanced merely for the purpose of explanation. The undeniable fact is that the machine does operate, both expansively and impulsively. When the expansion in the nozzles is complete, or nearly so, the fluid pressure in the peripheral clearance space is small; as the nozzle is made less divergent and its section enlarged, the pressure rises, finally approximating that of the supply. But the transition from purely impulsive to expansive action may not be continuous throughout, on account of critical states and conditions and comparatively great variations of pressure may be caused by small changes of nozzle velocity.

In the preceding it has been assumed that the pressure of supply is constant or continuous, but it will be understood that the operation will be, essentially the same if the pressure be fluctuating or intermittent, as that due to explosions occurring in more or less rapid succession.

A very desirable feature, characteristic of machines constructed and operated in accordance with this invention, is their capability of reversal of rotation. Fig. 1, while illustrative of a special case, may be regarded as typical in this respect. If the right hand valve be shut off and the fluid supplied through the second pipe, the runner is rotated in the direction of the dotted arrow, the operation, and also the performance remaining the same as before, the central ring being bored to a circle with this purpose in view. The same result may be obtained in many other ways by specially designed valves, ports or nozzles for reversing the flow, the description of which is omitted here in the interest of simplicity and clearness. For the same reasons but one operative port or nozzle is illustrated which might be adapted to a volute but does not fit best a circular bore. It will be understood that a number of suitable inlets may be provided around the periphery of the runner to improve the action and that the construction of the machine may be modified in many ways.

Still another valuable and probably unique quality of such motors or prime movers may be described. By proper construction and observance of working conditions the centrifugal pressure, opposing the passage of the fluid, may, as already indicated, be made nearly equal to the pressure of supply when the machine is running idle. If the inlet section be large, small changes in the speed of revolution will produce great differences in flow which are further enhanced by the concomitant variations in the length of the spiral path. A self-regulating machine is thus obtained bearing a striking resemblance to a direct-current electric motor in this respect that, with great differences of impressed pressure in a wide open channel the flow of the fluid through the same is prevented by virtue of rotation. Since the centrifugal head increases as the square of the revolutions, or even more rapidly, and with modern high grade steel great peripheral velocities are practicable, it is possible to attain that condition in a single stage machine, more readily if the runner be of large diameter. Obviously this problem is

facilitated by compounding, as will be understood by those skilled in the art. Irrespective of its bearing on economy, this tendency which is, to a degree, common to motors of the above description, is of special advantage in the operation of large units, as it affords a safeguard against running away and destruction. Besides these, such a prime mover possesses many other advantages, both constructive and operative. It is simple, light and compact, subject to but little wear, cheap and exceptionally easy to manufacture as small clearances and accurate milling work are not essential to good performance. In operation it is reliable, there being no valves, sliding contacts or troublesome vanes. It is almost free of windage, largely independent of nozzle efficiency and suitable for high as well as for low fluid velocities and speeds of revolution.

It will be understood that the principles of construction and operation above generally set forth, are capable of embodiment in machines of the most widely different forms, and adapted for the greatest variety of purposes. In my present specification I have sought to describe and explain only the general and typical applications of the principle which I believe I am the first to realize and turn to useful account.

What I claim is:

1. A machine adapted to be propelled by a fluid consisting in the combination with a casing having inlet and outlet ports at the peripheral and central portions, respectively, of a rotor having plane spaced surfaces between which the fluid may flow in natural spirals and by adhesive and viscous action impart its energy of movement to the rotor, as described.

2. A machine adapted to be propelled by a fluid, comprising a rotor composed of a plurality of plane spaced disks mounted on a shaft and open at or near the same, an inclosing casing with a peripheral inlet or inlets, in the plane of the disks, and an outlet or outlets in its central portion, as described.

3. A rotary engine adapted to be propelled by adhesive and viscous action of a continuously expanding fluid comprising in combination a casing forming a chamber, an inlet or inlets tangential to the periphery of the same, and an outlet or outlets in its central portion, with a rotor composed of spaced disks mounted on a shaft, and open at or near the same, as described.

4. A machine adapted to be propelled by fluid, consisting in the combination of a plurality of disks mounted on a shaft and open at or near the same, and an inclosing casing with ports or passages of inlet and outlet at the peripheral and central portions, respectively, the disks being spaced to form passages through which the fluid may flow, under the combined influence of radial and tangential forces, in a natural spiral path from the periphery toward the axis of the disks, and impart its energy of movement to the same by its adhesive and viscous action thereon, as set forth.

5. A machine adapted to be propelled by a fluid comprising in combination a plurality of spaced disks rotatably mounted and having plane surfaces, an inclosing casing and ports or passages of inlet and outlet adjacent to the periphery and center of the disks, respectively, as set forth.

6. A machine adapted to be propelled by a fluid comprising in combination a runner composed of a plurality of disks having plane surfaces and mounted at intervals on a central shaft, and formed with openings near their centers, and means for admitting the propelling fluid into the spaces between the disks at the periphery and discharging it at the center of the same, as set forth.

7. A thermo-dynamic converter, comprising in combination a series of rotatably mounted spaced disks with plane surfaces, an inclosing casing, inlet ports at the peripheral portion and outlet ports leading from the central portion of the same, as set forth.

8. A thermo-dynamic converter, comprising in combination a series of rotatably mounted spaced disks with plane surfaces and having openings adjacent to their central portions, an inclosing casing, inlet ports in the peripheral portion, and outlet ports leading from the central portion of the same, as set forth.

In testimony whereof I affix my signature in the presence of two subscribing witnesses.

NIKOLA TESLA.

Witnesses:
M. LAWSON DYER,
WM. BOHLEBER.

NEW INVENTIONS BY TESLA.

ADDRESS AT MEETING OF NEW YORK SECTION OF NATIONAL ELECTRIC LIGHT ASSOCIATION.

"My project was evidently far in advance of the times. Its progress was retarded and I was compelled to devote myself for a time to other inventions which appealed more to practical men. After years of careful thinking, I found that what the world needed most, and would most readily accept, was an efficient prime mover, a converter of heat into mechanical energy. This all the more as a new world is about to be explored, the world of waste. When you consider that in the manufacture of steel and iron in this country, some thirty million horsepower could be harnessed and a proportionate income derived from that power, all of which is wasted today, you can see what value a good convertor of thermal energy into mechanical energy would have.

"But to conceive that a prime mover is valuable and to get up one, are two different things. After some thought I finally came to the following argument: suppose a number of plates are moved through a fluid medium, the medium will, of course, be dragged along with the plates, and a certain frictional loss will be incurred. Inside the casing are arranged on a shaft a number of disks with openings and spokes and there are orifices of entrance on the sides to produce a perfect balance, and the usual arrangement of outlets. This system of disks being rotated, the water or air is sucked into the channel, is taken hold of and moves in a logarithmic spiral with very nearly the velocity of the system. It was perfectly well known that a fluid would be dragged by rotating surfaces, but somehow nobody realized the conditions for economic working, nor has any one properly grasped the principles which could be applied to propulsion. So it happens again that it is my good fortune to come to the rescue, and I have produced a highly economical way of compressing or pumping fluids." Mr. Tesla then gave a practical demonstration of the working of the principle in a model pump.

"This is one of the early forms of blower which I constructed" (referring to diagram). "That was constructed three years ago. It is a two-stage blower. Far more important than the pump blower or compressor is the turbine. Here is a simple structure, a casing with two entrances, disks arranged on the shaft and outlets in the center for the escape. In this instance the power is applied to one of the openings and the fluid moves with decreasing velocity toward the center until its energy is exhausted and transferred to the shaft. If the theory is correct, I am able to take out the entire energy of steam in one single stage. In the present turbine, sixty-five per cent is the limit of efficiency; theoretically I should be able to get ninety-nine per cent of the total energy of the steam on the shaft in these turbines. These turbines are simple, they have a great torque, far better than other turbines, and a machine will develop ten horsepower for every pound of weight. This principle can also be applied to the gas turbine."

Several slides showing two turbines coupled together were then projected on the screen and a new method of power measurement described. Mr. Tesla then dwelt on the advantages of these machines and showed a number which were constructed and in operation. They have no ducts, nozzles or such complications which cause so much trouble, and besides the machines are perfectly reversible, working with the same efficiency back or forth, making a valuable machine for driving boats, locomotives, automobiles, etc. The accompanying illustrations show two of these turbines.

"In this new invention we have a beautiful solution of many mechanical problems. We have a prime mover which is reversible, ideally simple, of enormous torque, incomparably greater than the turbine possesses, so I am looking for a revolution in mechanics from the application of this principle."

THE TESLA TURBINE

A MACHINE AS BIG AS A DERBY HAT THAT GENERATES 110 HORSEPOWER

BY

FRANK PARKER STOCKBRIDGE

THE WORLD'S WORK MARCH 1912 P. 543-548

WE FOLLOWED Dr. Nikola Tesla through the Waterside Power Station of the New York Edison Company — along narrow passages lined with huge electric switches, the turning of any one of which would throw a whole section of Manhattan into darkness or a blaze of light. We stumbled in the shadows of whirring dynamos, skirted great Corliss engines that seemed to rise from the very bowels of the earth beneath us and détoured past thundering turbines. Before the largest turbine we paused for a moment.

"Here," said Dr. Tesla, pointing to the huge machine, "is a triumph of engineering skill. This turbo-dynamo, the largest ever made, developing 30,000 horsepower, was built from plans worked out on paper. It was never tested until it was erected here and it worked perfectly from the first turning on of steam. That is engineering. But that is not what we are here for."

We pressed on until we reached an open space where a mechanic in blue jeans was wiping the oil and grease from a machine so tiny in comparison with the gigantic turbine we had just inspected that it seemed like a toy.

"Here it is," said the tall, thin man — or rather he shouted, for the noise of a hundred thousand horsepower of moving machinery is not conducive to free vocal expression. "Better take off your coats," he continued, "for it is a cold night and it gets pretty hot in here."

We followed his advice and example and stripped down to shirt-sleeves.

"Turn on the steam," said the inventor to the mechanic. The workman gave a valve a short turn. From inside the little machine, which seemed to be composed of two identical parts connected by a spiral spring, came a humming sound; the connecting spring began to revolve so rapidly that it looked like a solid bar of steel and the floor under our feet shook with rapid vibrations which died down. I glanced at a speed-gauge attached to what seemed to be the main shaft of the device and saw that it was registering 7,000 revolutions a minute. I looked up at the main steam gauge overhead and saw a pressure of ninety pounds to the square inch indicated.

Dr. Nikola Tesla, inventor of the alternating-current motor, and pioneer in research into high-tension electric currents generally, was demonstrating his latest invention — a steam turbine, different in principle from any heretofore in use and one which will take less room and less coal per horsepower than the best engines now running. "It's up to its normal speed now — about nine thousand revolutions," said Dr. Tesla, and the tachometer bore out his statement. "You see, for testing purposes, I have these two turbines connected by this torsion spring. The steam is acting in opposite directions in the two machines. In one, the heat energy is converted into mechanical power. In the other, mechanical power is turned back into heat. One is working against the other, and by means of this beam of light we can tell how much the spring is twisted and consequently how much power we are developing. Every degree marked off on this scale indicates twenty-two horsepower." We looked at the scale. The beam of light stood at the division marked "10."

"Two hundred and twenty horsepower," said Dr. Tesla. "We can do better than that." He opened the steam valves a trifle more, giving more power to the motive end of the combination and more resistance to the "brake" end. The scale indicated 330 horsepower. "These casings are not constructed for much higher steam pressures, or I could show you something more wonderful than that. These engines could readily develop 1,000 horsepower," he said, as we watched the turbine running smoothly, steadily, almost noiselessly except for that single clear, musical note.

Standing nearby was another and smaller machine of the same type, connected through a gear-box with a dynamo. The engine itself would almost go into an ordinary hat-box. At a signal from Dr. Tesla the mechanic turned on the steam. Instantly, without the fraction of a second's apparent delay, the dynamo was under full speed, and from the end containing the motor rose the same clear note, indicating a well-balanced machine running freely at its normal speed.

"This little turbine has developed 110 horsepower under tests," said Dr. Tesla. It was about the size of a derby hat.

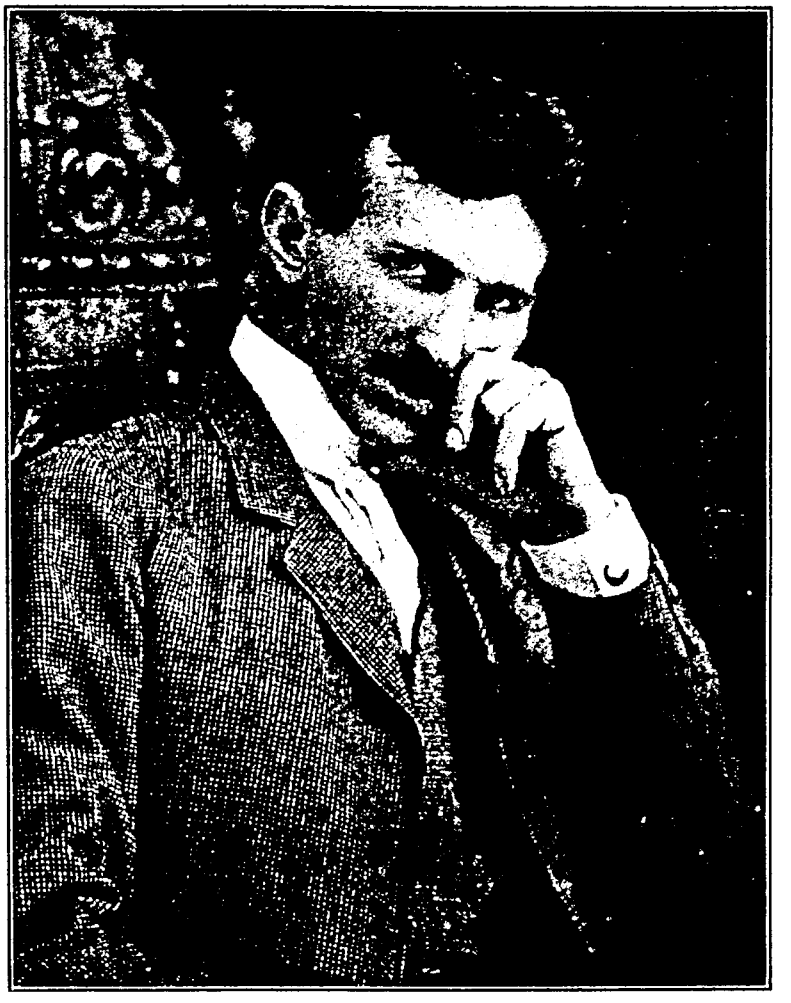
"Careful tests have shown that the single-stage turbine, running at 9,000 revolutions per minute, with a steam pressure of 125 pounds at the inlet, developing 200 brake horsepower, consumes

38 pounds of saturated steam per horse-power hour," said Dr. Tesla.

"But I can do better than that by compounding," he added. "The heat-drop under the conditions I named is only 130 British thermal units, and that is less than one third of the amount available under modern conditions of superheated steam and high vacuum. By compounding the turbines I shall get a steam consumption of not more than eight pounds per horse-power hour.

"The most efficient steam engines in America, big, slow-moving pumping engines working under ideal conditions and constant load, use about eleven pounds. I have undertaken a contract to produce one which will consume less than nine.

"The idea on which all steam engines — gas engines, too — have been built in the past was that there must be something solid and substantial for the steam to push against. The piston of a reciprocating engine and the blades and buckets of modern turbine engines are examples of what I mean. That idea has made them rather complicated devices, requiring careful fitting for efficient operation, great expense for repairs, and, especially in the case of turbines, great liability to



DR. NIKOLA TESLA

WHO, IN HIS SEARCH FOR AN ENGINE SUFFICIENTLY LIGHT AND POWERFUL TO OPERATE THE IDEAL FLYING MACHINE, HAS INVENTED A WONDERFUL LITTLE TURBINE MOTOR, FOR GENERAL USE, THAT IS AN ENTIRELY NEW APPLICATION OF MECHANICAL PRINCIPLES

damage. It has also made them bulky and heavy.

"What I have done is to discard entirely the idea that there must be a solid wall in front of the steam and to apply in a practical way, for the first time, two properties which every physicist knows to be common to all fluids (including steam and gas) but which have not been utilized. These are adhesion and viscosity.

"You know that water has a tendency to stick to a solid surface. That is the

THIS ABSURDLY SMALL ENGINE — TESLA'S SMALLEST MODEL — DEVELOPS 110 HORSE POWER

property of adhesion which every fluid — gas, steam, water, or whatever it be — possesses. You also know that a drop of water tends to retain its form, even against a considerable force, such as gravity. That is viscosity, the tendency to resist molecular separation, and all fluids have this property, too.

"It occurred to me that if I should take circular disks, mount them on a shaft through their centres, space them a little distance apart, and let some fluid under pressure, such as steam or gas, enter the interstices between the disks in a tangential direction, the fluid, as it moved, owing to these properties of adhesion and viscosity, would tend to drag the disks along and transmit its energy to them. It happened just as I had thought it would, and that is the principle of this turbine. It utilizes the very properties which cause all the loss of power in other turbines.

"Inside of the casings of those engines you saw — instead of buckets or blades or vanes on the edge of a wheel, there are simple disks of steel mounted on the shaft. In the two larger turbines these disks are eighteen inches in diameter and one thirty-second of an inch thick. There are twenty-three of them, spaced a little distance apart, the whole making up a total thickness of three and one half inches. The steam, entering at the periphery, follows a spiral path toward the centre, where openings are provided through which it exhausts. As the disks rotate and the speed increases, the path of the steam lengthens until it completes a number of turns before reaching the outlet — and it is working all the time. In the ordinary turbine the steam passes only around the periphery and the central portion of the wheel is useless. Moreover, every engineer knows that, when a fluid is used as a vehicle of energy, the highest possible economy can be obtained only when the changes in the direction and velocity of movement of the fluid are made as gradual and easy as possible. In previous forms of turbines more or less sudden changes of speed and direction are involved."

Later, in his laboratory in the Metropolitan Tower, discussing the commendations which eminent engineers, many of them with international reputations, have expressed concerning his turbine, Dr. Tesla summarized the points that make it a long step in advance in mechanical engineering.

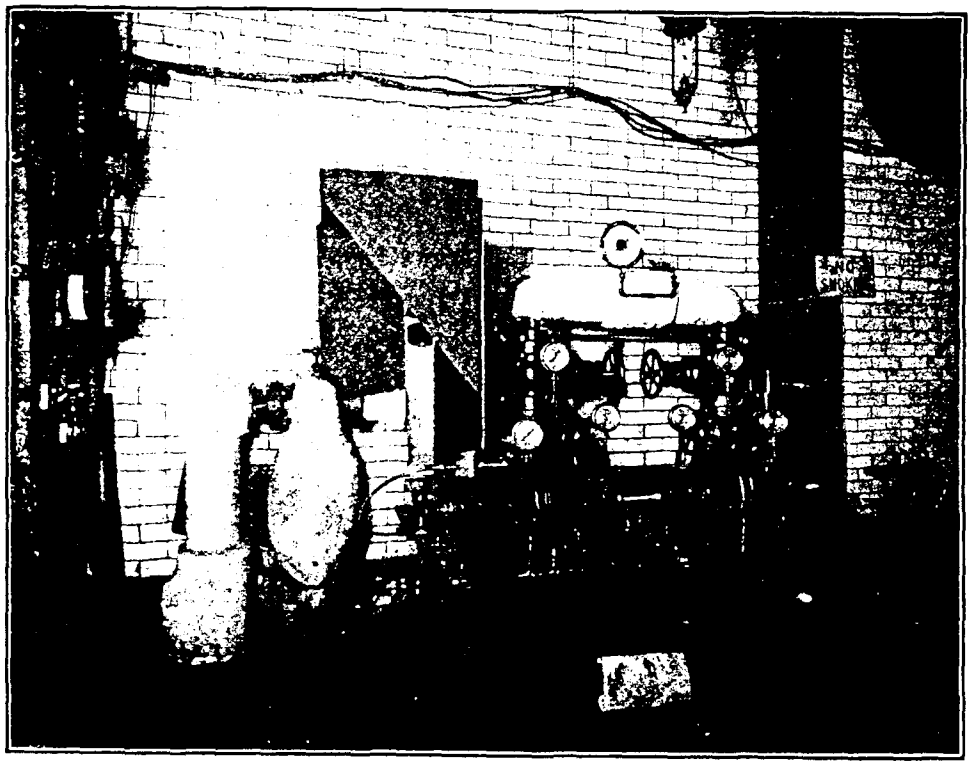
"To say nothing of it being a new application of mechanical principles," he said, "it has many decided advantages. First of these is its simplicity. It is comparatively inexpensive to construct, be-

cause nothing but the bearings need be accurately fitted, and exact clearances are not essential. Then there is nothing in it to get out of order and the disks can easily be replaced by any competent mechanic. It can be reversed without complex or cumbersome apparatus — all that is needed is a two-way valve to let the steam in at one side or the other, as desired. Reversing an ordinary turbine is next to impossible.

"My machine occupies, as you saw little space — the 110 horsepower turbine has disks only $9\frac{3}{4}$ inches in diameter — and in consequence it weighs very little. The lightest engines now in use weigh $2\frac{1}{2}$ pounds to the horsepower, while these, in their crudest forms, weigh less than that, and I expect to be able to produce 10 horsepower to the pound. Using gas instead of steam it gives most gratifying results, doing away with the complicated valves and springs of the prevailing types of gas engines. Another interesting application of it is as a pump, either for water or for air. The same disk arrangement is used, but the casing is so arranged, when built as a pump, that the fluid enters at the centre and is ejected at the periphery." He led the way into an adjoining room where a tiny turbine pump, with disks only three inches in diameter, operated by a one twelfth horsepower electric motor, was pumping 40 gallons of water a minute against a 9-foot head.

"How did you happen to turn your attention to mechanics instead of electricity?" I asked.

"I was a mechanical engineer before I ever took up electricity," replied Dr. Tesla. "I went into electric science years ago because I thought, in that direction, I was going to solve the problem I have been working on all my life — the production of an engine sufficiently light and powerful to operate the ideal flying machine. All my work in the wireless transmission of power, which has at-



TESTING THE SPEED, POWER, AND STEAM PRESSURE OF A TESLA TURBINE

THE PICTURE SHOWS TWO ENGINES, EACH CAPABLE OF PRODUCING 330 HORSE-POWER, WORKING AGAINST ONE ANOTHER FOR TEST PURPOSES, THE TORSION SPRING CONNECTING THE TWO BEING USED TO INDICATE THE AMOUNT OF POWER DEVELOPED

tracted more public attention than anything else I have ever done, was toward that end. I do not expect to build that ideal machine to-morrow, any more than I expect every steam engine in the world to be thrown into the scrap-heap because of this new application of mechanical principles, but such a flying machine will come some day, and meantime I have succeeded in developing something new in prime movers. I am young yet and have plenty of time ahead of me."

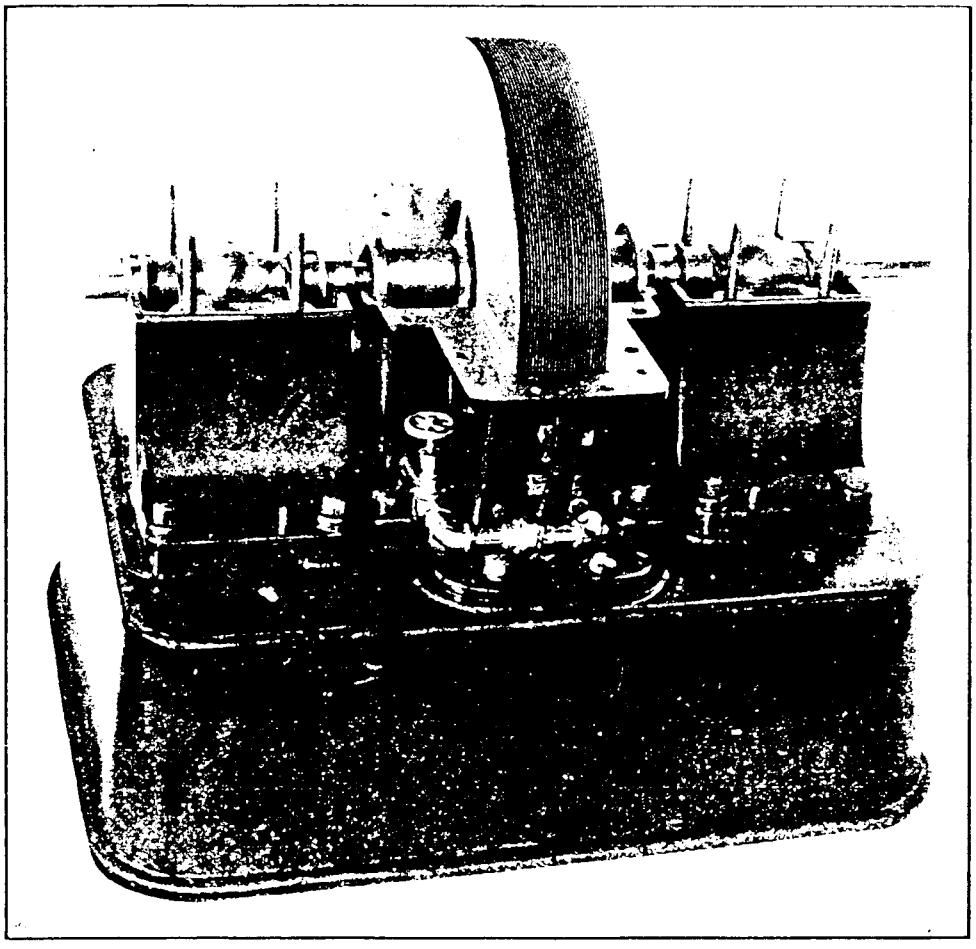
"My age? What do you think it is?" he asked.

"If I didn't know better, I'd say around forty," I ventured. "Fifty, for a guess."

"Fifty-four," was the answer.

"And you still expect to perfect your flying machine?"

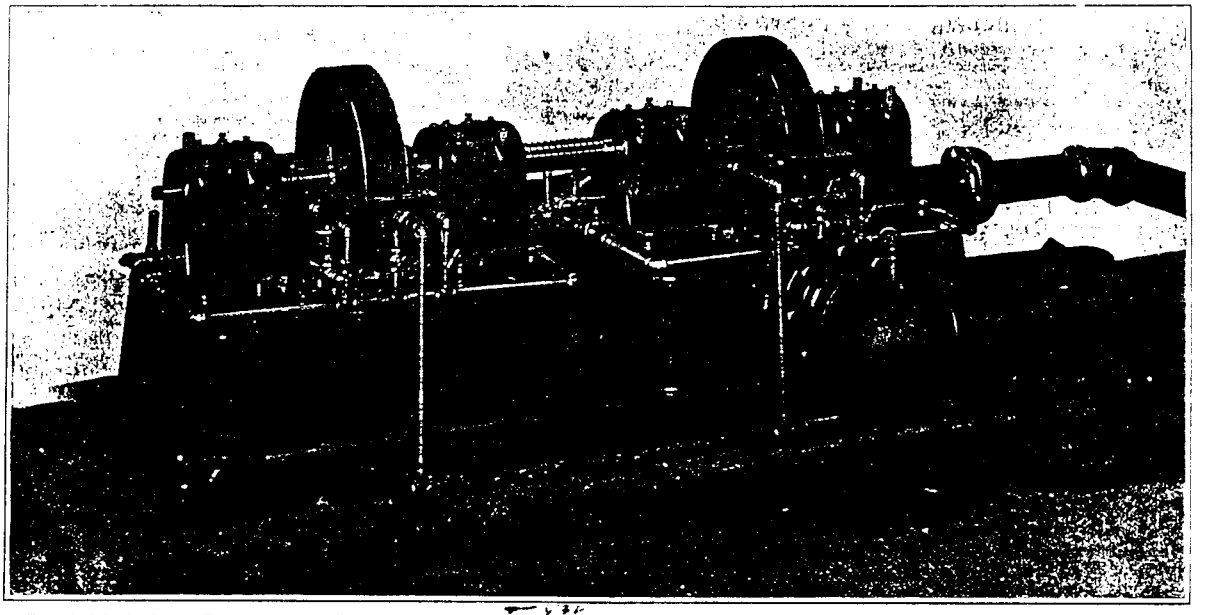
"Why not? I have half a century yet to live, if no accident happens. One of my grandfathers lived to be 118, the other past 100. One of my mother's grandfathers won a footrace at the age of 73. I hope it will not take me fifty years to perfect the flying machine, but if it does, I expect to be young enough at 104 to make a flight in it. The Tesla turbine will be on the market long before that, however."



A TESLA TURBINE WITH THE TOP OFF

SHOWING THE SERIES OF THIN DISKS BETWEEN WHICH THE STEAM PASSES AND WHICH, BY THE POWERS OF ADHESION AND VISCOSITY, THE STEAM DRAGS WITH IT IN ITS REVOLVING COURSE

I remembered that it was twenty-seven years ago that he had come over from Lika with the principle of the rotating field for alternating current motors already worked out, and began some mental calculations, which Dr. Tesla noticed.



The top half of casings is removed, showing two rotors. Each rotor consists of 25 disks ⅝-inch thick by 18 inches diameter. The steam enters at the periphery, and flows in spiral paths to exhaust at the center of the disks. The driving turbine is to the left, the brake turbine to the right. Between them is a torsion spring. The steam inlets are on opposite sides on the two rotors; the driving rotor moving clockwise. The torsion of the spring is automatically shown by beams of light and mirrors and the horse-power is read off a scale. At 9,000 revolutions per minute, with 125 pounds at the throttle and free exhaust, this turbine develops 200 horse-power. It weighs two pounds per horse-power.

The Tesla turbine testing plant at the Edison Waterside Station, New York.

The Tesla Steam Turbine

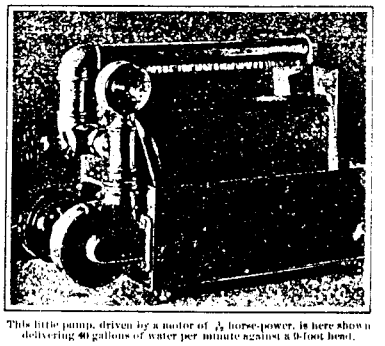
The Rotary Heat Motor Reduced to Its Simplest Terms

IT will interest the readers of the SCIENTIFIC AMERICAN to know that Nikola Tesla, whose reputation must, naturally, stand upon the contributions he made to electrical engineering when the art was yet in its comparative infancy, is by training and choice a mechanical engineer, with a strong leaning to that branch of it which is covered by the term "steam engineering." For several years past he has devoted much of his attention to improvements in thermo-dynamic conversion, and the result of his theories and practical experiments is to be found in an entirely new form of prime movers shown in operation at the Waterside station of the New York Edison Company, who kindly placed the facilities of their great plant at his disposal for carrying on experimental work.

By the courtesy of the inventor, we are enabled to publish the accompanying views, representing the testing plant at the Waterside station, which are the first photographs of this interesting motor that have yet been made public.

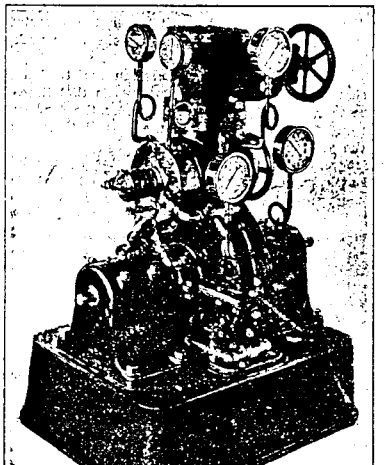
The basic principle which determined Tesla's investigations was the well-known fact that when a fluid (steam, gas or water) is used as a vehicle of energy, the highest possible economy can be obtained only when the changes in velocity and direction of the movement of the fluid are made as gradual and easy as possible. In the present forms of turbines in which the energy is transmitted by pressure, reaction or impact, as in the De Laval, Parsons, and Curtiss types, more or less sudden changes both of speed and direction are involved, with consequent shocks, vibration and destructive eddies. Furthermore, the introduction of pistons, blades, buckets, and intercepting devices of this general class, into the path of the fluid involves much delicate and difficult mechanical construction which adds greatly to the cost both of production and maintenance.

The desiderata in an ideal turbine group themselves under the heads of the theoretical and the mechanical. The theoretically perfect turbine would be one in which the fluid was so controlled from the inlet to the exhaust that its energy was delivered to the driving shaft with the least possible losses due to the mechanical means employed. The mechanically perfect turbine would be one which combined simplicity and cheapness of construction, durability, ease and rapidity of repairs, and a small ratio of weight and space occupied to the power delivered on the shaft. Mr. Tesla maintains that in the turbine which forms the subject of this article, he has carried the steam and gas motor a long step forward toward the maximum attainable efficiency, both theoretical and mechanical. That these claims are well founded is shown by the fact that in the plant at the Edison station, he is securing an output of 200 horse-power from a single-stage steam turbine with atmospheric exhaust, weighing less than 2 pounds per horse-power, which is contained within a space measuring 2 feet by 3 feet, by 2 feet in height, and which accomplishes these results with a thermal fall of only 130 B.T.U., that is, about one-third of the total drop available. Furthermore, considered from the mechanical standpoint, the turbine is astonishingly simple and economical in construction, and by the very nature of its construction, should prove to possess such a durability and freedom from wear and breakdown as to place it, in these respects, far in advance of any type of steam or gas motor of the present day.



This little pump, driven by a motor of ¼ horse-power, is here shown delivering 40 gallons of water per minute against a 6-foot head.

The turbine used as a pump.



This view shows one complete high pressure unit, with the steam throttle above, and below it the reversing valve and the compact turbine. Note the many gages used in the tests.

A 200-horse-power high-pressure turbine.

Briefly stated, Tesla's steam motor consists of a set of flat steel disks mounted on a shaft and rotating within a casing, the steam entering with high velocity at the periphery of the disks, flowing between them in free spiral paths, and finally escaping through exhaust ports at their center. Instead of developing the energy of the steam by pressure, reaction, or impact, on a series of blades or vanes, Tesla depends upon the fluid properties of adhesion and viscosity—the attraction of the steam to the faces of the disks and the resistance of its particles to molecular separation combining in transmitting the velocity energy of the motive fluid to the plates and the shaft.

By reference to the accompanying photographs and line drawings, it will be seen that the turbine has a rotor *A* which in the present case consists of 25 flat steel disks, one thirty-second of an inch in thickness, of hardened and carefully tempered steel. The rotor as assembled is 3½ inches wide on the face, by 18 inches in diameter, and when the turbine is running at its maximum working velocity, the material is never under a tensile stress exceeding 50,000 pounds per square inch. The rotor is mounted in a casing *D*, which is provided with two inlet nozzles, *B* for use in running direct and *B'* for reversing. Openings *C* are cut out at the central portion of the disks and these communicate directly with exhaust ports formed in the side of the casing.

In operation, the steam, or gas, as the case may be, is directed on the periphery of the disks through the nozzle *B* (which may be diverging, straight or converging), where more or less of its expansive energy is converted into velocity energy. When the machine is at rest, the radial and tangential forces due to the pressure and velocity of the steam cause it to travel in a rather short curved path toward the central exhaust opening, as indicated by the full black line in the accompanying diagram; but as the disks commence to rotate and their speed increases, the steam travels in spiral paths the length of which increases until, as

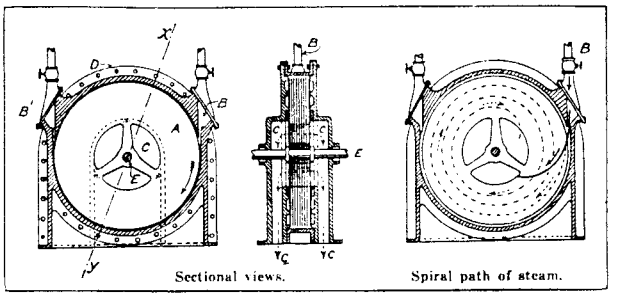
In the case of the present turbine, the particles of the fluid complete a number of turns around the shaft before reaching the exhaust, covering in the meantime a lineal path some 12 to 16 feet in length. During its progress from inlet to exhaust, the velocity and pressure of the steam are reduced until it leaves the exhaust at 1 or 2 pounds gage pressure.

The resistance to the passage of the steam or gas between adjoining plates is approximately proportionate to the square of the relative speed, which is at a maximum toward the center of the disks and is equal to the tangential velocity of the steam. Hence the resistance to radial escape is very great, being furthermore enhanced by the centrifugal force acting outwardly. One of the most desirable elements in a perfected turbine is that of reversibility, and we are all familiar with the many and frequently cumbersome means which have been employed to secure this end. It will be seen that this turbine is admirably adapted for reversing, since this effect can be secured by merely closing the right-hand valve and opening that on the left.

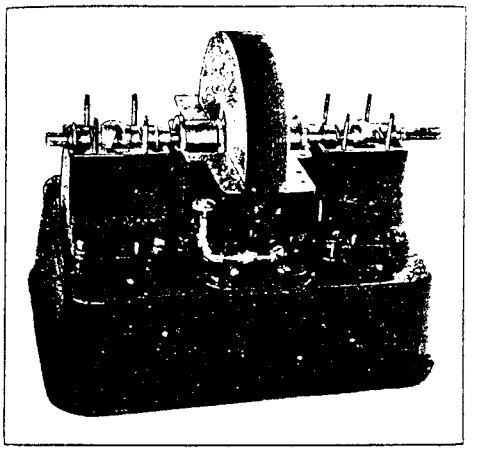
It is evident that the principles of this turbine are equally applicable, by slight modifications of design, for its use as a pump, and we present a photograph of a demonstration model which is in operation in Mr. Tesla's office. This little pump, driven by an electric motor of 1/12 horse-power, delivers 40 gallons per minute against a head of 9 feet. The discharge pipe leads up to a horizontal tube provided with a wire mesh for screening the water and checking the eddies. The water falls through a slot in the bottom of this tube and after passing below a baffle plate flows in a steady stream about ¾ inch thick by 18 inches in width, to a trough from which it returns to the pump. Pumps of this character show an efficiency favorably comparing with that of centrifugal pumps and they have the advantage that great heads are obtainable economically in a single stage. The runner is mounted in a two-part volute casing and except for the fact that the place of the buckets, vanes, etc., of the ordinary centrifugal pump is taken by a set of disks, the construction is generally similar to that of pumps of the standard kind.

In conclusion, it should be noted that although the experimental plant at the Waterside station develops 200 horse-power with 125 pounds at the supply pipe and free exhaust, it could show an output of 300 horse-power with the full pressure of the Edison supply circuit. Furthermore, Mr. Tesla states that if it were compounded and the exhaust were led to a low pressure unit, carrying about three times the number of disks contained in the high pressure element, with connection to a condenser affording 28½ to 29 inches of vacuum, the results obtained in the present high-pressure machine indicate that the compound unit would give an output of 600 horse-power, without great increase of dimensions. This estimate is conservative.

The testing plant consists of two identical turbines connected by a carefully calibrated torsion spring, the machine to the left being the driving element, the other the brake. In the brake element, the steam is delivered to the blades in a direction opposite to that of the rotation of the disks. Fastened to the shaft of the brake turbine is a hollow pulley provided with two diametrically opposite narrow slots, and an incandescent lamp placed inside close to the rim. As the pulley rotates, two flashes of light pass out of the same, and by means of reflecting mirrors and lenses, they are carried around the plant and fall upon two rotating glass mirrors placed back to back on the shaft of the driving turbine so that the center line of the silver coatings coincides with the axis of the shaft. The mirrors are so set that when there is no torsion on the spring, the light beams produce a luminous spot stationary at the zero of the scale. But as soon as load is put on, the beam is deflected through an angle which indicates directly the torsion. The scale and spring are so proportioned and adjusted that the horse-power can be read directly from the deflections noted. The indications of this device are very accurate and have shown that when the turbine is running at 9,000 revolutions under an inlet pressure of 125 pounds to the square inch, and with free exhaust, 200 brake horse-power are developed. The consumption under these conditions of maximum output is 38 pounds of saturated steam per horse-power per hour—a very high efficiency when we consider that the heat-drop, measured by thermometers, is only 130 B.T.U., and that the energy transformation is effected in one stage. Since about three times this number of heat units are available in a modern plant with superheat and high vacuum, the above means a consumption of less than 12 pounds per horse-power hour in such turbines adapted to take up the full drop. Under certain conditions, however, very high thermal efficiencies have been obtained which demonstrate that in large machines based on this principle, in which a very small slip can be secured, the steam consumption will be much lower and should, Mr. Tesla states, approximate the theoretical minimum, thus resulting in nearly frictionless turbine transmitting almost the entire expansive energy of the steam to the shaft.



Details of turbine.



This turbine, whose rotor consists simply of a set of flat disks 18 inches in diameter, develops 200 brake horse-power on test.

Turbine with upper half of casing removed.

Engineering News, October 12, 1911, 448-449

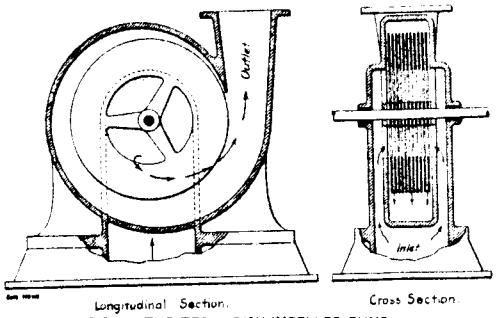


FIG. 1. THE TESLA DISK-IMPELLER PUMP.

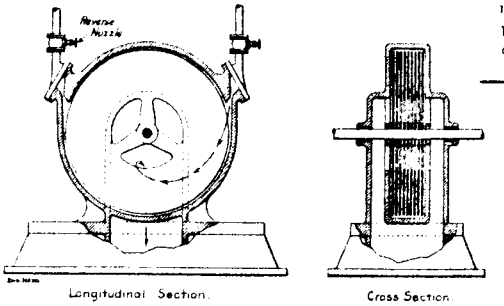


FIG. 2. THE TESLA DISK WHEEL IN A STEAM TURBINE.

The Tesla Steam Turbine.

A novel design of steam turbine was mentioned before a local meeting of the National Electric Light Association in New York City, on May 15, 1911, by Mr. Nikola Tesla, whose name has become famous in connection with the rotating magnetic field and other alternating-current developments. Several different types of power apparatus, both driving and driven, embodying the underlying principle of this turbine design have now been built and tested in the Tesla laboratories, including a turbine which has been running at the Waterside Station of the New York Edison Company for several months. The general results of this early work have recently been made public by Mr. Tesla.*

The following outline of the designs and the underlying principle running through them all has been made after a conversation with Mr. Tesla and a study of the apparatus. The fundamental principle underlying these several designs appears to be broadly applicable to apparatus wherein a fluid moves some device or the device causes the fluid to move. This includes hydraulic, steam and internal combustion motors, hydraulic and air pumps, etc.

The study of this most interesting line of designs may be approached through the case of either prime movers or driven devices. For the purposes of this article it is perhaps simpler to consider first a pump for a fluid like air, gas, vapor, or liquid. Fig. 1 shows, more or less diagrammatically, a rotary pump in which a series of smooth, flat disks revolves in a casing, with a volute delivery passage. The disks are fastened to a driving shaft but have central openings which serve as inlets for the fluid. On rotating the shaft and disks, the fluid film in contact with the disks is set in motion, even with perfectly smooth disks, on account of the molecular adhesion between disk and fluid. The fluid between disks is also dragged along by the molecular attraction between particles of the fluid (viscosity). The motion of each point of the disk being circular, it is evident that the particles of fluid receive impelling forces which are always tangential to the circular paths. The successive points on the disk that impel given particles of fluid are at increasing distances from the center. The fluid being mechanically unconstrained by walls or vanes except in an axial direction is free to travel in spiral paths from the axis to the periphery.

The fluid, in traversing the space from inlet to periphery, may follow a long spiral of several turns or a short one of part of a turn, depending on the quantity of fluid that is allowed to escape from the outlet. With unrestricted flow from the casing, there is little resistance to flow in a radial direction (that is from the backing up and development of pressure), and the tangential slip between disk and fluid is large. The pressure in the casing depends on the velocity with which the particles of fluid leave the periphery of the disk; the maximum pressure with the exit throttled is then proportional to the speed of the disks, the velocity head being converted to pressure head in the volute passage of the casing. The power absorbed is proportional to the square of the slip.

To one seeing such a piece of apparatus in action for the first time, the capacity of a small machine is surprising, especially in the absence of all vanes and projections of every sort which have heretofore been deemed necessary to force the fluid along. A moment's consideration, however, shows that the whole surface of each disk is effective in impelling the fluid along (solely by molecular drag), and the useful surfaces may be made very great in a machine of modest dimensions, in direct contrast with other analogous apparatus where only the relatively small area of buckets, blades, or other projections is effective.

Consideration also shows that any buckets or projecting parts that a disk might carry would constrain the fluid to travel in paths less natural or free, with the consequent development of impact and eddy friction which may be expected to decrease the efficiency over that of simple disks depending for their hold on the fluid only on natural molecular forces. In the case of the Tesla disks, the particles of fluid may be mentally pictured as rolling along on their spiral paths in orderly procession, held to the disks by some gravitation or force; whereas in the case of a disk with projections, the fluid would be pushed and crowded along by impact, with consequent disturbances.

It has been found, as theory would indicate, that the quantity of fluid discharged off the disks is proportional to the area of the disks; that is, the capacity of such machines increases nearly directly with their length along the shaft and about as the square of their diameter, the discrepancies arising naturally from the casing not having impelling surface but adding to the diameter and length of the machine.

The spacing of the disks in such a machine would depend on the conditions under which it had to operate—increasing with the viscosity and diameter and decreasing with the allowable slip. The aim of the designer would be to deliver the fluid from the disks at not much below peripheral speed under normal load conditions. When limitations of practical design would prevent the

*For early statements of these developments see "Electrical Review and Western Electrician," Sept. 9 and 30, 1911; "The Scientific American," Sept. 30, 1911.

securing of desired pressures with one simple set of disks the multiplication of stages is easily accomplished. The fluid then would pass from the exit passages of one stage to the inlet of a second, and so on.

The operations of a pump, as above described, are in general reversible for the production of power from a fluid moving with considerable velocity or under pressure. If a fluid under pressure, but of low velocity, enters the casing of the device shown in Fig 1 in order to flow along the volute of decreasing cross-section and through the inter-disk spaces to what is now the outlet, it must constantly accelerate, converting its pressure head to velocity head. This moving fluid will exert a pull on the disks on account of molecular adhesion and viscosity. If the shaft were blocked so that it could not rotate, the particles of fluid would take short spiral paths from the volute to the outlet.

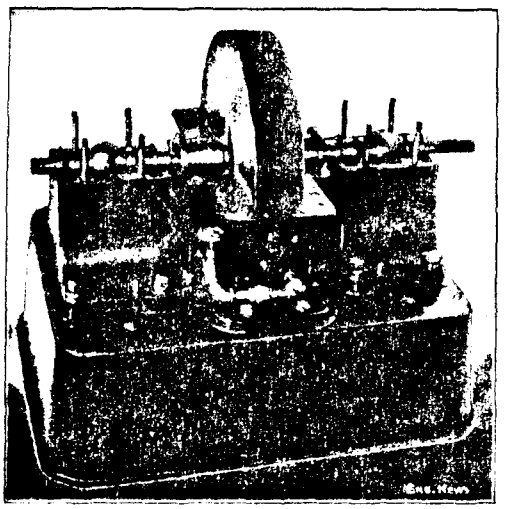


FIG. 3. A 200-HP TESLA TURBINE WITH THE UPPER CASING SECTION REMOVED.

If the disks are allowed to rotate, however, the particles of fluid in contact with the disks would be subjected to a force preventing travel along the shortest spirals to the outlet. The resultant of the centrifugal force exerted by the disk and that coming from the velocity of the steam constrains the fluid then to follow a longer spiral path. This reminds one of the counter E. M. F. developed in the armature windings of an electric motor opposing the impressed voltage.

It is evident that the torque developed by the disk increases with the difference in peripheral velocity of the disk and of the fluid in contact. As in the analogous case of the pump, the torque rises as the square of the slip. When the shaft runs free, without load, the speed rises and the centrifugal counter force on the fluid, traveling in very long spirals (almost concentric circles), would cause the casing pressure to rise nearly to the supply pressure, the difference being only that required to do the work of supplying energy losses.

Such a prime mover would give its maximum output at about 50% average slip, but the maximum efficiency would come with a comparatively small slip, the actual figure depending upon the fluid employed, on the working conditions, and on the mechanical limitations of design.

The greatest interest in the Tesla design, as outlined above, probably centers in its application to a heat motor, such as the Tesla steam turbine. While a device like Fig. 1 would operate with steam, it probably would be advisable for obvious reasons to modify the mechanical features—for instance, to suppress the volute passage in the casing, coming down to the simple construction indicated in Fig. 2.

If the steam is expanded in a diverging nozzle it may be expanded over any of the usual ranges of pressure drop, the heat energy then being converted into kinetic energy. The high velocity steam escaping from the nozzle is caused to impinge tangentially on the edges of the disks in order to escape, it has to take a spiral path from the periphery of the disks to the center openings. The velocity energy of the steam is utilized through the molecular drag on the surface of the disks. With such a nozzle it is seen that the machine is essentially of the so-called "impulse" type of turbine. The arrangement shown in Fig. 2 has a very simple and convenient means of reversing, it merely being necessary to provide a duplicate nozzle discharging against the opposite diameter of the disks and in the opposite direction. When the machine is at rest or running slowly, as in starting, the steam takes a short path from the nozzle to the exhaust and develops a comparatively large torque, since this would be proportional to the square of the difference in velocity of steam and disk. As the machine speeds up, the difference in velocity between steam and disks decreases and the centrifugal forces tend to lengthen the spiral path, so that a given quantity of steam may make several revolutions before finally passing out the exhaust.

It is not necessary, however, to expand the steam before it reaches the disks, and in place of the nozzles shown there may be simple ports. The machine then apparently will operate as a reaction type of turbine, the steam expanding as it flows in its path from port to exhaust. The expanding steam might develop a slight reactive thrust against the disk, but it would depend probably more for its influence on the peculiar action of increasing the velocity of the steam in small increments as it flows along, and absorbing the kinetic energy as fast as developed, in driving the disk.

In Figs. 3 and 4, a steam turbine tested at the Waterside Station of the New York Edison Company is shown. The rotor consists of 25 disks, 18 inches in diameter. The assembled unit, Fig. 3, occupies a floor space some 20 x 35 ins. and

Fig. 5. A 110-HP Tesla Steam Turbine.

it stands some 5 ft. high. The numerous gages seen in Fig. 3 were attached for test purposes. With steam at 125 lbs. gage and exhausting to atmosphere, 200 HP. was developed with a speed of 9,000 r. p. m. The steam consumption under these conditions was about 38 lbs. per HP.-hr.

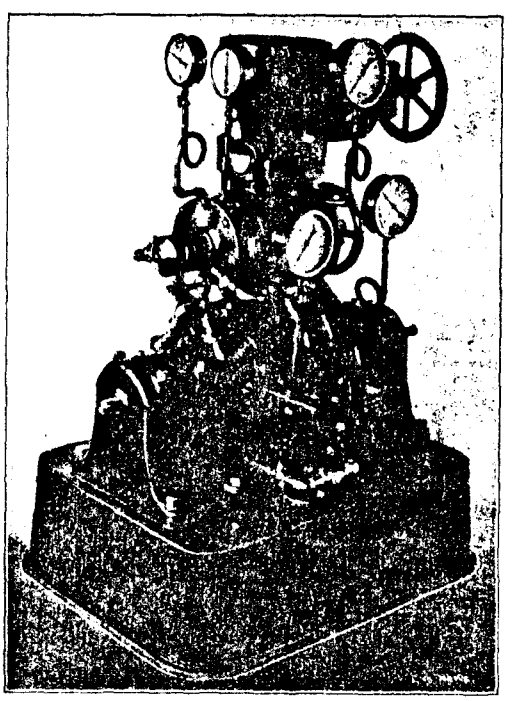


FIG. 4. A 200-HP TESLA STEAM TURBINE AS TESTED AT THE WATERSIDE STATION OF THE NEW YORK EDISON CO.

Mr. Tesla states that with moderate superheat and the degree of vacuum ordinarily obtainable in a turbine plant, the consumption can be reduced to 10 or 12 lbs. per HP.-hr. The weight of the unit as shown was about 400 lbs., giving a unit weight of 2 lbs. per HP.

Through refinements in design in addition to the increase of capacity secured with superheat and vacuum, Mr. Tesla expects that the weight may be reduced to as little as ¼-lb. per HP. capacity and still allow of designs which will have rotational speeds low enough for direct connecting to the majority of services.

One of the interesting possibilities of the design of turbine shown is that of self-regulation. It has already been shown how the counter pressure due to the rotation of the disks amounted nearly to that of the impressed fluid when running idle. Since the centrifugal head increases as the square of the number of revolutions, and as with available materials great peripheral velocities of the disks are possible, a turbine may be designed which will not run away, the peripheral speed being limited to that value which corresponds to the maximum velocity of the fluid which can be developed.

The main principle of design may be use for an internal-combustion motor. The small machine shown in Fig. 5 has been operated with gaseous fuel burned in an auxiliary chamber and the products of combustion cooled by injecting steam or water spray. This gives a mixture of superheated steam and gases which leave the combustion chamber under high pressure, but reduced in temperature so that they may be led directly to the disks, serving in the place of steam from a boiler. The machine shown developed 110 HP., and it is stated that only the small sized shaft prevented pushing the load higher. The products of combustion instead of being cooled by the formation of superheated steam may be expanded in an insulated nozzle, the temperature falling with the reduction in pressure and the increase in velocity. At the exit of the nozzle, the temperature could be sufficiently reduced so that the gases could be caused to impinge on the disks without injuring them.

Engineering, p.637
November 10, 1911

BLADELESS TURBINES.

FROM a recent issue of the *Scientific American* it appears that Mr. Nikola Tesla has revived a method of steam-turbine construction which recalls one tried here some nine or ten years ago. The idea was patented by Mr. E. C. Thrupp in 1901, but Mr. D. J. Smith, of 58, Compton-street, E.C., informs us that at a still earlier date he assisted an Australian inventor to experiment with a similar type. The Tesla turbine is represented diagrammatically by Figs. 1, 2, and 3, which we reproduce from the *Scientific American*. As will be seen, its mechanical simplicity is extreme. The whole machine consists of a series of 25 thin discs 18 in. in diameter and $\frac{1}{25}$ in. thick mounted side by side on a shaft, with a small clearance between them. Steam is admitted tangentially through a nozzle on the right, and after flowing through the interspaces between the discs, escapes at the centre, causing the rotor to rotate by its frictional drag on the discs. To reverse the direction of rotation, the nozzle on the right is closed, and that shown on the left opened.

Mechanically nothing could be simpler, but unfortunately this mechanical simplicity is purchased at the expense of hydraulic inefficiency, as any of the various inventors, who have experimented on the device, could have ascertained had they applied to someone with a competent knowledge of theoretical mechanics. That the system is likely to be defective in this respect is obvious from the consideration that in a perfect turbine the steam must expand adiabatically. Now, in turbines of the type under consideration, the whole impelling force is due to the fact that the fluid slides over the surfaces of the discs with friction, against which it does work, and this frictional work is returned to the steam as heat. The whole impulsion of the rotor accordingly is dependent on the wasting of work by friction. If there were no friction, the steam would expand adiabatically, but would do no work, whereas, in the case of an ordinary turbine, could adiabatic expansion be secured, the efficiency would be a maximum. It can be shown, in short, that the maximum possible hydraulic efficiency of such a turbine is 50 per cent., and any experiments showing a better result than this can safely be discarded, though some allowance may be necessary for the fact that an hydraulic efficiency of 50 per cent. may, in a compound turbine, owing to the reheat factor, correspond to a somewhat higher thermodynamic efficiency, since the work wasted in friction at the high-pressure end of such a turbine is partially recoverable in subsequent stages.

Obviously, the system of maintaining a motion and doing work by the frictional drag of a fluid is not confined to cases of rotation, and the mathematical aspect of the question is, perhaps, more simply presented if we consider a case of rectilinear motion. Thus, suppose a stream of fluid moving at v ft. a second to be discharged into a tank mounted on wheels. Let the latter at the outset be held fast, then, although the jet exercises a frictional drag on the tank, no work is done, since motion of the latter is prevented. The frictional drag on the tank is equal to the momentum destroyed per second; or if W denote the weight discharged per second, the

thrust on the tank is given by the relation $T = \frac{W}{g} r$.

At the same time the kinetic energy destroyed per second is $W \frac{r^2}{2g}$. Next assume the speed of the jet and the speed of the tank are both increased by the amount s. Then assuming, as before, that the fluid merely collects in the tank, the frictional drag is unaltered, since the fluid enters with an absolute velocity $r + s$, and remains there with an absolute velocity s. Hence the frictional drag impelling the tank is again $T = \frac{W}{g} r$, and the useful work done per second in propelling the tank is $T s = \frac{W}{g} r s$.

The kinetic energy supplied by the jet is now, however, $\frac{W}{g} \frac{(r + s)^2}{2}$.

Hence the efficiency of the device is

$$\frac{W}{g} r s \div \frac{W}{g} \frac{(r + s)^2}{2} = \frac{2 r s}{(r + s)^2}$$

By differentiation, it is easily seen that this is a maximum when $r = s$, in which case the efficiency is $\frac{1}{2}$. Finally, let the speed of the jet be still further increased by an amount y, whilst the speed of the tank remains constant at s. The fluid is now delivered from the tank with an absolute velocity $s + y$, and a relative velocity y. The case, therefore, is exactly analogous to the Tesla turbine. The frictional drag remains the same as before—viz., $T = \frac{W}{g} r$, and the work done is still $\frac{W}{g} r s$.

The kinetic energy supplied is now, however, $W \frac{(r + s + y)^2}{2g}$, so that the efficiency is $\frac{2 r s}{(r+s+y)^2}$, which is less than when the whole of the fluid remained, and moved with the tank. In that case the maximum possible hydraulic efficiency was 50 per cent., so that when the fluid is delivered as it is in the case of the Tesla turbine, the hydraulic efficiency must be less than $\frac{1}{2}$. Of course, the motion in the latter is not rectilinear, but the argument above set forth applies identically, if for linear momentum we substitute moment of momentum, for frictional drag frictional torque, and for linear velocity angular velocity.

To make an efficient turbine of the device it would be necessary to provide vanes. We should then get a construction analogous to the "Jumbo" turbine patented and experimented with by Sir Charles Parsons many years ago. This was abandoned because it was found that any solid particles which might accidentally get into the turbine were kept in a constant state of flux between the rotor and the standing portion of the machine. They were carried in with the steam and flung out again by the centrifugal force, and the wear was consequently excessive.

The Engineering Magazine, Dec. 1911, 414-16

THE TESLA STEAM TURBINE

A STEP TOWARD THE MAXIMUM ATTAINABLE EFFICIENCY OF A MOTOR OF THE STEAM TURBINE TYPE

Engineering News—Scientific American

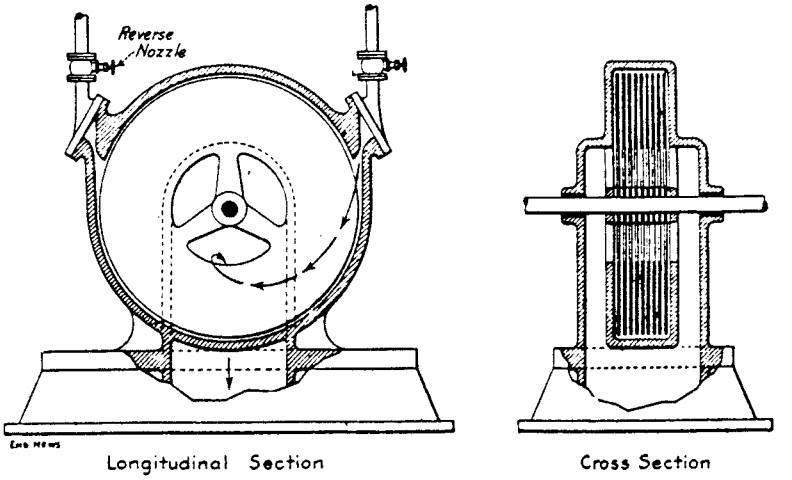


FIG. I. THE TESLA DISK WHEEL IN A STEAM TURBINE

CONSIDERABLE interest has been taken by the scientific press in the latest invention which Mr. Nikola Tesla has placed before the public. Mr. Tesla's name will always be connected with the contributions he has made toward the advancement of electrical science, but it must be remembered that his training and inclinations have always been toward the "steam engineering branch of mechanical engineering. The result of several years' practical experiments in improvements in thermo-dynamic conversion is found in an entirely new form of prime mover now in operation at the Waterside Station of the New York Edison Company.

"A brief description of Tesla's steam turbine," the *Scientific American* says, "consists of a set of flat steel disks mounted on a shaft and rotating within a casing, the steam entering with high velocity at the periphery of the disks, flowing between them in free spiral paths, and finally escaping through exhaust ports at their center. Instead of developing the energy of the steam by pressure, reaction, or impact, on a series of blades or vanes, Tesla depends upon the fluid properties of adhesion and viscosity—the attraction of the steam to the faces of the disks and the resistance of its particles to molecular separation combining in transmitting the velocity energy of the motive fluid, steam or water, to the plates and the shaft."

The actual operation of the turbine is described by the *Engineering News* as follows:—

"If the steam is expanded in a diverging nozzle, it may be expanded over any of the usual ranges of pressure drop, the heat energy then being converted into kinetic energy. The high velocity steam escaping from the nozzle is caused to impinge tangentially on the edges of the disks. In order to escape, it has to take a spiral path from the periphery of the disks to the center openings. The velocity energy of the steam is utilized through the molecular drag on the surface of the disks. With such a nozzle it is seen that the machine is

essentially of the so-called 'impulse' type of turbine. The arrangement shown in Fig. 1 has a very simple and convenient means of reversing, it merely being necessary to provide a duplicate nozzle discharging against the opposite diameter of the disks and in the opposite direction. When the machine is at rest or running slowly, as in starting, the steam takes a short path from the nozzle to the exhaust and develops a comparatively large torque, since this would be proportional to the square of the difference in velocity of steam and disk. As the machine speeds up, the difference in velocity between steam and disks decreases and the centrifugal forces tend to lengthen the spiral path, so that a given quantity of steam may make several revolutions before finally passing out the exhaust."

In Figure 2, a steam turbine tested at the Waterside Station of the New York Edison Company is shown. The rotor consists of 25 disks, 18 inches in diameter. The assembled unit occupies a floor space some 20 × 35 inches and it stands some 5 feet high. With steam at 125 lbs. gage and exhausting to atmosphere, 200 H.P. was developed with a speed of 9,000 r. p. m. The steam consumption under these conditions was about 38 lbs. per H.P.-hr. Mr. Tesla states that with moderate superheat and the degree of vacuum ordinarily obtainable in a turbine plant, the consumption can be reduced to 10 or 12 lbs. per H.P.-hr. The weight of the unit as shown was about 400 lbs., giving a unit weight of 2 lbs. per H.P.

Through refinements in design in addition to the increase of capacity secured with superheat and vacuum, Mr. Tesla expects that the weight may be reduced to as little as ¼-lb. per H.P. capacity and still allow of designs which will have rotational speeds low enough for direct connecting to the majority of services.

One of the interesting possibilities

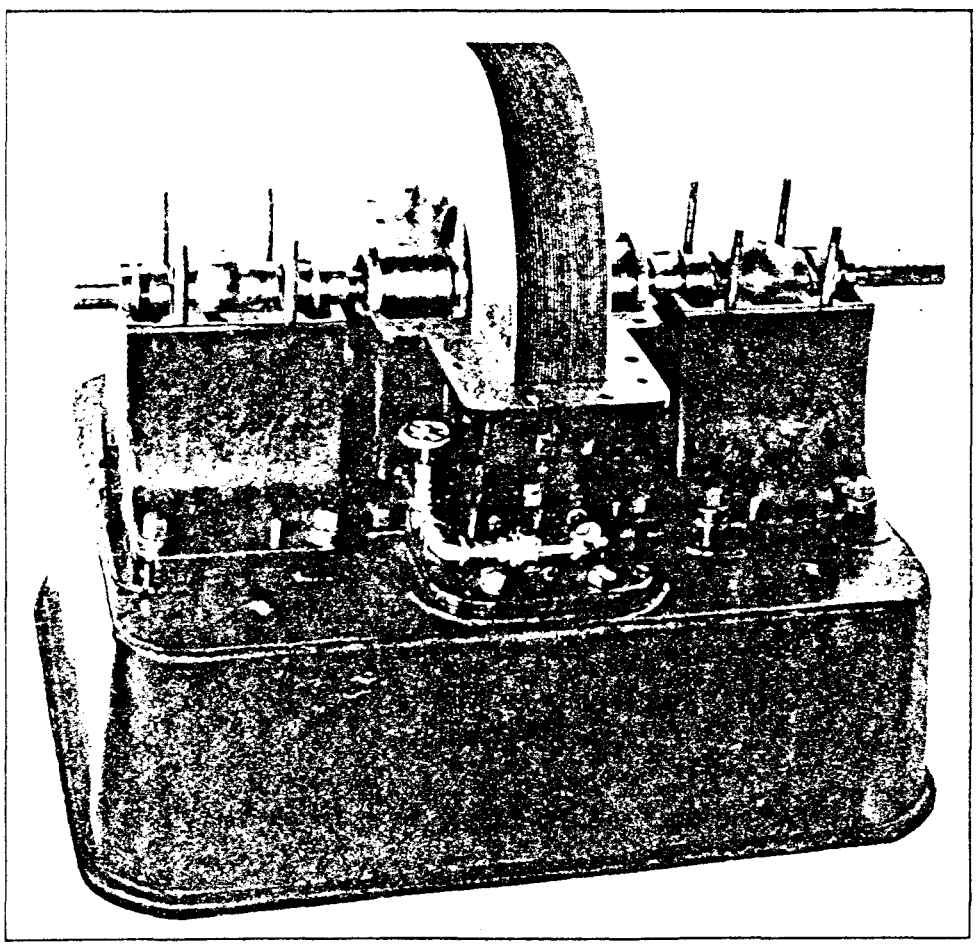


FIG. 2. A 200-H.P. TESLA TURBINE WITH THE UPPER CASING REMOVED

of the design of turbine shown is that of self-regulation. It has already been shown how the counter pressure due to the rotation of the disks amounted nearly to that of the impressed fluid when running idle. Since the centrifugal head increases as the square of the number of revolutions, and as with available materials great peripheral velocities of the disks are possible, a turbine may be designed which will not run away, the peripheral speed being limited to that value which corresponds to the maximum velocity of the fluid which can be developed.

THEORY OF THE TESLA TURBINE.

IN our issue of November 10, 1911, we gave a short account of the Tesla turbine, and showed that even under ideal conditions the hydraulic efficiency of a turbine of this kind cannot possibly exceed 50 per cent. In a paper published last June in the *Revue de Mécanique*, and since reproduced in a number of the French technical journals, Professor E. Mérigeault, of the École Nationale des Mines de St. Etienne, combats this conclusion, maintaining the contrary view, that the efficiency may be very high, and will be the higher the higher the friction. This extraordinary conclusion is arrived at in an article with numerous equations occupying in all over seven pages of the journal in which it appeared. Nevertheless it merely demonstrates once more that the value of the result finally delivered by the mathematical process depends entirely on the legitimacy of the original hypotheses to which it is applied. It is here that Professor Merigeault has gone wrong, and as his results are being quoted by those who are apparently too impressed with his analysis to venture to question his premises, it may be well if we point out exactly the nature of the fundamental error serving as the basis for his mathematical superstructure. To clarify ideas, we reproduce from ENGINEERING, *loc. cit. supra*, three views illustrating the construction of the Tesla turbine. It consists of a series of twenty-five discs, 18 in. in diameter, mounted side by side on a shaft and having a small clearance between them. Steam is admitted tangentially by the nozzle on the right, and flowing through the interspace between the discs, finally escapes at the centre, carrying the rotor round by its frictional drag on the surface of the discs. The appliance is, it will be seen, a true turbine. The potential energy of the steam is converted into kinetic energy, and this, again, into useful work. The steam enters the moving element at a high speed, and passing through it without change of pressure, is delivered at a reduced velocity at the inner periphery.

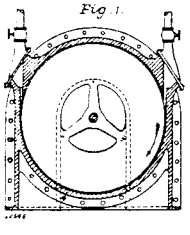


Fig. 1.

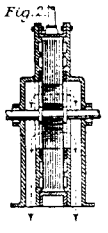


Fig. 2.

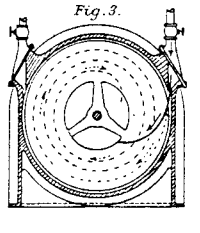


Fig. 3.

The mechanical analogue on which Professor Merigeault bases his reasoning satisfies none of these conditions. It is not merely not analogous to the Tesla turbine, but is not analogous to any turbine of any kind, but to a pressure engine. He assumes, in fact, that the Tesla turbine can, for the purpose of mathematical treatment, be replaced by the tank A, fixed pipe B, and moving pipe C. The latter is supposed to slide without friction and without leakage in the horizontal part of B, and to be connected in any suitable way to the mechanism to be driven. The pipe B may, for the purpose of argument, be considered frictionless, while C is a rough pipe. As the water flows through C, the frictional drag on its rough walls provides the force needed to drive the mechanism to which it is supposed to be connected. Then by a series of equations Professor Merigeault demonstrates that the higher the friction between the water and the wall of the moving element the greater is the efficiency of the appliance. This, however, is sufficiently obvious without any mathematics at all. Assume, for example, that the friction is so great that there is no slip between the moving water and the wall. The two will therefore move together exactly as if C were closed with a solid plug, and the efficiency then is unity. The appliance has, however, at the same time become a water-pressure engine of a very common type.

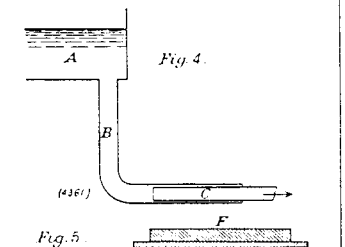


Fig. 4.

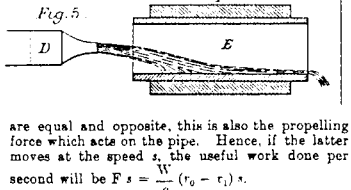


Fig. 5

If, without going to the extreme of assuming no slip between the fluid and the walls, we admit that slip exists, we are obviously working in the same condition as if the pipe C were smooth, but closed with a leaky plug, and the efficiency will obviously be smaller the greater the slip or the greater the leakage. In short, Professor Mérigeault's analogue is not a turbine at all, but a water-pressure engine with a leaky plunger, and obviously the efficiency will be higher the smaller the leakage—that is to say, the smaller the slip and the greater the friction. In fact, the fluid passes through his moving element without change of velocity, but merely with change of pressure. The true analogy to the Tesla turbine is represented in Fig 5, where a jet of water issuing from a nozzle D enters a pipe E sliding without friction in the sleeve F. As in Professor Mérigeault's analogue, the pipe E constitutes the moving element, and is to be coupled in any appropriate manner to the machine to be driven. Falling to the bottom of this pipe, the water travels on, its velocity being continually reduced by the friction until it finally escapes at the outer edge as indicated. The frictional drag between the water and the pipe provides the force necessary to overcome the resistance of the machine driven. In the conditions stated the pressure of the fluid is atmospheric at both ends of E, which is, in fact, traversed without change of pressure, the energy required to drive it being obtained at the expense of the kinetic energy of the jet.

The efficiency of this device is easily calculated.

Let v_0 be the absolute velocity of the fluid as it issues from D, and v_1 its final absolute velocity as delivered at the further end of the pipe E. Then if W be the weight of the water discharged per second, the change of momentum effected per second by the friction of the pipe is $\frac{W}{g}(v_0 - v_1)$.

Hence the force F, which has acted on the fluid so as to change its momentum by this amount, is $F = \frac{W}{g}(v_0 - v_1)$, and since action and reaction are equal and opposite, this is also the propelling force which acts on the pipe. Hence, if the latter moves at the speed s, the useful work done per second will be $F\,s = \frac{W}{g}(v_0 - v_1)\,s$.

Now v_1 must always be greater than s, or the water could not escape from the delivery end. Assume that in the limit it may be equal to s, a condition which can be approximated to by using a sufficiently large pipe. Then, in this limiting condition, the useful work done is $\frac{W}{g}(v_0 - s)\,s$, which is easily seen to be a maximum when $s = \frac{v_0}{2}$. Adopting this value, the useful work done becomes $\frac{W}{4}\frac{v_0^2}{g}$, whilst the kinetic energy expended is $\frac{W}{2}\frac{v_0^2}{g}$. Hence, in the stated limiting condition of $v_1 = s$, the maximum possible efficiency is 50 per cent. That the efficiency will be reduced if v_1 is equal to s is easily shown as follows:—Keeping s constant, assume v_0 to be increased to $v_0^1 = v_0 + y$. Then, by reducing the length of the pipe to compensate for the increase of friction with the increased velocity, we can make $v_1^1 = v_1 + y = s + y$, since v_1 was equal to s.

Hence the propelling force acting on E is now

$$F = \frac{W}{g}\left(v_0^1 - v_1^1\right) = \frac{W}{g}\left(v_0 + y - s - y\right) = \frac{W}{g}(v_0 - s),$$

or exactly the same as before.

Hence, s being constant, the useful work done is also exactly the same as before, but the energy expended has gone up from

$$\frac{W\,v_0^2}{2g} \text{ to } \frac{W(v_0)^2}{2g}$$

The efficiency is therefore reduced by making v_1 greater than s. It cannot be less than s, or the fluid could not escape, and since, when it has a limiting value equal to s the efficiency is only 50 per cent., the hydraulic efficiency of any turbine of the Tesla type must in practice be less than 50 per cent.

As stated, the Tesla turbine is of the impulse type. It hardly seems feasible to construct a reaction turbine operating by frictional drag, but were it possible to do this, the efficiency would be even lower than it is with the impulse form. The frictional drag and the reaction of the jet would, in a reaction machine, act in opposite directions and mutually oppose each other.

Engineering
4/30/15, 486-87

"THEORY OF THE TESLA TURBINE."

TO THE EDITOR OF ENGINEERING.

SIR,—I would like to point out certain flaws in the argument of last week on the Tesla turbine. Although the professor you mentioned used a very bad analogue to illustrate his work on the subject, and your analogue is slightly better than his, neither of you have got down to the bed rock of the whole matter.

You will see what I mean directly you consider the possibilities of allowing the exhaust water from your moving pipe to go into a second moving pipe, and so on. It then becomes obvious that by having the first pipe move nearly as fast as the water from the nozzle, and the second pipe move nearly as fast as the first pipe, and so on, that a much higher efficiency than 50 per cent. is possible with such a system. Now, this action is almost a true analogue of what happens in the Tesla turbine (I will show later the respect in which it differs from it), for in the Tesla turbine we have high velocity liquid firstly giving up a small part of its energy to the outer parts of the discs, and then gradually moving to the inner parts of the discs, which, of course, have a smaller circumferential velocity than the outer parts. So far as the motion of liquid between the discs is concerned, we "may" say that we have elemental rings of liquid which are gradually shrinking in radius, and which have each of them a "circulation" or a moment of momentum (angular momentum) which is being gradually given up to the discs.

Now we come to an important difference between the above analogue and the action of the Tesla turbine. Every engineer knows that if we have a cylinder of liquid rotating on an axis, that the pressure of the liquid is greater near the walls of the cylinder than at a point on the axis; he would explain it by saying that "the centrifugal force on the liquid causes it," and, of course, he is quite right. Now it cannot be difficult to see that if we have an elemental ring of liquid "circulating" (we will say, for argument's sake, circulating in a hollow annulus which envelope it), and if such ring be lessened in radius by some exterior means (say the hollow annulus is elastic, and that it contracts in radius), then work must have been done on the ring of liquid by the exterior means; in fact, the work may be said to be done "against the centrifugal force." The ring of moving liquid must, therefore, have gained in kinetic energy; as a matter of fact, the circumferential velocity of the liquid composing the ring is now greater than it was before, the law being that $v \propto \frac{1}{r}$, where v is the circumferential velocity of the liquid, and r is the radius of the ring of it. (Those who have studied hydrodynamics will, of course, say that they knew this before, when I tell them that it is merely a case of the "conservation of the velocity potential of the circuit.") This consideration makes it quite obvious that in the Tesla turbine there is a lot of energy transferred to the discs which was not initially kinetic energy at the outer periphery of the rotor, and that the pressure of the liquid entering the outer parts of the discs must be considerably greater than the exhaust pressure. The analogue which I mentioned is therefore incomplete, in that it has no action analogous to this last-mentioned action.

Yours faithfully,
WALTER C. M. PETTINGILL, B.Sc.

47, Francis road, Edgbaston, Birmingham,
April 19, 1915.

[We are not quite certain, from our correspondent's letter, whether he does or does not contend that the limiting efficiency of a radial-flow friction-driven turbine is greater than that of the axial flow analogue adopted in the article criticised. As a matter of fact, it is not difficult to show that the limit of efficiency is the same in both cases.

In what follows, we shall assume the fluid to be incompressible, and will neglect the head necessarily expended in accelerating the radial velocity of flow. Then in any vortex the energy lost by 1 lb. of fluid in crossing an annulus of breadth Δr and mean radius r is

$$\frac{r}{g}\cdot\frac{dv}{dr}\cdot\Delta r + \frac{v^2}{rg}\Delta r,$$

In the case of a free circular vortex the total energy of the fluid is the same at all radii, and the above expression is equal to zero.

In a Tesla turbine we must equate this expenditure of energy to the work done in the annulus, both useful and wasted. Hence we get

$$\frac{r}{g}\frac{dv}{dr}\Delta r + \frac{v^2}{rg}\Delta r = f(v-s)^n\,r\,\Delta t,$$

where $f(v-s)^n$ denotes the frictional resistance opposing motion, s is the velocity of the disc surface at radius r and $r\,\Delta t$ is the distance moved tangentially by the fluid whilst crossing the annulus. Noting that $\frac{dr}{dt}$ is the radial velocity, and $s = r\omega$, where ω is the angular velocity of the discs, the above equation can be integrated when $n = 1$. The result is, however, too complicated to be convenient, and is unnecessary for our present purpose.

The useful work done is obviously

$$f(v-s)^n s.$$

This will be larger the smaller n is. For fluid friction the least possible value of n is unity, in which case the flow is viscous. This fact shows the advantage of keeping the interspace between the discs very small, so as to damp out turbulence. With $n = 1$ the useful work will be a maximum when s is $\frac{v}{2}$, in which case the efficiency of the annulus is

$$\eta = \frac{\text{Useful work done in annulus}}{\text{Energy expended in annulus}} = \frac{f(v-s)\,s}{f(v-s)\,v} = \frac{\frac{v^2}{4}}{v^2} = \frac{1}{2} \text{ when } s = \frac{v}{2}.$$

Hence, assuming it to be possible that the surface speed of the disc at each radius is one-half the fluid velocity, the total efficiency will have the limiting value of 0.5, the useful work done in each successive annulus being then one-half the energy accounted for in that annulus.

Of course, with an elastic fluid the reheat factor must be borne in mind, an hydraulic efficiency of 0.5 being equivalent to an efficiency ratio of about 0.845.

It is of some interest to note that if the friction were "solid friction," and thus independent of the relative velocity of motion, the turbine might have nearly unit efficiency by making the discs so rough that there was no slip, and at the same time making the ratio of the outer to the inner periphery very large.—ED. E.]

Engineering
5/28/15, 609-10

"THEORY OF THE TESLA TURBINE."

TO THE EDITOR OF ENGINEERING.

SIR,—With reference to your editorial note on my letter in your issue of April 30, I am quite in agreement with your differential equations (in fact, your statement, $\frac{r}{g}\frac{dr}{dr}\Delta r + \frac{v^2}{rg}\Delta r = 0$ for a free circular vortex leads on integration to a previous statement of mine that $r \propto \frac{1}{v}$ for a free circular vortex); but I would point out, with all due respect, that although the 50 per cent. limit you mention is true for either your analogue or for ring discs in which the outer periphery is only slightly greater than the inner periphery, it is "not" true when the ratio between the peripheries is considerable (say 3 to 1). I did not give the full solution in my letter, because I think it is possible to form a good conception of the action without mathematics. Suppose it granted (as you suggest) that we have a turbine in which there is a 50 per cent. slip all through, my comment on such a case is that the efficiency would be improved by having more discs, and these closer together, and by allowing the rotor to run at a higher speed.

However, on integrating the equation you mention when $n = 1$, we have the circumferential velocity

$$v = \omega r + \frac{2k\omega}{gf}\cdot\frac{1}{r} + A\cdot\frac{1}{r}\cdot e^{+\left(\frac{gf}{2k}r^2\right)},$$

where A is an arbitrary constant, k is the radial velocity of the water towards the centre at a radius of 1 ft., f is the friction of 1 lb. of water when moving through the plates at 1 ft. per second, and ω is the angular velocity of the rotor. Due to incompressibility of the water, we can also write radial velocity towards centre u

$$= -\frac{dr}{dt} = \frac{k}{r},$$

and the flux of water per second $= 2\pi k\rho$ (where ρ is the density) for unit width of the flow.

This shows as a possible case (when A is zero)—

$$v - \omega r = \frac{2k\omega}{gf}\cdot\frac{1}{r}.$$

Of course $(v - \omega r)$ is slip at radius r, and since $u = \frac{k}{r}$, we see that in this case a fluid particle describes "on the plates" an equiangular spiral, which is certainly interesting.

This being a fairly simple case, let us see what is the corresponding efficiency when the outer periphery $= R$, and the inner periphery $= \frac{R}{3}$.

The torque transmitted to the rotor equals the angular momentum of liquid coming in per second minus the angular momentum of liquid going out at the exhaust per second:

i.e., torque $= \frac{2\pi k\rho}{g}\left\{R\left(\omega R + \frac{2k\omega}{gf}\cdot\frac{1}{R}\right) - \frac{R}{3}\left(\omega\frac{R}{3} + \frac{2k\omega}{gf}\cdot\frac{3}{R}\right)\right\}$

$$= \frac{16}{9}\frac{\pi k\rho\omega R^2}{g}$$

Hence the energy per second transmitted to the rotor

$$= \frac{16}{9}\frac{\pi k\rho\omega^2 R^2}{g}.$$

The kinetic energy of incoming water per second is:—

$$\frac{1}{2}\cdot\frac{2\pi k\rho}{g}\left(\omega R + \frac{2k\omega}{gf}\cdot\frac{1}{R}\right)^2,$$

which, if k is small in comparison with other quantities, practically equals

$$\frac{\pi k\rho\omega^2 R^2}{g}$$

We will now find the potential energy per second. Pressure at inlet (above atmospheric) is:—

$$P = \frac{1}{g}\int_{\frac{R}{3}}^{R} \frac{v^2 \rho}{r}\, d\tau + \ell\rho \int_{\frac{R}{3}}^{R} \frac{A}{r}\, d\tau + \frac{\rho}{g}\int_{\frac{R}{3}}^{R} \frac{du}{dt}\, d\tau.$$

but

$$v = \omega\left(r + \frac{2k}{g}\cdot\frac{1}{r}\right),$$

and

$$\frac{du}{dt} = \frac{du}{dr}\cdot\frac{dr}{dt} = -u\frac{du}{dr} = \left(-\frac{k}{r}\right)\left(-\frac{k}{r^2}\right)$$

Hence

$$P = \frac{\rho\omega^2}{g}\int_{\frac{R}{3}}^{R}\left(r + \frac{4k}{g}\cdot\frac{1}{r} + \frac{4k^2}{g^2}\cdot\frac{1}{r^3}\right) d\tau + \ell\rho k \log 3$$

$$+ \frac{\rho k^2}{g}\int_{\frac{R}{3}}^{R}\frac{1}{r^3}\, d\tau = \frac{\rho\omega^2}{g}\int_{\frac{R}{3}}^{R} r\, d\tau \text{ (practically, when } k$$

is small enough to make other terms negligible),

$$= \frac{4}{9}\cdot\frac{\rho\omega^2 R^2}{g} \text{ (practically, when } k \text{ is small enough}$$

to make other terms negligible),

or

$$P \times 2\pi k = \frac{8}{9}\cdot\frac{\pi k \rho \omega^2 R^2}{g},$$

or total incoming energy

$$= \frac{17}{9}\cdot\frac{\pi k \rho \omega^2 R^2}{g},$$

when k is small enough in comparison with certain other quantities—a condition which unfortunately means making a large turbine for the power developed.

Hence, unless we take into account the frictional losses at the casing of the turbine, which have not been previously mentioned, the efficiency may be nearly $\frac{16}{17}$ when the ratio between the peripheries is 3 to 1.

There is a way of getting rid of the friction at the outer casing, but, even so, I am not advocating the use of the turbine unless in a greatly modified form.

Yours faithfully,
WALTER C. M. PITTINGILL, B.Sc.
47, Francis-road, Edgbaston, Birmingham.
May 5, 1915.

[In his interesting letter our correspondent points out that if the constant of integration in the equation we gave is made zero, the Tesla turbine may have a very high efficiency. On the other hand, we gave reasons for concluding that the efficiency of any particular annulus could not exceed 50 per cent., and hence that the efficiency of the whole disc could not exceed this limit. There is thus a discrepancy which must be due to a false assumption somewhere.

In our argument we assumed that the expenditure of energy in an annulus was independent of the speed of the disc. As an *average*, this is certainly true, since the energy expended is fixed solely by the initial and final conditions.

Nevertheless, the distribution of the total energy over the different annuli will depend on the speed, and it is, perhaps, here that the origin of the discrepancy is to be found. On the other hand, to obtain our correspondent's result, the constant A has to be zero, and not merely very small. Possibly there is a critical condition involved. It will be seen that with A zero the actual motion of the fluid consists of Rankine's free vortex superimposed on his forced vortex, the motion of the water relatively to the plates being a free vortex.

If A has any sensible value, the conditions would appear to be entirely altered, and the distribution of energy will apparently then depend almost entirely on the term including this coefficient. This appears to be a conclusion fairly deducible from the numerical magnitude of the quantities involved.

Let us for convenience of calculation take all quantities in the C.G.S. system of units, and conceive an annulus of radius r, breadth Δr, and the thickness δ.

The total resistance suffered by the fluid in the annulus is

$$\frac{6\mu\pi}{\delta}(v - s)\, r\, \Delta r \text{ dynes per sq. cm.},$$

μ being the viscosity.

The total work done in the annulus in the time dt is therefore

$$\frac{6\mu\pi}{\delta}(v - s)\, r\, \Delta r\, dt, \text{ ergs.}$$

If W be the weight passing per second, we have then

$$W dt\left(\frac{v}{g}\frac{dv}{dr} + \frac{v^2}{rg}\right)\Delta r = \frac{6\mu\pi}{\delta}(v - s)\, r\, \Delta r\, dt.$$

Putting $s = \omega r$ this gives

$$v = \omega r + \frac{\omega W \delta}{3\pi\mu g}\cdot\frac{1}{r} + \frac{A}{r}\, e^{-\frac{3\pi\mu g}{W\delta}r^2}.$$

For water the numerical value of μg may be taken as 10.

Suppose the plates to be 1 mm. apart, so that $\delta = 1$ mm., and that the inner radius of the discs is 10 cm., and also that the radial velocity here is 7.5 cm. per second. Then

$$W = 7.5 \times 2\pi \times 10 \times \tfrac{1}{10} = 75 \times 2\pi.$$

Hence

$$\frac{3\pi\mu g}{W\delta} = 2.$$

Hence

$$v = \omega r + \frac{\omega}{2r} + \frac{A}{r}\, e^{-2r^2}$$

It will be obvious that if A have any sensible value whatever, the velocity in any annulus will be practically independent of ω, since e^{-2r^2} is an enormous quantity. Even at the inner radius it is e^{-20}, a number requiring nearly 9d digits to express. It was this fact that led us to observe in giving the equation originally that its integral was too complicated to be convenient.

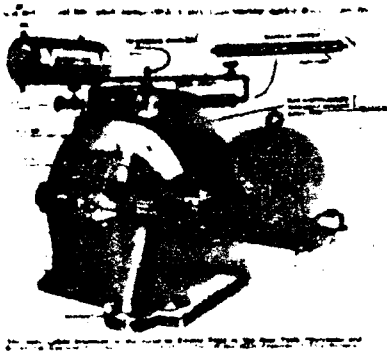
The head lost in passing from inlet edge to discharge edge is equal to

$$\int_{r_0}^{r_1} \frac{v^2}{rg}\, dr.$$

Hence when A is not zero it is impracticable to evaluate it even by approximate integration. The problem, it is clear, requires further elucidation, but there seems no obvious method of solution applicable to the case of A greater than zero. Another point which arises is that in the case of a steam-turbine it is mechanically impossible to run the discs at the speed required to reduce A to zero. The rim velocity required for a condensing turbine would be of the order of 2800 ft. per second.—ED. E.]

Popular Science, September 1955, 230-32

Cardboard Blower Works Like Tesla's Lost Turbine

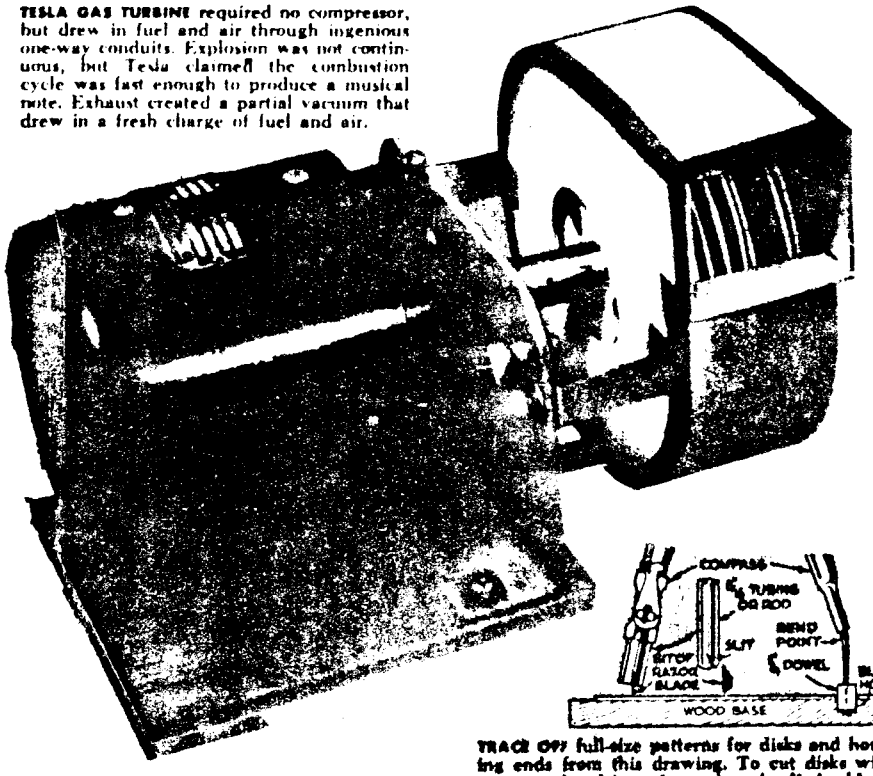


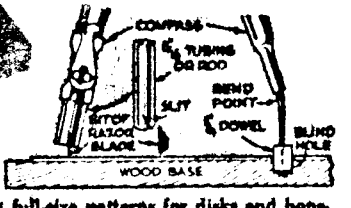
By Walter E. Burton

DECADES before skin friction began to pester airplane designers, Nikola Tesla harnessed it to produce power. Tesla, the Croatian-born engineer who gave us alternating current and the induction motor, was after a more efficient engine to spin his generators.

The Tesla turbine had no buckets or vanes on its rotor, but only a number of smooth disks closely spaced on a shaft.

TESLA GAS TURBINE required no compressor, but drew in fuel and air through ingenious one-way conduits. Explosion was not continuous, but Tesla claimed the combustion cycle was fast enough to produce a musical note. Exhaust created a partial vacuum that drew in a fresh charge of fuel and air.





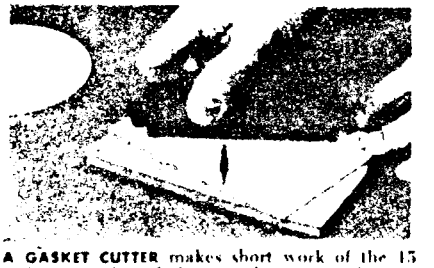
TRACE OFF full-size patterns for disks and housing ends from this drawing. To cut disks with compass, bend its point and angle slit for blade so that both are vertical when legs span 2½". Clamp if possible. Punch ¼" shaft hole first. Slip this on the dowel covering the compass point, and set into hole in base. Cut part way through from one side, then flip over.

THIS HOMEMADE BLOWER, which delivers a respectable air stream on modest power, is based on the principle of Tesla's vaneless turbine. You can build it of cardboard.

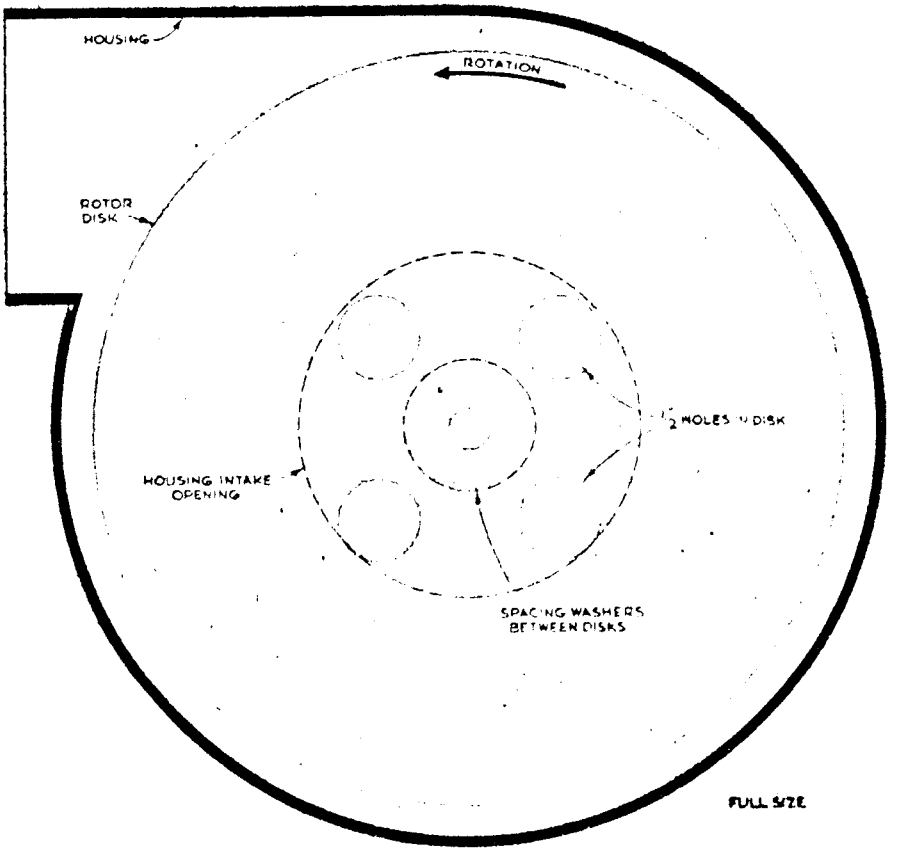
The power jet was directed against the edges of the disks, exhausting through holes in them near the shaft.

It works as a blower. Using the same disk rotor, you can build a quiet, effective little blower. It will ventilate a darkroom, cool electronic or projection equipment, or dry negatives, photo prints, or painted objects.

Made of cardboard, the blower shown produced a moderately strong breeze when mounted on a 1/30-hp. motor that runs at 1,525 r.p.m. It hurls air off the



A GASKET CUTTER makes short work of the 15 disks. The shaft hole must be accurately centered. If you use a compass, as in the drawing, punch the center and swing the cut from it.



rims of its blades, drawing it in through the holes that serve as exhaust ports.

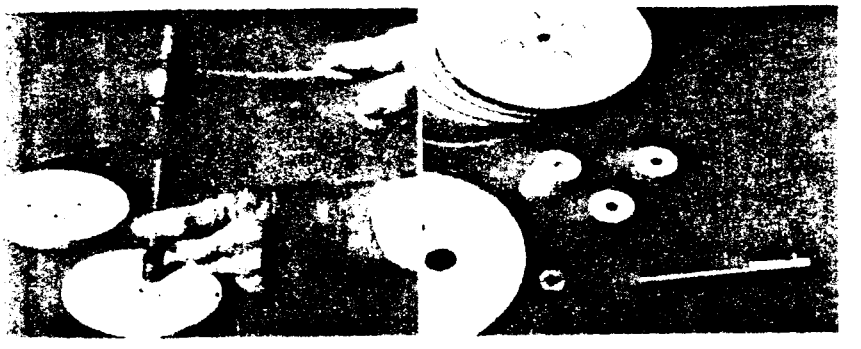
Powerhouse in a hat. Tesla's first model was so small and light for its output as to be a "powerhouse in a hat." Built in Long Island in 1906, it weighed less than 10 lbs., was about six inches in its greatest dimension, and developed 30 hp. when connected to a source of compressed air.

Its rotor consisted of eight flat metal disks 6" in diameter, spaced 1/32" apart on a 1" shaft. The housing had to be 1/64" bigger inside than the disks, for they expanded almost that much at their top speed of 35,000 r.p.m.

Blowtorch turbine. To dispense with the boiler, Tesla adapted his idea to an internal-combustion turbine, shown in the drawing on page 230 from a 35-year-old contemporary magazine. By the use of one-way conduits of his own design, Tesla achieved a pulse-jet effect fast enough to produce continuous rotation.

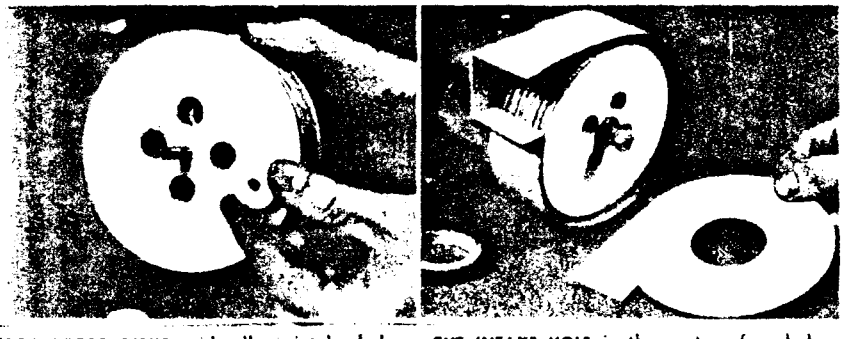
Bigger Tesla steam turbines were also built. One made in 1911 turned up 110 hp. with disks only 10" in diameter. But a 500-kw. generating unit with 60" disks suffered from metal fatigue and other ills that beset all turbines.

Because other turbines were already highly developed, it was bypassed. Except for use in an early auto speedometer, in which a driven disk partly rotated a spring-loaded one placed close to it, the principle has lain dormant. END



AIR INLET HOLES are best cut with a punch like this, but can be made with a razor blade. Lay them out on one disk, cut out, and use it as a guide for spacing holes in the others.

BLOWER SHAFT shown was turned to fit a small motor. For an experimental setup, clamp the disks on a ¼" bolt with the head cut off. Chuck in an electric drill or a drill press.



STACK ROTOR DISKS with all air-intake holes aligned. A cardboard washer, small enough not to block these, goes between each two disks. Sandpaper off any burrs or ragged edges.

CUT INTAKE HOLE in the center of each housing end. Glue and tape a 1¾"-wide strip between them. Tin-can brackets are riveted to inside housing end and bolted to the motor.

July 24, 1962 W. B. McLEAN 3,045,428

VORTEX GAS TURBINE

Filed July 6, 1960

FIG. 3.

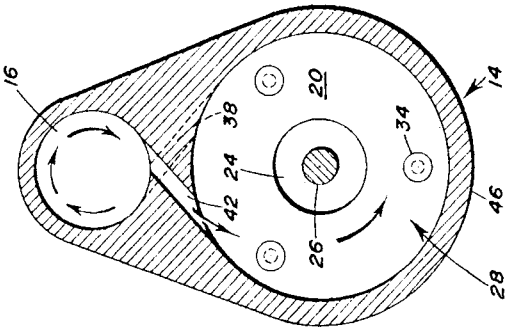


FIG. 1.

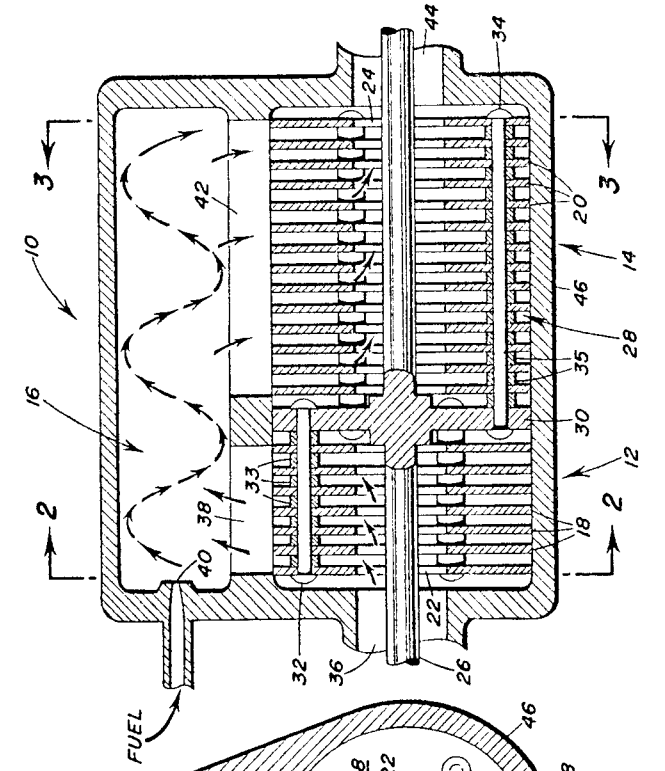
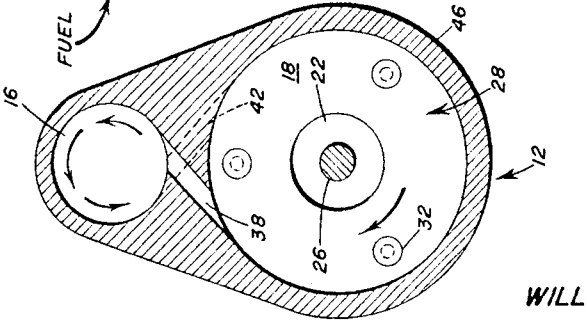


FIG. 2.



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INVENTOR.

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ATTORNEY

United States Patent Office

3,045,428

Patented July 24, 1962

3,045,428

VORTEX GAS TURBINE

William B. McLean, 510 Lexington St., China Lake, Calif., assignor of fifty percent to Walter G. Finch, Baltimore, Md.

Filed July 6, 1960, Ser. No. 41,147

6 Claims. (Cl. 60—39.75)

This invention relates generally to power plants, and more particularly it pertains to gas turbines of a continuous combustion type.

In 1903, Nikola Tesla developed a novel type of steam turbine, for which he was granted U.S. Patent No. 1,061,206, on May 6, 1913. The Tesla type turbine as a gas pump and as a gas motor has the advantage of simplicity of construction and the possibility of using materials which could not be incorporated into the blade structure of a normal gas turbine.

The principles for operation of a continuous combustion turbine are well known. Such a turbine essentially consists of a serial connection of a compressor element, and a motor element between which combustion gases are formed which drive the elements and provide useful output power.

An object of this invention, therefore, is to provide a simple axial inlet and discharge gas turbine in which the combustion of the fuel mixture takes place remotely of the blades for easy starting and stability of burning over a wide variation of flow rates.

Another object of the present invention is to provide a gas turbine power plant of the Tesla disk type having separate circular disk chambers, and a circular combustion chamber therebetween intercoupled by tangential passages.

Another object of this invention is to provide a vortex combustion chamber for a gas turbine which requires no additional moving parts.

Still another object of this invention is to provide a gas turbine having separate compressor and motor elements, and a separate combustion chamber, whereby the back pressure thereof can be arranged for the best efficiency.

And another object of the invention is to provide a Tesla turbine with refractory blades and a remote combustion chamber, premitting high combustion temperatures to be used with consequent high operating efficiency.

And yet another object of the invention is to provide a unitary rotor gas turbine utilizing centrifugal feed, centripetal discharge of a medium for a vortex type combustion chamber.

These and other objects and attendant advantages of this invention will become more readily apparent and understood from the accompanying specification and single sheet of drawings in which:

FIG. 1 is a longitudinal section drawing of a vortex gas turbine incorporating features of this invention;

FIG. 2 is a cross section taken along line 2—2 of FIG. 1; and

FIG. 3 is a cross section taken along line 3—3 of FIG. 1.

Referring now to the details of the drawings in FIG. 1, there is shown generally a vortex type gas turbine arrangement 10. This vortex type gas turbine arrangement 10 consists of a novel combination of a turbine 14, a pump 12, and a vortex combustion chamber 16.

The pump 12 and turbine 14 each consist of a series of equally spaced flat disks 18 and 20 with central openings 22 and 24, respectively, carried on a common shaft 26.

These disks 18 and 20 are arranged to rotate freely in a closely encompassing cylindrical chamber 28, with the shaft 26 being supported on suitable bearings (not shown). The disks 18 and 20 are attached by means of bolts 32 and 34, respectively, to a solid barrier plate 30 secured to or integrally formed with the shaft 26. The disks 18 and 20 are maintained in spaced relationship by collars 33 and 35, respectively, located between each pair of adjacent disks.

As the shaft 26 rotates, an oxidizer gas is drawn into the pump 12 along the shaft 26 through an intake port 36 by viscous friction and centrifugal force emparted by the disks 18. The oxidizer gas, in this case, air, is forced through an opening 38 which communicates tangentially from the periphery of the disks 18 into the combustion chamber 16, as best illustrated in FIG. 2.

The combustion chamber 16 is of cylindrical shape. The opening 38 enters the combustion chamber 36 in tangential relationship, preferably reversed or crossed over as shown so that the arrows depicting air flow in the chambers 28 and 16 rotate in opposite directions.

As shown best in FIG. 1, the air in the chamber 16 will have a spiral motion. Combustible fuel is injected in the air stream, preferably axially and near the upstream end of the chamber 16, through a jet 40. The air fuel mixture is ignited by suitable means, such as a spark plug (not shown) or the fuel and oxidizer gas may be of the hypergollic type requiring no ignition means.

In any case, the combustion of the air fuel mixture is in the form of a vortex sheet as the air or oxidizer gas enters through the opening 38 and meets and burns the fuel injected by the jet 40. The expanded gas or products of combustion leaves the chamber 16 by a second tangential opening 42 as shown by the arrows in FIG. 3 and impinge in centripetal fashion against the disks 20 of turbine 14 in their direction of rotation.

By reason of its expansion, the gas or products of combustion flowing into turbine 14, will have greater energy than the air entering through the pump 12. This expanded gas causes the disks 20 to continue to rotate and the gas, with its energy expended, leaves the turbine through an exhaust port 44 located concentrically with the shaft 26. The Tesla type turbine 14 will reach its maximum operating efficiency when the rotating speed is sufficiently high to produce a back pressure equal to about half of the inlet pressure. A measure of control is here provided by varying the number of disks 18 and 20, and, therefore, the relative sizes of the pump 12 and turbine 14. It will be noted this in no way changes the size of the combustion chamber 16.

A greater number of disks 20 are used in the turbine 14 than disks 18 in the pump 12, and, therefore, the torque developed by the out-flowing gas will be greater than the torque required to rotate the disks 18 which are pumping the incoming air. Also, because the peripheral speeds of the pump 12 and turbine 14 are the same, the pressure drop across the small gap between the barrier plate 30 and the wall 46 of the chamber 28 will be small and leakage by this path will be insignificant. Useful power is taken off by suitable connection to the shaft 26.

It will occur to those skilled in the art that by choosing refractory metals or ceramics for the simple disks 18 and 20, passages or openings 36, 38, 42, and 44 and chambers 16 and 28, higher temperature operation is permissible without the expense and difficulty of elaborate fabrication of buckets and fins. Further, the simple rotary structure of the turbine arrangement permits higher speed operation of the turbine 12 and pump 14. Either technique increases the horsepower output per pound of weight of the operating of the turbine arrangement, a very important factor in present flying vehicles.

It is to be noted that the disks 20 of the turbine 14 can be flat plates formed of flat, punched sheets of tantalum or tungsten. Tantalum disks 20 will allow the rotor to run at high speeds, whereas tungsten or ceramic disks will permit high temperature operation. The turbine 14 will reach its maximum efficiency when the

rotating speed of the rotor is sufficiently high to produce a back pressure equal to about half of the inlet pressure. Also, the vortex combination chamber **16**, as designed, has great stability of burning of the air-fuel mixture over a wide variation in flow rates and pressures, which facilitates the easy starting of the turbine arrangement **10**.

Obviously many modifications and variations of the present invention are possible in light of the above teachings. It is, therefore, to be understood that within the scope of the appended claims the invention may be practiced otherwise than as specifically described.

What is claimed is:

1. A gas turbine, comprising, structure having a pair of laterally parallel spaced cylindrical shaped co-extensive chambers therein, one of said chambers having an inlet port for an oxidizer medium and an outlet port for exhaust gases arranged at opposite ends thereof, integral rotary means including a shaft extending through said inlet and outlet ports and having equally spaced equally sized disks positioned transversely along the longitudinal axis thereof and having central openings therein, together with a barrier element for dividing said one chamber into a pump section and a turbine section, with the number of said disks in said turbine section being greater than the number of said disks in said pump section, means for maintaining said disks in equally spaced relationship in said pump and turbine sections, said disks and barrier element being of circular shape and of slightly smaller diameter than said one chamber, said structure having a first passageway tangentially connecting said pump section to said second chamber and an other passageway tangentially connecting said turbine section to said other chamber and being arranged in a reversed direction from said first passageway and spaced along the longitudinal axis of said structure from said first mentioned passageway, and means for injecting fuel into said other chamber at the end thereof corresponding to said inlet port to said one chamber for mixing with said oxidizer medium to form a fuel mixture for burning thereof in said other chamber.

2. A turbine as recited in claim 2, wherein said disks in said pump section and turbine section are formed of refractory metals as well as ceramics.

3. A turbine as recited in claim 1, wherein said disks in said pump section are formed of refractory metals, while said disks in said turbine section are formed of tantalum.

4. A turbine as recited in claim 1, and means for igniting the oxidizer-fuel mixture in said other chamber.

5. A unitary rotor type gas turbine, comprising, structure having a pair of laterally parallel spaced and co-extensive cylindrical chambers therein, one of said chambers having an inlet port for an oxidizer medium and an outlet port for exhaust gases arranged axially at opposite ends thereof, rotary means including a shaft extending through said inlet and outlet ports and having equally spaced equally sized disks with central openings about said shaft positioned transversely along the longitudinal axis thereof, together with a barrier element for dividing said one chamber into a pump section and a turbine section, with the number of said disks in said turbine section being greater than the number of said disks in said pump section, means for maintaining said disks in equally spaced relationship in said pump and turbine sections, said disks and barrier being of slightly smaller diameter than said one chamber, said structure having at least one tangentially arranged passageway connecting said pump section to said other chamber and at least one other tangentially arranged passageway connecting said turbine section to said other chamber and being arranged in a reversed direction from said first mentioned tangentially arranged passageway between said pump section and said other chamber, and means for injecting fuel into said other chamber at the end thereof corresponding to said inlet port to said second chamber, whereby upon rotational movement of said rotary means, an oxidizer medium is drawn into said pump section through said inlet port and central openings of said disks and is forced by viscous friction and centrifugally feed outwardly through said tangentially arranged passageway between said pump section and other chamber where by vortex action, said oxidizer medium is mixed with the injected fuel to form fuel mixture, which upon ignition and burning thereof, the liberated expanding gases are drawn through said tangentially arranged passageway between said other chamber and turbine section to drive said spaced disks therein by viscous friction, with said liberated exhaust gases being centripetically discharged between said disks in said turbine section to said outlet port thereof.

6. A gas turbine, comprising, structure having a pair of laterally parallel spaced cylindrical shaped co-extensive chambers therein, one of said chambers having an inlet port for an oxidizer medium and an outlet port for exhaust gases arranged at opposite ends thereof, integral rotary means including a shaft extending through said inlet and outlet ports and having spaced disks positioned transversely along the longitudinal axis thereof and having central openings therein, together with a barrier element for dividing said one chamber into a pump section and a turbine section, means for maintaining said disks in spaced relationship in said pump and turbine sections, said structure having a first passageway tangentially connecting said pump section to said other chamber and a second passageway tangentially connecting said turbine section to said other chamber and being arranged in a reversed direction from said first passageway and spaced along the longitudinal axis of said structure from said first mentioned passageway, and means for injecting fuel into said other chamber at the end thereof corresponding to said inlet port to said one chamber for mixing with said oxidizer to form a fuel mixture for burning thereof in said other chamber.

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Laminar Flow Over an Enclosed Rotating Disk

By S. L. SOO,[1] PRINCETON, N. J.

Laminar flow over an enclosed rotating disk was studied to reduce the inconsistency between previous theoretical and experimental results. Unlike the case of a disk in an infinite fluid medium, the friction-moment coefficient of the enclosed disk is proportional to Re^{-1} in the laminar range and $Re^{-1/4}$ in the turbulent range. The latter is accurate for the range of disk diameter to gap ratios between 200 and 50. The former correlation, instead of being an approximation according to simple shear as has been suggested, is shown to be true even when recirculation exists. The deviation from Re^{-1} relation in the laminar range is shown to be due to the inertia effect of recirculation. The significance of inertia effects in addition to that due to centrifugal force has been pointed out and corrections have been presented. Radial outflow has been shown to be more effective than radial inflow for turbine-disk cooling. It has been shown that rotation of the shroud provides an added sealing effect for a centrifugal machine.

Nomenclature

The following nomenclature is used in the paper:

a_n, a_n' = coefficients in series expansion of W
A = Re_z^2
b_n, b_n' = coefficients in series expansion of V
c_n, c_n' = coefficients in series expansion of U
C = constant of integration
D = diameter of disk, ft
k_s = friction-moment coefficient
m = net flow rate, slug/sec
M = friction moment, ft-lb (one side)
n = rpm
p = pressure, psf
r = radial co-ordinate or radius as defined, ft
r_s = disk radius, ft
Re_z = Reynolds number based on $\omega z_0^2/\nu$
Re_r = Reynolds number based on $\omega r_s^2/\nu$ or nr_s^2/ν as stated
u = radial component of velocity, fps
U = dimensionless velocity as defined
v = peripheral component of velocity, fps
V = dimensionless velocity as defined
w = axial component of velocity, fps
W = dimensionless velocity as defined
z = axial co-ordinate, ft
z_0 = gap between disk and housing, ft
ζ = dimensionless axial co-ordinate
ζ_2 = $1 - \zeta$
θ = angle, rad
μ = viscosity, lb/ft sec
ν = kinematic viscosity, ft²/sec
ρ = density, slug/cu ft
τ = shear stress, lb/ft²
ψ = stream function as defined, ft/sec
ω = angular velocity, rad/sec
', ", etc. = first, second, etc., derivatives of the variable with respect to ζ

Introduction

Studies of the problem of friction and heat transfer for a disk rotating in an infinite fluid medium already have been made thoroughly (1–6).[2] Experimental measurements of disk friction in a finite housing have shown that the case of an enclosed disk is quite different from the case of a disk in an infinite medium, although many of the theoretical correlations follow the trend of that for an infinite system (7–9). The study reported here was made to obtain a better understanding of the phenomena associated with the enclosed rotating disk.

Problems of importance in engineering which can be generalized in the case of flow over an enclosed rotating disk include windage losses, air cooling of turbine disks (10), pedestal bearings with center feed of lubricant (11), and leakage flow over the shroud of a centrifugal pump or compressor. The analysis presented in this paper is restricted to the case of incompressible laminar flow; an approximate solution for turbulent flow has been given in the discussion.

Only a limited amount of numerical computation was possible in connection with the present study. However, the method is suitable for solutions by automatic computers in cases where the applications justify their use.

Formulation of the Problem

The system shown in Fig. 1 consists of a disk or plate rotating at constant angular velocity ω in an incompressible fluid and situated at a distance z_0 from a stationary plate or boundary. Symmetrical with the axis of rotation the fluid flows radially inward or outward at a mass rate of flow m.

The equation of continuity and the momentum equations for steady flow may be expressed in cylindrical co-ordinates as

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad \ldots\ldots [1]$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] \quad \ldots\ldots [2]$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right] \quad \ldots\ldots [3]$$

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Manuscript received at ASME Headquarters, March 4, 1957. Paper No. 57—SA-28.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

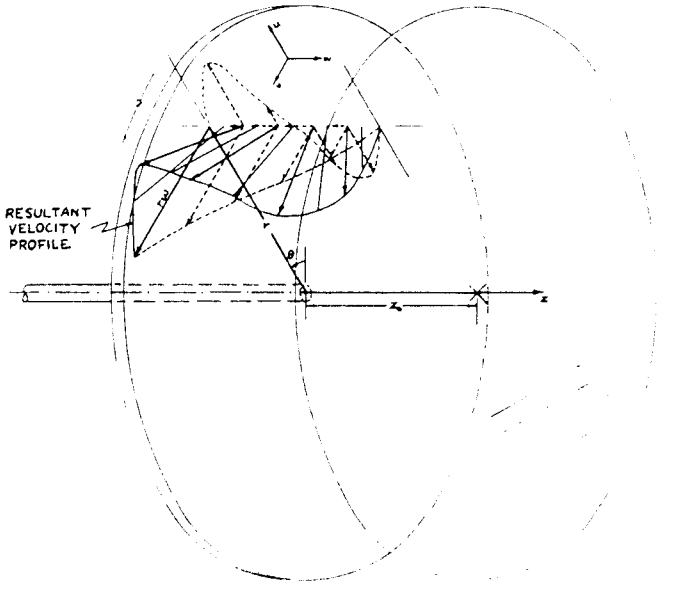


FIG. 1 CO-ORDINATE SYSTEM OF AN ENCLOSED ROTATING DISK, SHOWING VELOCITY PROFILE AT RADIUS r AT THE CONDITION OF NO NET RADIAL FLOW

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right] \quad [4]$$

with the following boundary conditions

$$\left.\begin{aligned} &u(r, 0) = 0; \quad u(r, z_0) = 0; \quad 2\pi\rho\int_0^{z_0} ru\,dz = m; \\ &v(r, 0) = r\omega; \quad v(r, z_0) = 0; \\ &w(r, 0) = 0; \quad w(r, z_0) = 0; \end{aligned}\right\} \quad [5]$$

$m > 0$ for radial outward flow;
$m < 0$ for radial inward flow

The flow rate m may be assumed to be small in all the practical cases mentioned previously.

APPROXIMATE SOLUTION FOR SMALL m

Introducing the usual boundary-layer approximations, Equations [2] to [4] can be simplified to

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\frac{\partial^2 u}{\partial z^2} \quad [6]$$

$$\frac{u}{r}\frac{\partial rv}{\partial r} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} \quad [7]$$

$$\frac{\partial p}{\partial z} = 0 \quad [8]$$

Equation [1] can be satisfied by the stream function ψ defined by

$$w = \frac{1}{r}\frac{\partial r\psi}{\partial r}, \quad u = -\frac{\partial \psi}{\partial z} \quad [9]$$

Further, in cases where m may be considered small, the substitutions

$$\psi = \frac{\omega z_0^2}{\nu} r\omega z_0 W(\zeta) + \frac{m}{2\pi\rho}\frac{U(\zeta)}{r} \quad [10]$$

and

$$v = r\omega V(\zeta) \quad [11]$$

where

$$\zeta = z/z_0 \quad [12]$$

into Equations [6] and [7] enables separation of the variables. This leads to

$$W^{iv} + 2AWW''' - 2VV' = 0 \quad [13]$$

$$V'' + 2AW'V - 2AWV' = 0 \quad [14]$$

$$U^{iv} - 2AWU''' - 2AWU'' = 0 \quad [15]$$

where

$$A = \left(\frac{\omega z_0^2}{\nu}\right)^2 = \mathrm{Re}_z^2 \quad [16]$$

and Re_z is the Reynolds number of the system based on z_0. The velocity components are given by

$$u = -\mathrm{Re}_z r\omega W' - \frac{mU'}{2\pi\rho z_0 r} \quad [17]$$

$$w = 2\mathrm{Re}_z \omega z_0 W \quad [18]$$

$$v = r\omega V \quad [19]$$

together with the boundary conditions

$$\left.\begin{aligned} &W(0) = W(1) = 0 \\ &V(0) = 1, \; V(1) = 0 \\ &U'(0) = U'(1) = 0 \\ &U(1) - U(0) = 1 \end{aligned}\right\} \quad [20]$$

FIRST-ORDER APPROXIMATION

A set of simplified solutions can be obtained when one takes Re_z as arbitrarily small. This is equivalent to neglecting all inertia effects except that due to centrifugal force as in reference (7). In other words, if one takes A equal to zero in Equations [13] to [15], the solution reduces to

$$U = 3\zeta^2 - 2\zeta^3 \quad [21]$$

$$U' = 6\zeta - 6\zeta^2 \quad [22]$$

$$V = 1 - \zeta \quad [23]$$

$$W = -\frac{\zeta^2}{20} + \frac{7}{60}\zeta^3 - \frac{\zeta^4}{12} + \frac{\zeta^5}{60} \quad [24]$$

$$W' = -\frac{\zeta}{10} + \frac{7}{20}\zeta^2 - \frac{\zeta^3}{3} + \frac{\zeta^4}{12} \quad [25]$$

The results are shown by solid lines in Figs. 2, 3, 4, and 5 for comparison with the solution given later for finite values of A.

Equation [23] shows that simple shear exists in the peripheral direction across the gap z_0. Essentially, this simplified solution constitutes a first-order approximation. The results, however, are good enough for most engineering requirements.

The resisting moment M due to friction can be obtained by integrating the shearing unit stress over the area of the disk. The shearing stress is

$$\tau = \mu\frac{\partial v}{\partial z} = -\mu\frac{r\omega}{z_0} \quad [26]$$

and the moment of one side is given by

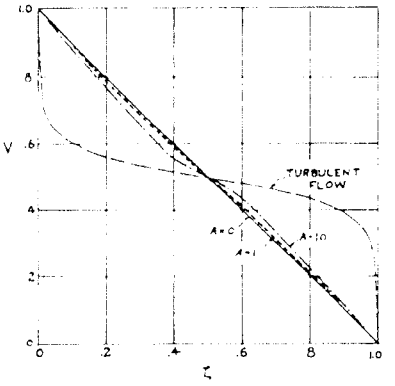


FIG. 2 DIMENSIONLESS PERIPHERAL VELOCITY V ($v = r\omega V$, $A = \mathrm{Re}_z^2$)

$$M = 2\pi \int_0^{r_s} \tau r^2 dr = \frac{\pi}{2} \frac{\mu\omega}{z_0} r_s^4 \quad \ldots\ldots\ldots [27]$$

The moment coefficient of friction (8) is

$$M \Big/ \left[\int_0^{r_s} \int_0^{2\pi} \frac{1}{2} \rho (r\omega)^2 r \cdot r \, d\theta \, dr \right]$$

$$k_s = \frac{M}{\pi\omega^2\rho r_s^5/5} = \frac{5}{2\mathrm{Re}_z} \left(\frac{z_0}{r_s} \right) = \frac{5}{4\mathrm{Re}_r} \left(\frac{D}{z_0} \right) \quad \ldots\ldots [28]$$

where

$$\mathrm{Re}_r = \omega r_s^2/\nu$$

and $\quad \ldots\ldots\ldots\ldots\ldots\ldots [29]$

$$D = 2r_s$$

Equation [28] has been plotted against $\mathrm{Re}_r = nr_s^2/\nu$ in Fig. 6 for comparison with the experimentally determined curve (7, 8). Other theoretical curves (2, 7) are included for comparison. Reference (7) neglected momentum terms other than that due to centrifugal force but considered the axial component to be of similar order as the peripheral component.

SECOND-ORDER APPROXIMATION

Including the effect of finite values of A in Equations [13], [14], and [15], while still considering only small values of m, may be considered as a second-order approximation. The solution of this case requires series expansions from $\zeta = 1$ to $\zeta = 1/2$. Equilibrium requires that

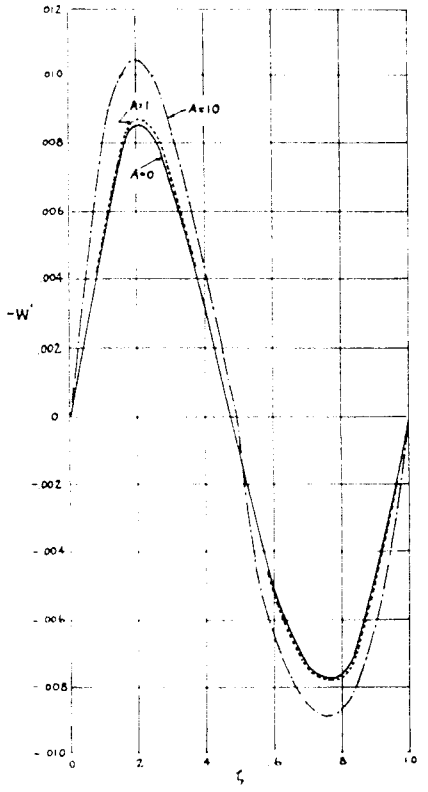


FIG. 4 DIMENSIONLESS RADIAL VELOCITY W'. RADIAL VELOCITY DUE TO ROTATION $= -\mathrm{Re}_z \, r\omega W'$, $A = \mathrm{Re}_z^2$

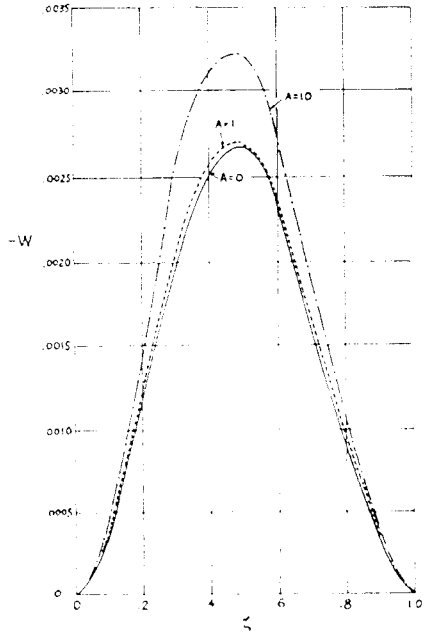
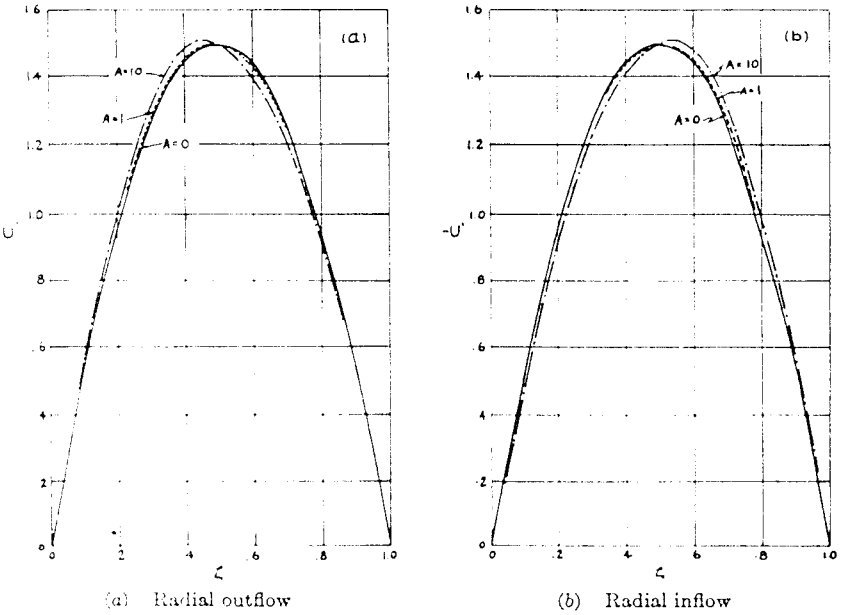


FIG. 3 DIMENSIONLESS AXIAL VELOCITY W ($w = 2\mathrm{Re}_z \, \omega \, z_0 \, W$, $A = \mathrm{Re}_z^2$.)



(a) Radial outflow (b) Radial inflow

FIG. 5 DIMENSIONLESS RADIAL VELOCITY U'. RADIAL VELOCITY DUE TO NET FLOW $= -\dfrac{mU'}{2\pi\rho z_0 r}$, $A = \mathrm{Re}_z^2$

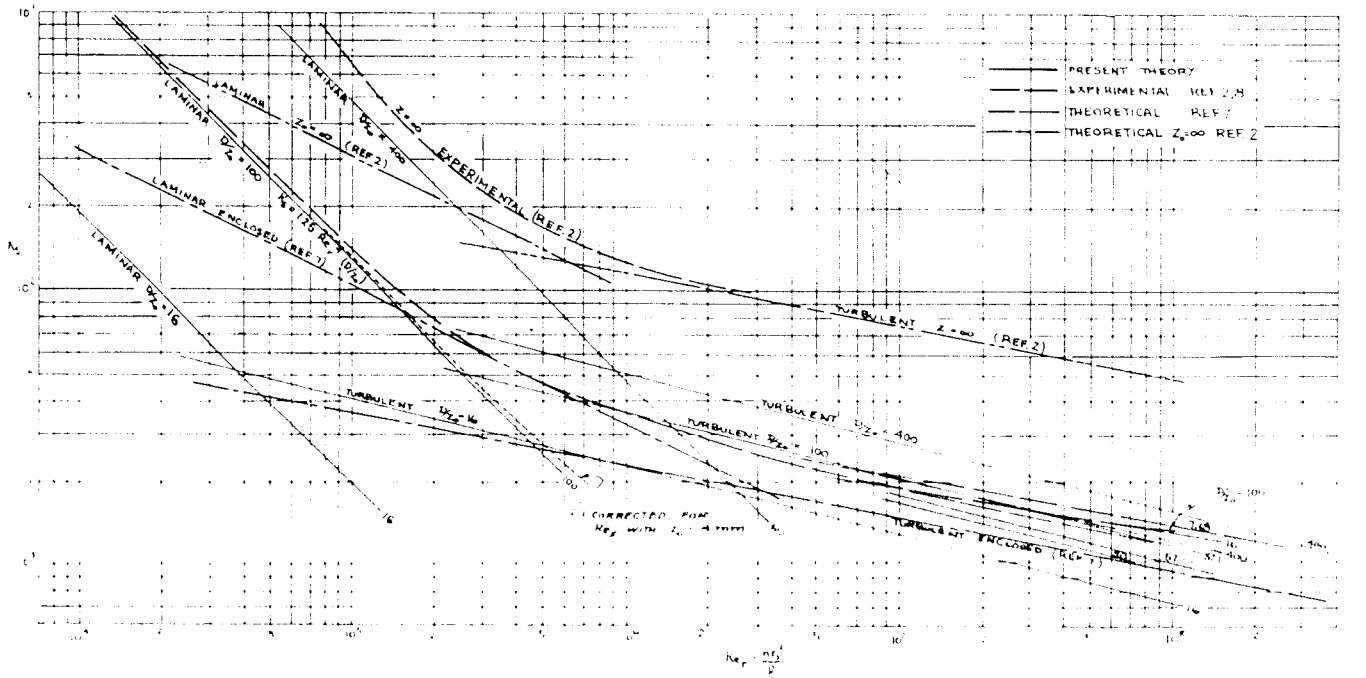


FIG. 6 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS
[n = rpm, k_m is defined as the ratio of friction moment on one side and $\rho\omega^2 b r^5/5$.]

$$V'(0) = V'(1) \quad [30]$$

since the same torque must be transmitted even if a disk and its housing are reduced to narrow rings of control volume of radius dr. In other words, with small mass flow, the angular momentum flux from the space between the disks must be small and therefore the friction torques on the two disks must be equal. This suggests the introduction of

$$\zeta_2 = 1 - \zeta \quad [31]$$

for $1/2 < \zeta < 1$, into Equations [13], [14], and [15] which become

$$W_2^{iv} + 2AW_2W_2''' + 2V_2V_2' = 0 \quad [32]$$

$$V_2'' - 2AW_2'V_2 + 2W_2V_2' = 0 \quad [33]$$

and

$$U_2^{iv} + 2AW_2U_2''' + 2AW_2'U_2'' = 0 \quad [34]$$

The subscript 2 denotes functions of ζ_2. Using series expansions

$$W = \Sigma\, a_n\zeta^n, \quad W_2 = \Sigma\, a_n'\zeta_2^n \quad [35]$$

$$V = \Sigma\, b_n\zeta^n, \quad V_2 = \Sigma\, b_n'\zeta_2^n \quad [36]$$

Equations [13], [14], [32], and [33] can be solved simultaneously by determining the coefficients a_n, b_n, a_n', and b_n' from the following boundary conditions:

Equation [30] requires that

$$\Sigma\, \frac{b_n}{2^n} = \frac{1}{2} \quad [37]$$

At $\zeta = \zeta_2 = 1/2$ the requirements are

$$W\left(\frac{1}{2}\right) = W_2\left(\frac{1}{2}\right) \quad [38]$$

$$W'\left(\frac{1}{2}\right) = W_2'\left(\frac{1}{2}\right) \quad [39]$$

$$W''\left(\frac{1}{2}\right) = W_2''\left(\frac{1}{2}\right) \quad [40]$$

$$W'''\left(\frac{1}{2}\right) = W_2'''\left(\frac{1}{2}\right) \quad [41]$$

Equations [15] and [34] can be expressed as

$$U''' - 2AWU'' = C \quad [42]$$

$$U_2''' + 2AW_2U_2'' = C \quad [43]$$

where C is the constant of integration which must be determined.

From the known functions W and W_2, Equations [15] and [34] can be solved for U and U_2 by using the series expansion

$$U = \Sigma\, c_n\zeta^n, \quad U_2 = \Sigma\, c_n'\zeta_2^n \quad [44]$$

with the boundary conditions

$$U(0) = 0, \quad U_2(1) = 1 \quad [45]$$

$$U\left(\frac{1}{2}\right) = U_2\left(\frac{1}{2}\right) \quad [46]$$

$$U'\left(\frac{1}{2}\right) = U_2'\left(\frac{1}{2}\right) \quad [47]$$

$$U''\left(\frac{1}{2}\right) = U_2''\left(\frac{1}{2}\right) \quad [48]$$

The coefficients of the foregoing series expansions are presented in the Appendix.

Numerical solutions of the simultaneous Equations [37] to [41] and [46] to [48] determine the coefficients of the series expansion of U, V, and W. Determination up to b_4 calls for the solution of five simultaneous equations of second order. As the value of A increases, more terms have to be taken into account which increases the order of the algebraic equations accordingly.

Figs. 2 to 5 show the results of solutions for the cases $A = 1$ (dotted line) and $A = 10$ (dash-dot line). The peripheral component of velocity in this case is represented by

$$v = r\omega(1 + b_1\zeta + \ldots)$$

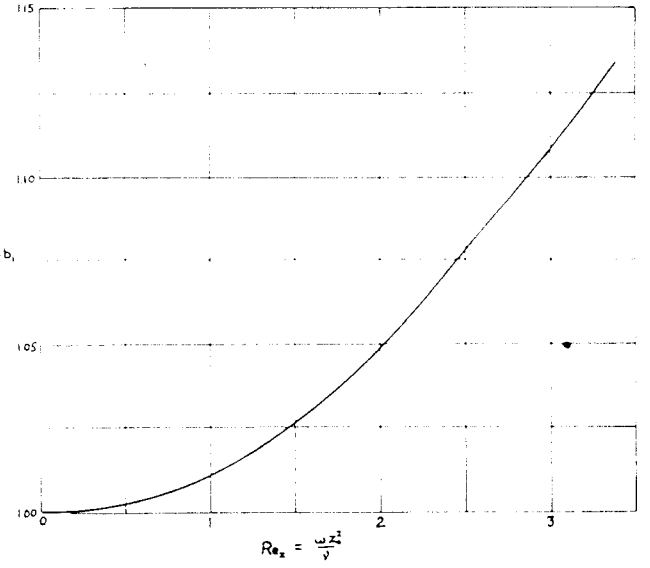


FIG. 7 COEFFICIENT b_1 OR INERTIA CORRECTION FACTOR OF MOMENT COEFFICIENT (SHEAR AT THE WALL = $b_1 r \omega / z_0$)

Values of the coefficient b_1 are shown in Fig. 7. These are the correction coefficients of k_s for various values of Re_z within the laminar-flow range; at large values of Λ, this correction for moment coefficient becomes increasingly significant.

Discussion

1 The boundary-layer solution obtained in the foregoing appears justified by comparison with the experimental results, Fig. 6, for small spacing. The first-order approximation for $Re_z \to 0$ differs from the solution in reference (7) only in that the latter accounts for $\partial^2 w / \partial z^2$. Equations [17] to [19] show $\partial^2 w / \partial z^2$ to be small when compared to $\partial^2 u / \partial z^2$ and $\partial^2 v / \partial z^2$ since w is of the order of the gap z_0 while u and v are of orders of the radius r. The emphasis of $\partial^2 w / \partial z^2$ while disregarding the momentum contribution other than that due to centrifugal force leads to the analogous condition of a disk in an infinite medium. The flow condition over an enclosed disk, however, is basically different from the case of an infinite medium; any effort in trying to get similarity between the two cases is not likely to lead to significant result. For wider disk spacing the correction of inertia effect becomes increasingly significant, the limiting case is when the spacing is infinite, in which case an exact solution is given in reference 4, and the inertia effects are accounted for in an exact manner.

2 In the case of the first-order approximation, the fluid is under simple shear r across the gap. The result obtained here is a generalization of the simple case presented in reference (1). The centrifugal force induces the recirculation u outward at the disk surface and inward at the surface of the housing. The induced w being negative means flow from near the housing toward the disk.

3 The second-order approximations for finite values of Re_z show the significance of the inertia of the fluid in modifying the conditions of simple shear. The inertia effects tend to increase the shear at the wall, Fig. 2. The curve labeled turbulent flow in Fig. 2 shows a boundary layer based on the $1/7$ power velocity-distribution law. This indicates the manner by which inertia effects aid the transition from a laminar to a turbulent boundary layer through instability (5). The presence of a point of inflection has been suggested as a necessary and sufficient condition for instability (13). The effect of finite values of Re_z also can be seen in Fig. 4. In the limit, the increased effects of inertia and instability transform the motion u into that of turbulence with a thin boundary layer (7).

4 Simple peripheral shear across the gap is a very good approximation, Fig. 6, when the gap z_0 is very small compared to the radius r. The moment coefficient for this case is

$$k_s = \frac{5}{4 Re_r} \left(\frac{D}{z_0} \right)$$

In the experimental case mentioned previously, $Re_z = 0.118$ for water at $Re_r = \nu r_s^2 / \nu = 10^6$, the correction for the second-order effect of Re_z is very small (dotted line in Fig. 6). Therefore Equation [28] is a very good approximation for the condition in pedestal bearings. Fig. 7 shows that the first-order approximation would lead to 15 per cent error when Re_z is equal to 3.8. Therefore a correction may be necessary in the case of gas-turbine disks and centrifugal-compressor shrouds.

On the basis of the $1/7$ power velocity-distribution law (2), it can be shown with the present method that, where the turbulence dissipation in the core is small when compared to that due to shear at the walls, the moment coefficient in the turbulent regime is given by, for turbulent boundary layer of half of the thickness of the spacing

$$k_s = 0.0206 \, Re_r^{-1} \left(\frac{D}{z_0} \right)^{1/4} \quad \ldots\ldots\ldots\ldots [49]$$

(Appendix) which also checks closely with the experimental results, Fig. 6. The independence of the gap in the transition from the laminar to the turbulent range as represented in reference (7) is due mainly to the similarly prescribed motion for all sizes of gap by neglecting momentum in the boundary layer other than that due to centrifugal force while considering viscous forces. It is inconceivable that, for similar disk, housing, fluid, and speed, similar motion occurs whether the gap is 0.01 in. or 1 in. Comparison with experimental results of reference (8) shows that reference (7) provides reasonable over-all approximations only for $D/z_0 < 50$; i.e., for wide spacing. On the other hand, Equation [49] tends to give too high a value of k_s at large D/z_0 (>200) and too low at small D/z_0 (<50). Hence reference (7) provides a good approximation for wide gap, while for small gap, other velocity laws should be taken in the derivation of Equation [49] because, below certain values of D/z_0, the solution should reduce to the case of $z_0 = \infty$ (1, 4).

A proper explanation of the continuity of the experimental curve is probably that for the range $Re_r > 10^5$ represented in Fig. 6, transition from laminar to turbulent boundary layer exists in the space between the disk and the housing. In other words, laminar motion always exists for a solid disk in a housing. The similarity of the dimensionless correlation of reference (7) to that of the case of a disk in an infinite medium (1) can be attributed physically as due to overemphasis of the axial motion while neglecting momentum other than that due to centrifugal force. The axial motion is one of the main motions in the latter case but in the former case, its effect is small when the gap is small. The proportionality to $Re^{-1/2}$ in the laminar case of reference (7) should not be attributed to recirculation; recirculation occurs even for very low Reynolds numbers. The approximation in Equation [49], however, is good only when turbulent dissipation in the core is small when compared to that at the walls.

From similarity between friction and heat transfer (12), the heat transfer from an enclosed rotating disk should follow a trend similar to the friction characteristics. However, the measurements reported in reference (14) do not substantiate this fact.

5 The accuracy of both of the foregoing approximations can be seen by substituting Equations [21] to [25] into Equations [2] and [3]. The criterion is that the quantity

$$\frac{30m}{\pi\rho z_0 r^2 \omega \mathrm{Re}_z} < 1$$

for high accuracy. Fortunately this is true in almost all practical cases.

6 When the net flow is small, the recirculation-velocity component is but little affected by the net flow. W and U are independent in the case $\mathrm{Re}_z \to 0$ so the actual value of u can be obtained by superposition. For instance, in the case of radial outflow, there is no recirculation for any radius smaller than r_1 given by

$$r_1^2 = \frac{45m}{\pi\omega\rho z_0 \mathrm{Re}_z} \quad [50]$$

The radial flow velocity at $r = r_1$ is

$$u = \frac{1}{60} r_1 \omega \mathrm{Re}_z \left[\frac{2U'}{3} - 60\,W'\right] \quad [51]$$

The trend is shown in Fig. 8. Similarly, for radial inflow there is no recirculation below r_1'

$$(r_1')^2 = \frac{30m}{\pi\rho z_0 \omega \mathrm{Re}_z}$$

where m is the strength of radial inflow. The possible cases of flow over a rotating disk are shown diagrammatically in Fig. 9 when there is no casing at the outer edge. It is seen that, without net flow, recirculation takes place over the whole radial dimension. With net flow less than the foregoing limiting values, local recirculation takes place in the form of a torus.

7 For substantial values of Re_z, U' is modified by W as shown in Fig. 5. The distortion will be more pronounced at higher values of A. In the case of radial outflow, such modification tends to increase the radial shear rate at the disk side while, in the case of radial inflow, such modification tends to increase the radial shear rate at the housing side.

8 According to both items 7 and 6, in general, radial outflow over the disk increases the shear at the surface of the disk and decreases the shear at the housing in comparison with the case of no net radial flow. This trend is quite favorable in the source flow cooling of a gas-turbine disk. The cooling of a radial-inflow turbine disk by radial inflow of cool air will be less effective. This effect can be seen further in Fig. 9.

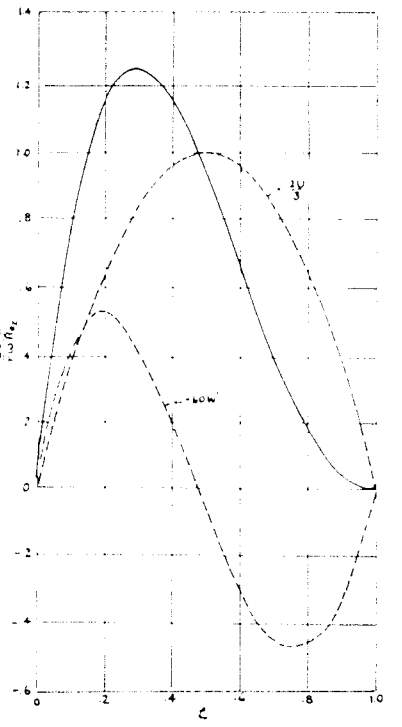


FIG. 8 LIMITING CASE OF SOURCE FLOW AND ROTATION AT RADIUS r_1

9 The average radial pressure variation between any radius r_0 and r can be shown to be

$$\frac{p - p_0}{\frac{1}{2}\rho r^2\omega^2} \leqq -\frac{3}{\pi}\left(\frac{m}{\rho z_0 \omega r^2}\right)\left(\frac{1 - 0.07896\,\mathrm{Re}_z^2}{\mathrm{Re}_z}\right)\ln\left(\frac{r}{r_0}\right) + \frac{3}{10\pi^2}\left(\frac{m}{\rho z_0 \omega r^2}\right)^2\left(\frac{r^2}{r_0^2} - 1\right) + \frac{2}{5}(1 + 0.001095\,\mathrm{Re}_z^2)\left(1 - \frac{r_0^2}{r^2}\right) \quad [52]$$

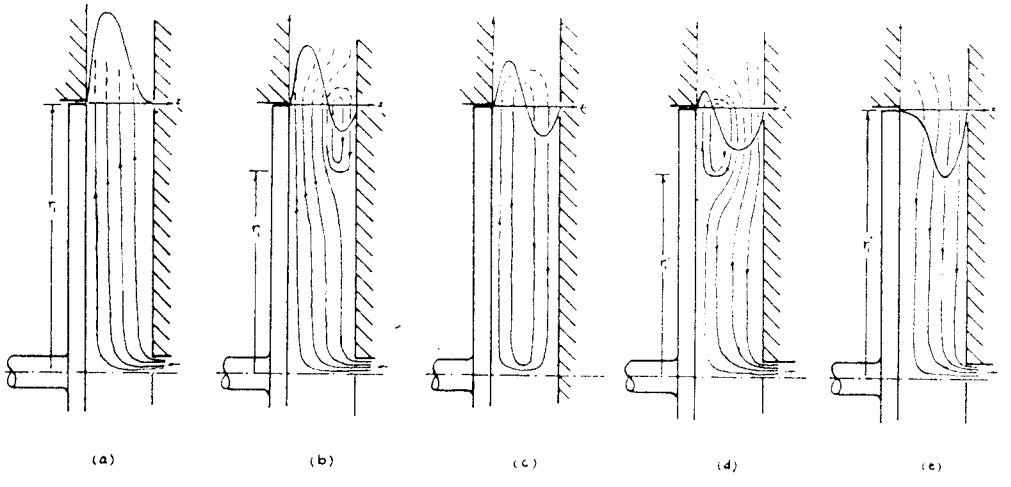


FIG. 9 RADIAL FLOW OVER ROTATING DISK AT VARIOUS FLOW CONDITIONS WHEN THERE IS NO CASING AT THE OUTER EDGE

[(*a*) Radial outflow, with limiting flow at rim; (*b*) radial outflow, flow quantity less than limiting value at rim; (*c*) no net flow, pure recirculation; (*d*) radial inflow, flow quantity less than limiting value at rim; (*e*) radial inflow, with limiting flow at rim.]

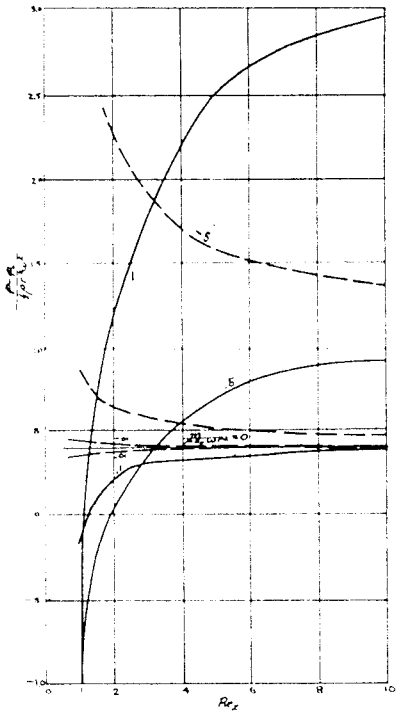


FIG. 10 AVERAGE PRESSURE DIFFERENCE BETWEEN DISK AND HOUSING FOR $r/r_0 = 10$
($m > 0$ for radial outflow, $m < 0$ for radial inflow.)

The first term on the right-hand side accounts for friction imparted on radial net flow, the second term accounts for diffusion (radial outward flow) or acceleration (radial inward flow), and the third term represents the radial pressure variation due to centrifugal effect. For very small Re_z

$$\frac{p - p_0}{\frac{1}{2}\rho r^2\omega^2} = -\frac{3}{\pi \mathrm{Re}_z}\left(\frac{m}{\rho z_0 \omega r^2}\right)\ln\frac{r}{r_0} + \frac{3}{10\pi^2}\left(\frac{m}{\rho z_0 \omega r^2}\right)^2\left(\frac{r^2}{r_0^2} - 1\right) + \frac{2}{5}\left(1 - \frac{r_0^2}{r^2}\right) \quad [53]$$

The last term of Equation [53] represents the minimum average radial pressure distribution over an enclosed rotating disk. Equation [53] was plotted as shown in Fig. 10, for both inward ($m < 0$) and outward flow ($m > 0$). It can be seen that rotation always tends to reduce the leakage flow of a centrifugal pump or compressor, but the minimum inlet pressure required to carry out radial outflow cooling really depends on the combination of the parameters Re_z and $m/\rho z_0 \omega r^2$. In the case of radial outflow, as in cooling a gas-turbine disk, the diffusion in the radial flow tends to raise the static pressure, while fluid friction tends to decrease the pressure due to the increase in velocity (15). The data shown in Fig. 10 for large values of $m/\rho z_0 \omega r^2$ tends to be overoptimistic due to instability of flow under the adverse pressure gradient. The centrifugal force tends to attenuate the boundary layer at the disk surface. At the housing, the recirculation tends to aggravate boundary-layer separation.

CONCLUSIONS

1 The solution obtained here for an enclosed rotating disk is valid. The friction moment coefficients

$$k_z = \frac{5}{4\mathrm{Re}_r}\left(\frac{D}{z_0}\right) \text{ for laminar flow}$$

$$k_z = 0.0206\ \mathrm{Re}_r^{-1/4}\left(\frac{D}{z_0}\right)^{1/4} \text{ for turbulent flow}$$

are quite accurate for small gaps between the disk and housing.

2 For large values of Re_z which are usually associated with wide gaps, the friction moment obtained from simple shear theory can be 15 per cent too small at values of Re_z of the order 4. The second-order effect of Re_z must be considered if a closer approximation is required.

3 The method developed for including the second-order effect of Re_z can be carried out with automatic computers.

4 Radial outflow enables very effective cooling of a disk surface.

5 Rotation provides an added sealing effect of the shroud of a centrifugal machine.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. Robert C. Dean, Jr., Ingersoll-Rand Corporation, Phillipsburg, N. J., and reviewers for their suggestions.

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Appendix

DETERMINATION OF COEFFICIENTS OF SERIES EXPANSION

The coefficients determined from Equations [13] and [15] are

$a_0 = 0$, $b_0 = 1$

$a_1 = 0$, b_1

a_2, $b_2 = 0$

a_3, $b_3 = -2Aa_2/3$

$a_4 = b_1/12$, $b_4 = -Aa_2(3 + b_1)/6$.

$$a_5 = b_1^2/60, \qquad b_5 = -Ab_1(1 + 6a_3b_1)/30$$

$$a_6 = -Aa_2(1 + 3a_2)/90, \qquad b_6 = -A(3b_1^2 + 8Aa_2^2)/180$$

$$a_7 = -Aa_2(3 + 9a_3 + 8b_1)/630, \quad b_7 = -A^2a_2(-1 - 18a_2 + 30a_3 - 5a_2b_1)/315$$

The coefficients determined from Equations [32] and [33] are

$$a_0' = 0, \qquad b_0' = 0$$

$$a_1' = 0, \qquad b_1' = -b_1$$

$$a_2', \qquad b_2' = 0$$

$$a_3', \qquad b_3' = -b_3,$$

$$a_4' = 0, \qquad b_4' = -b_4$$

$$a_5' = -b_1'^2/60 \qquad b_5' = -b_5$$

$$a_6' = Aa_2^2/30, \qquad b_6' = -b_6$$

$$a_7' = (3Aa_2'a_3' + 2b_1'b_3')/210, \qquad b_7' = -b_7$$

The basic unknowns b_1, a_2, a_3, and a_2' and a_3' can be determined by simultaneous solution of Equations [37] to [41]. Equations [45] to [48] provide the information needed to determine the unknown coefficients C, c_2, and c_2' in the following

$$c_0 = 0, \qquad c_0' = 1$$

$$c_1 = 0, \qquad c_1' = 0$$

$$c_2, \qquad c_2'$$

$$c_3 = C/6, \qquad c_3' = -C/6$$

$$c_4 = 0, \qquad c_4' = 0$$

$$c_5 = Ac_2a_2/15, \qquad c_5' = -Ac_2'a_2'/15$$

$$c_6 = A(2c_2a_3 + Ca_2)/60, \qquad c_6' = -A(2c_2'a_3' + Ca_2')/60$$

$$c_7 = A(c_2b_1 + 6Ca_3)/630, \qquad c_7' = -A(c_2'b_1' + 6Ca_3')/630$$

Derivation of Moment Coefficient in Turbulent Range

Assuming the validity of $1/7$ velocity law (1) at the wall and taking the velocity of the core as one half of the disk velocity, the shear stress at the wall is given by

$$\tau_0 = 0.03955\,\rho\left(\frac{r\omega}{2}\right)^{7/4}\nu^{1/4}z_0^{-1/4}$$

The moment coefficient

$$k_s = \frac{2\pi\int_0^{r_s}\tau r^2 dr}{\pi\rho\omega^2 r_s^5/5} = 0.0206\left(\frac{D}{z_0}\right)^{1/4}\mathrm{Re}_r^{-1/4}$$

Discussion

R. E. Nece[3] and J. W. Daily.[4] The writers wish to compliment the author on his contribution in generalizing the solution for the case of simple laminar flow between a rotating smooth disk and a closely spaced fixed boundary. His findings and conclusions regarding the details of velocity distribution, flow circulation, and heat-transfer effects appear physically reasonable and it is expected they can be verified by suitable experiments.

The author also discusses turbulent flow and the regime he describes as the transition to turbulence. The writers believe this needs further clarification, and, in view of their own experiments with smooth and rough enclosed disks which are currently under way at the M.I.T. Hydrodynamics Laboratory, are prompted to make the following comments.

In this paper, the author's discussions for both laminar and turbulent flow are concerned only with the case of close axial clearance between a rotating disk and a stationary boundary. In such instances, the flow regime is characterized by converged boundary layers; that is, the boundary-layer thickness on both rotating and stationary walls is constant over the radius and equal to $z_0/2$. For both laminar and turbulent flow, there is a second regime obtained with decreasing D/z_0 (increasing axial clearance) such that for constant

$$\mathrm{Re}_r = \frac{\omega r_s^2}{\nu}$$

the boundary layers, which are a function of Re_r and therefore have the same thickness for different z_0 values, will be relatively thinner leaving a "core" of finite width which rotates at approximately one half the velocity of the disk. Furthermore, for a turbulent boundary layer, the thickness will vary over the radii of both the disk and stationary boundary.

In general, it should be expected that as Reynolds number, Re_r is increased from a low value, the successive flow regimes encountered may be:

1 Laminar flow with converged boundary layers.
2 Laminar flow with rotating core.

Then, following the transition from laminar to turbulent flow with an attendant initial increase in boundary layer thickness:

3 Turbulent flow with converged boundary layers.
4 Turbulent flow with rotating core.

Of course, the transition from the laminar to turbulent regime is not likely to be sharp because, since the local Reynolds number varies as radius squared, turbulence no doubt occurs first at the periphery and works progressively inwards. This has been demonstrated experimentally for a free disk rotating in air in the author's reference (13).

The author's Equation [29] applies to regime No. 1, his Equation [49] to No. 3. The relations from reference (7) of the author's Bibliography plotted on the author's Fig. 6 represent solutions for regimes Nos. 2 and 4. These relations in the author's notation are

2nd regime
$$k_s = \frac{1.06}{(\mathrm{Re}_r)^{1/2}} \qquad [54]$$

4th regime
$$k_s = \frac{0.0248}{(\mathrm{Re}_r)^{1/5}} \qquad [55]$$

with

$$\mathrm{Re}_r = \frac{\omega r_s^2}{\nu}$$

It will be noted that Equations [54] and [55] do not predict an effect of axial clearance.

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* It should be noted that the author's Fig. 6 is plotted using

$$\mathrm{Re}_r = \frac{\mathrm{rpm} \times r_s^2}{\nu}$$

whereas ω instead of rpm is used in the Reynolds numbers in his equations. Parenthetically, it might be noted that the equations corresponding to [54] and [55] for a "free disk" ($z_0 = \infty$), which in the author's notation are

$$k_s = \frac{1.53}{(\mathrm{Re}_r)^{1/2}} \quad \text{and} \quad k_s = \frac{0.058}{(\mathrm{Re}_r)^{1/5}}$$

respectively (using ω in the Reynolds number), appear to be plotted incorrectly in the author's Fig. 6.

The question arises as to the magnitude and range of Reynolds numbers over which these several regimes might exist, and also, of course, as to the accuracy of the equations for calculating the disk-friction torque. Briefly, the facts of the situation seem to be the following:

1 For turbulent flow, all published torque formulas have been derived using the physical model of a rotating core, corresponding, therefore, to the high Reynolds-number range and regime No. 4.

2 Between the laminar flow, regime No. 2 and the high Reynolds-number turbulent-flow range, a "transition" curve is sometimes observed, although up to now regime No. 3 has not been explicitly tagged.

3 Contrary to the predictions of the previously published formulas, torque measurements by the writers for regimes No. 2 and No. 4, as well as published measurements for regime No. 4 (see author's Fig. 6 and reference 8) have indicated variations with the axial-clearance ratio such that as D/z_0 increases, the torque coefficient decreases.

4 The author's Equation [49] indicates increasing torque coefficient with increasing D/z_0.

In view of these observations, it appears that the range over which regime No. 3 may exist will depend on the axial clearance ratio, decreasing with decreasing values of D/z_0 (increasing clearance). M.I.T. experiments indicate for D/z_0 less than about 30, this regime may not be distinguished from a laminar-to-turbulent transition range of the usual sort. As to the lack of agreement between published formulas and experiment for the wide clearance regimes Nos. 2 and 4, an important shortcoming of all the theoretical analyses has been the omission of the effect of wall friction at the cylindrical portion of the enclosing housing on the loss of momentum and kinetic energy. It is concluded that an adequate theoretical development for torque equations must be based on a more realistic physical model than has been used heretofore.

R. C. Dean, Jr.[5] The author is to be complimented on presenting an analysis of the laminar disk-friction and boundary-layer problem which, within the knowledge of the writer, is more complete and authentic than any other in the literature for the case of a small gap between the disk and casing. The solution yields flow patterns which show evidence of circulation in the clearance gap which bears some resemblance to the Taylor-Gortler rings found in the gap between a rotating cylinder and stationary coaxial cylindrical casing. The work of Kaye and Elgar[6] shows that these vortexes have a significant influence on heat transfer, and presumably also on friction, in the cylindrical case and would be expected similarly to influence the heat transfer from and friction of a rotating disk in a closely spaced casing by promoting a vigorous mixing of the flow.

Kaye and Elgar's investigation demonstrated that the flow between cylindrical walls could occur in four regimes depending upon the mass flow and rotative speed of the inner cylinder. These regimes are characterized by the flow being laminar or turbulent with or without the vortexes. The writer suggests that the problem of the flow between a disk and housing might be investigated profitably by using visualization and hot-wire techniques to reveal the detailed space-time characteristics of the flow. These characteristics may aid immeasurably, as in Elgar's case, to explain the variations in integrated parameters as shown in the author's Fig. 6.

M. A. Santalo[7] and W. A. Wilson.[8] There are two distinct approaches to the solution of fluid-flow problems in engineering devices. This paper is a creditable example of an elegant mathematical treatment based on the general form of the Navier-Stokes equations. These fundamental relationships have been simplified by "introducing the usual boundary-layer approximations," and further simplified by restricting attention to the case of very small Reynolds numbers and very small integrated radial flows. The method then yields precise solutions for the cases consistent with these assumptions.

The second approach is also classic; it involves positing the essential kinematic characteristics of the solution, frequently a one-dimensional description or symmetry in velocity profiles. The kinematic parameters of the hypothesis are then adjusted to satisfy certain gross limitations embodied in the continuity, momentum, and energy relationships. Solutions thus obtained are susceptible to indefinite refinement by applying the method simultaneously to smaller and smaller subdivisions of the process. In the limit, of course, the latter method coincides with the former, but in practice and in spirit the two methods are quite distinct. The second is based on the assumption of the possibility of a direct physical apprehension of the phenomenon and the first on confidence in being able to cull from the general differential equations those terms of negligible consequence.

The first method holds out the promise of definitive solutions, either analytical or numerical. In fluid-mechanics problems we usually must be content with severe restrictions on the range of applicability of these solutions. In the present instance the addition to our understanding of flow over a rotating disk is limited to the correction factor presented in Fig. 7. The author suggests the applicability of this correction to flows over gas-turbine disks and centrifugal-compressor shrouds. As a matter of fact, the Re_z range, 0 to 3.5, covered by Fig. 7 probably covers the laminar-flow regime for which the curve is valid, but typical applications in turbines and compressors lie far outside it. (For example if $\omega(D/2) = 1000$ fps, $z_0 = 1/16$ in. and the fluid is air at 200 F, then $Re_z = 78.5$.)

Of greater practical importance is the turbulent regime. The author presents some conclusions about this regime in his Discussion. Although we are not led in detail through the reasoning by which he comes to these conclusions it is fairly clear that this reasoning is of the second kind; i.e., he posits the $1/7$ power velocity-distribution "law." Such use of analogy can be very useful indeed. A paper by Hisao Jimbo[9] examines several aspects of the problem of flow over a rotating disk starting with friction laws developed by Schultz-Grunow in reference (7) of the paper, and the present paper. His conclusions which were reasonably substantiated by experiment have several practical implications.

There are two specific questions which the writers would like to direct to the author:

1 Are the slight asymmetries of the velocity profiles in Figs. 3, 4, and 5 attributable to some physical phenomenon or to the simplifications introduced into the basic equations? In particular, for the case $A = 0$, $m = 0$, and $D/z \gg 1$ the asymmetry indicated is very surprising.

2 What is the definition of "average pressure difference" as used in the caption of Fig. 10? If the assumption $\partial p/\partial z = 0$ ap-

[5] Head, Advanced, Engineering Department, Ingersoll-Rand Company, Phillipsburg, N. J. Mem. ASME.

[6] "Modes of Adiabatic and Diabatic Fluid Flow in an Annulus with an Inner Rotating Cylinder," by Joseph Kaye and L. C. Elgar, Research Laboratory of Heat Transfer in Electronics, Massachusetts Institute of Technology, Cambridge, Mass., Report No. RLHTE-13, February 15, 1957.

[7] Assistant Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

[8] Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

[9] "Investigation of the Interaction of Windage and Leakage Phenomena in a Centrifugal Compressor," ASME Paper No. 56–A-47.

plies then $p = f(r)$ only and there would seem to be nothing to average. For the case of $m = 0$ and the symmetrical profiles of Fig. 2 one can conclude that at $z = z_0/2$

$$\frac{p - p_0}{\frac{1}{2}\rho r^2\omega^2} = 0.25$$

not 0.4 as indicated in Fig. 10.

A final comment is a school-masterish plea for more consideration of the readers in the preparation of curves. Fig. 6 which presumably relates the theory presented to other theories and to experimental work is virtually unintelligible to the writers.

Author's Closure

The author wishes to thank the discussers for their interest and criticism.

The four flow regimes pointed out by Professors Nece and Daily are certainly enlightening. While all these conditions tend to occur to a given disk, depending on the physical dimensions and properties, these are the four predominating cases. Their suggestion of the study including wall friction at the cylindrical portion of the enclosing housing is a worthy one in an effort toward a better understanding of disk friction problem. No doubt their calculations on the cases of free disks were right as a direct conversion, but the author has made a further conversion, based on the definition of moment coefficient given by Equation [28] in order to make comparison with the experimental results presented in reference (8).

The remarks contributed by Dr. Dean are valuable and instructive in supplementing this present study. His suggestion for experimental work will certainly lead to significant understanding.

The restriction as suggested by Professors Santalo and Wilson is really not there; as long as the flow is laminar, the method presented is not restricted to $\mathrm{Re}_s = 3.5$, nor $\mathrm{Re}_s = 78.5$. Even if calculations were carried out to the latter figure, one can always point out to an example in which Re_s is a larger quantity. In fact, the last paragraph of Introduction suggested these possibilities. The following might serve to answer their specific questions:

1 The viscosity profiles in Fig. 3 are not symmetric because centrifugal force always acts radially outward. The axial components in Fig. 4 are also not symmetric; Figs. 3 and 4 are related by continuity. The case of $A = 0$ in Fig. 5 is symmetric because it is simply Poiseuille flow.

2 The simplification as presented by Equation [8] means variation of pressure in the axial direction is negligible as compared to variations in the radial direction. In other words

$$\frac{\partial p}{\partial r} \simeq \frac{d\bar{p}}{dr}$$

where $\bar{p}$ means averaging in the axial direction, or Equation [52] was obtained from Equation [6] in the following manner

$$-\frac{1}{\rho}\frac{d\bar{p}}{dr} = \frac{\partial}{\partial r}\left[\frac{1}{z_0}\int_0^{z_0} u^2 dz\right] + \frac{1}{z_0}\int_0^{z_0} w\frac{\partial u}{\partial z}dz - \frac{1}{r} \times \left[\frac{1}{z_0}\int_0^{z_0} v^2 dz\right]$$

In their calculation in arriving at $(p - p_0)\Big/\left(\frac{1}{2}\rho r^2\omega^2\right) = 0.25$. they have neglected contributions from the radial component of velocity.

With regard to their final comment, the author wishes to assure the readers that better pictures will be prepared next time.

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An Analytical and Experimental Investigation of Multiple Disk Pumps and Compressors

An unconventional type of turbomachinery employing a rotor composed of smooth disks is described and the history and motive principle of such devices briefly recounted. An experimental program of research on such devices is described, together with a description of the design and testing of an experimental water pump, a blower, and an air compressor using multiple disk rotors.

Data resulting from the testing of the devices are presented in summary form with discussions of the conclusions which the data allow.

A "first-approximation" analysis is presented of the flow processes within the rotor and of the limiting performance and efficiency of multiple disk pumps. Solution of the resulting equations, found by digital computation, is presented for a wide range of geometry and of flow variables, in terms of dimensionless parameters. A brief discussion is given of possible applications of multiple disk pumps, compressors and blowers, and of the information now available for the design of the devices.

Nomenclature

- b = distance between adjacent disks
- f = friction factor, further defined in context
- p = pressure
- ΔP_e = pressure change in entrance passage
- ΔP_r = pressure change across rotor from inner radius to position x
- ΔP_s = pressure change in diffuser
- ΔP_t = $\Delta P_e + \Delta P_r + \Delta P_s$, total pressure change across entire pump
- Q = volume flow rate
- r = radial position coordinate, from center of disks
- T = torque
- u = velocity component in r-direction
- v = velocity component in θ-direction
- W = work per unit mass
- x = r/r_o, dimensionless radial position coordinate
- y = $v/\Omega r_o$, dimensionless velocity coordinate in θ-direction
- η = efficiency, further defined in context
- Ω = angular velocity of disks
- ρ = density of fluid
- τ = shear stress, further defined in context
- θ = angular position coordinate

Subscripts

- i = value at inner radius
- o = value at outer radius

Introduction

IN CONVENTIONAL turbomachinery, momentum exchange between the fluid and the rotor is effected by providing blades or shaped passages to deflect and constrain the flow in the rotor. The resulting force is due to the integral of the pressure distribution over the surface of the rotor. In some turbomachinery, however, such as the conventional drag turbine, force on the rotor is due to the integral of the shear stress over the rotor surface and no constraining blades or passages are present.

There exists a type of turbomachinery rotor essentially different from conventional bladed or vaned rotors and also very different from the conventional drag turbine rotor, both in configuration and in effects. This is the multiple disk rotor, consisting of a number of thin, smooth, flat, parallel disks arranged normal to a shaft and fastened rigidly to it with small spaces between the disks. Holes or slots are provided in the disks near the shaft. This type of rotor, with appropriate housings, can be used to form hydraulic and gas turbines, hydraulic pumps, and gas compressors. A photograph of such a rotor is presented in Fig. 1.

In the pump application of the multiple disk rotor, the rotor is enclosed in a housing provided with bearings, housing-to-rotor seals, a diffusion scroll, and appropriate fluid supply cavities. Fluid enters the pump rotor through the holes or slots near the

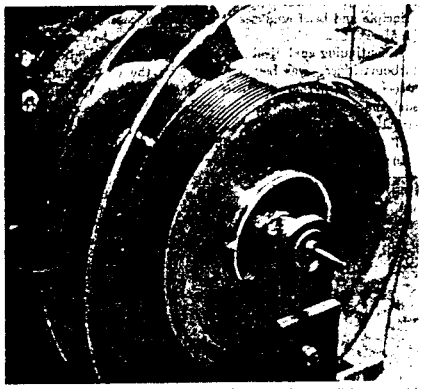

Fig. 1 A low-pressure air compressor (or blower) using disk rotor, with a lucite diffusing housing

shaft, then enters the spaces between the disks in an approximately radial direction. Because of the shear stress exerted on the fluid by the disks, the fluid is accelerated tangentially which in turn causes centrifugal forces which accelerate the fluid radially toward the disk periphery. Consequently, the fluid follows a "spiral" path relative to a fixed coordinate system while between the disks and exhausts into the diffusion scroll at a high velocity with both radial and tangential components. During passage through the rotor, the fluid reacts on the multiple disk system requiring that external torque be supplied to the pump shaft by a driving engine or motor. The fluid exhausts from the diffuser scroll at a higher energy than that at which it enters the pump, the increased energy being primarily in the form of enthalpy. The net useful result is an increase in the pressure of the fluid between the pump inlet and outlet.

In the compressor or blower application of the multiple disk rotor, the operational principle is the same but the fluid handled is gaseous rather than liquid. As in the case of conventional turbomachinery, a blower would be expected to perform in very nearly the same manner as a pump since the fluid density is not greatly changed within the blower, but a compressor with a high compression ratio would incur compressibility effects within the rotor.

The origin of the idea for such a rotor is somewhat obscure. The first definitive United States patents involving the rotor were issued to N. Tesla [1][1] in 1913, covering both turbine and pump applications of the rotor. The inventor did not seriously attempt to exploit the devices and the principle has received little attention in published engineering investigation. A public demonstration of a multiple disk turbine was given by Tesla prior to the issue of patents and was widely reported in the semitechnical press of that time [2, 3]. Several writers offered explanations of the motive principle and brief analyses of the flow process within the rotor [4, 5].

A continuing analytical and experimental investigation of disk turbomachinery was begun in 1958 by the writer and several associates with the objectives of determining the feasibility of such machinery. The investigation resulted in the design, construction, and testing of a variety of experimental air turbines and air compressors, and a liquid pump using multiple disk rotors.[2] All of the machines performed the functions for which they were intended. The test results indicate that multiple disk pumps and compressors are feasible, may offer attractive performance in certain special applications (but not where conventional pumps are satisfactory), and are worthy of further investigation. The analyses indicate the upper limits of performance which can be approached by multiple disk turbomachinery and the manner in which the upper performance limits vary with geometrical and operational parameters. This paper presents a summary of test results from a pump, an air blower, and an air compressor using multiple disk rotors and a first-approach type of analysis of the performance of such machinery.

[1] Numbers in brackets designate References at end of paper.

[2] The investigation reported herein was sponsored by Texas Instruments, Inc., Dallas, Texas.

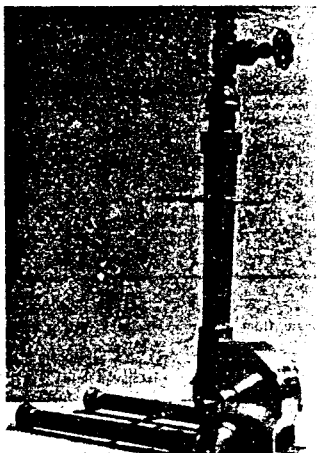

Fig. 2 A water pump using a multiple disk rotor, with a diffuser incorporated for testing

An Experimental Multiple Disk Pump

Fig. 2 shows a multiple disk pump while Fig. 3 shows the rotor of the pump. The rotor was 5.0 in. in diameter, composed of 34 disks, each 0.020-in. thick, and with 0.020-in. disk spacing. In this machine, the disks were made with a 1.0-in-dia central hole; the rotor was thus hollow, with no shaft through it. The disks were held together, spaced, and stiffened by eight small throughbolts and an inlet spider at each side of the rotor provided attachment to shafts external to the rotor. Water entered from appropriate chambers in the pump housing, through the inlet spiders, into the central hollow portion of the rotor, and finally into the spaces between the disks. At the outer periphery of the rotor the water entered a spiral diffusing scroll and left the pump through a final straight diffusing pipe. Bronze bearings were used, with no external lubrication. Carbon seals were provided between the rotor and the housing to minimize leakage between the two. All other details such as packings, couplings, and piping were conventional.

The pump was driven by a variable speed electric motor cradled to form a dynamometer for testing. Water at approximately 100 deg F was the pumped fluid. Appropriate data were collected to yield the performance summarized in Table 1, in which representative values only are presented. Minor inconsistencies in the data presented are due to experimental inaccuracies.

The flow was unstable over some ranges of performance; only values at which stability obtained are reported. The instability is believed due to separation and resulting cavitation within the rotor, perhaps abetted by cavitation at the inlet spiders. A complete description of the pump, the test conditions, and the resulting data and performance have been documented in a thesis concerning the machine [6]. In Table 1, the efficiency reported is the overall pump efficiency, defined as the ratio of the increase in the hydraulic energy of the fluid through the pump to the shaft work supplied to the pump; the difference in the two quantities is manifested primarily by a temperature rise in the liquid pumped.

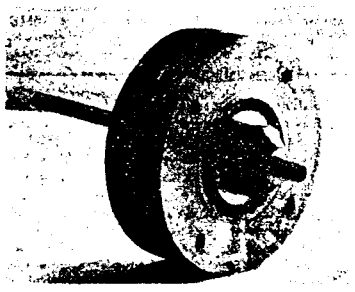

Fig. 3 The rotor of the pump shown assembled in Fig. 2

Table 1 Representative performance data from multiple disk water pump, operated at atmospheric pressure and approximately 100 deg F at pump inlet

Speed, rpm	Discharge pressure, psig	Volume flow rate, gal/min	Efficiency %
3500	47.2	0.0	...
3500	46.7	10.1	6.1
3500	41.4	24.8	16.9
3500	39.2	34.8	21.0
3500	37.1	[illegible]	20.1
3500	28.8	41.8	13.8
5500	92.0	0.0	...
5500	87.0	12.0	7.1
5500	82.5	25.2	13.4
5500	76.5	26.6	12.8
5500	42.8	34.2	7.9
5500	39.4	41.1	8.8
5500	30.2	46.2	7.3

Table 2 Representative performance data from multiple disk air blower, operated at atmospheric pressure and approximately 100 deg F at blower inlet

Speed, rpm	Discharge pressure, in. water at 60 deg F	Volume flow rate, cfm free air
6000	10.5	0
6000	10.0	28
6000	8.0	84
6000	6.0	122
6000	2.0	178
7500	16.0	0
7500	12.0	88
7500	8.0	144
7500	2.0	228
9000	22.0	0
9000	18.0	76
9000	12.0	160
9000	2.0	280
10000	28.0	0
10000	24.0	68
10000	20.0	124
10000	16.0	174
10000	12.0	224
10000	6.0	300
10000	2.0	348

Experimental Multiple Disk Blower and Compressor

A multiple disk blower was constructed using a rotor of 18 disks, each 10-in. diameter, spaced 1/16-in. apart. The disks were mounted on a solid shaft with drilled holes each 1/2-in. diameter drilled parallel to the shaft as air inlet holes. The rotor was inclosed in a housing providing a spiral diffusion scroll around the rotor periphery. As the first machine constructed to determine feasibility of the multiple disk principle for blowers, the blower was crude with respect to mechanical design details and developed excessive vibration at 11,000 rpm, preventing testing above about 10,500 rpm. No housing-to-rotor seals were provided.

Table 2 presents data representative of the performance of the blower. Operation was very stable over the entire usable range and the noise level was exceptionally low for the rotor speeds used. Testing was conducted using a multiple disk air turbine as the motive power for the blower; no shaft power measurements were made, which prevented calculation of the overall efficiency of the blower. Through the use of performance determined for the driving turbine, the highest overall (hydraulic) efficiency of the blower was closely estimated to be 17 percent which occurred at 10,500 rpm and at a flow rate of 140 cfm.

A multiple disk compressor was constructed with a rotor 6.0 inches in diameter, composed of 77 disks each 0.020-in. thick and with 0.020-in. space between adjacent disks. The disks were rigidly mounted to a solid shaft and kidney-shaped air inlet holes were provided near the shaft. The rotor was stiffened with small throughbolts. The aluminum housing was provided with a spiral diffusion scroll at the rotor periphery. Ball bearings were

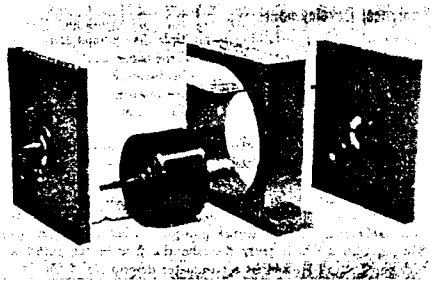

Fig. 4 A disassembled view of an air compressor using a disk rotor

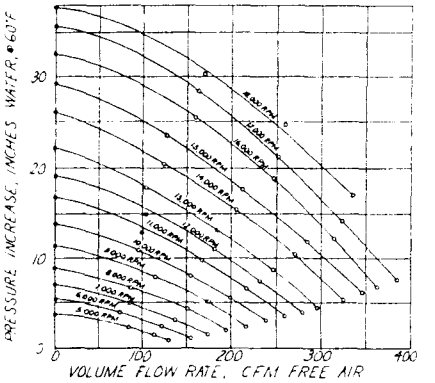

Fig. 5 The pressure-volume flow rate characteristics of experimental multiple-disk air compressor at selected rotational speeds

used with oil-mist lubrication and carbon housing-to-rotor seals were provided. Fig. 4 presents a disassembled view of the compressor.

A multiple disk air turbine was used as the driving unit during testing of the compressor. Fig. 5 presents a summary of the performance data for the machine over the range of speed attained in the test program. No operational difficulties were encountered during approximately 300-hr test time and the noise level was exceptionally low. The air flow was stable over the entire range for which performance is reported.

The compressor was designed to operate at speeds to 30,000 rpm but such speeds were not reached due to limitations of the compressed air supply for the driving air turbine in the test facility. After the collection of the data reported herein, further testing of the compressor was accomplished at higher speeds and included the determination of the power supplied to the compressor enabling calculation of the overall compressor efficiency. The work was not complete at this writing but preliminary results indicate that efficiency increases with shaft speed at every volume flow rate and that efficiency is approximately 23 percent at 20,000 rpm and a volume flow rate of 180 cfm, this being the highest efficiency at that speed. Details of the construction of the compressor, the test techniques, and complete results are contained in a recent thesis [7].

Analytical Development

Several analyses pertaining to multiple disk pumps and compressors have been made in support of the experimental work reported herein. These analyses were in the spirit of a feasibility study and attempted to establish to a reasonable degree the upper limits of performance of such machinery which might be approached by real devices.

The most informative of the analyses, from the standpoint of a compromise between accuracy of the mathematical model and simplicity of the solution, is given here. In the analysis, flow between a single pair of disks is considered; thus it is assumed that the flow in all disk spaces is similar. This condition is reasonably attained in real disk pumps. In lieu of the viscous flow equations which truly describe the flow in the rotor, a "friction factor" is used as a parameter descriptive of the frictional properties of the flow passage, and the factor is assumed to have an average constant value throughout the space between two disks. The use of such a factor has been justified by experimental work [8] conducted on the flow between parallel disks; the assumption of an average constant value of the factor between the disks is somewhat less defensible and is doubtlessly a distortion of fact.

The analysis therefore accounts for the flow conditions within the rotor in a "first-approximation" manner. Further, the analysis considers the performance of an overall pump, including the rotor, inlet passages, and diffuser. In the analytical model, the reversible aspects of all of the pump elements (except the rotor) are considered, that is, no allowance for frictional losses is included in the analysis.

The analysis therefore establishes only the upper limits of performance of a disk pump which can be approached to a greater or lesser degree in real machines according to the losses in the machines due to irreversibility of the inlet and diffusion processes, "windage" frictional losses between the rotor and the housing, and the departure of the rotor flow processes from that represented by the mathematical model of the rotor flow process used. In spite of these facts, the analysis gives insight into the gross features of the flow mechanism and defines the manner in which pump performance varies with geometrical and operational design parameters.

The results are presented in dimensionless form and therefore describe an infinity of geometrical and flow parameter combinations. The results are believed applicable to low pressure blowers as well as liquid pumps but are of doubtful value for compressors in which compressibility effects are significant.

Statement of Idealized Problem

The steady flow of an incompressible fluid is considered, supplied to a pump consisting of two disks with a single flow passage between the disks. The fluid is considered to be accelerated from the stagnation state to a purely radial velocity of whatever magnitude required by continuity to admit the flow into the space between the disks. This acceleration is considered to take place reversibly, and at the expense of the fluid pressure. The tangential component of the fluid at the inner radius of the pump rotor is zero. The fluid is assumed to be admitted uniformly over the inner circumference of the space between the disks.

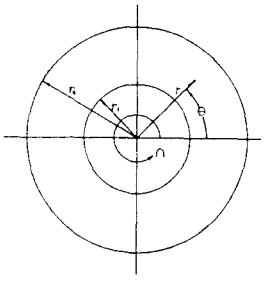

Fig. 6 Plan form of a disk and the coordinate system

The fluid is assumed to occupy the entire space between the disks and to constitute a two-dimensional axially symmetric flow in the plane of the disks. The disks are plane, smooth, parallel, and rotating with constant angular velocity.

At the outer radius, the fluid is assumed to be collected from the rotor and decelerated to a stagnation condition by a reversible diffuser. Fig. 6 depicts the plan form of the disks and a coordinate system.

Analysis

Under the idealizations made the equations of motion for the fluid between the disks can be written. Since all derivatives with respect to time and θ are zero, the components of acceleration are

$$r\text{-direction: } u\frac{du}{dr} - \frac{v^2}{r}$$

$$\theta\text{-direction: } u\frac{dv}{dr} + \frac{uv}{r}$$

Forces on a fluid element are considered to be surface forces only; that is, gravitational forces are ignored. Surface forces are due to pressure gradients and viscous shear stresses.

The force on a fluid element due to the pressure gradient is zero in the θ-direction because of axial symmetry. In the radial direction, the force due to the pressure gradient is found from consideration of a fluid element to be

$$r\text{-direction: } -\frac{dp}{dr}\,br\,d\theta\,dr.$$

Viscous forces are accounted for by expressing the shear stress at surfaces of contact between the fluid and the disks in terms of a constant factor f according to the conventional hydraulic treatment of bulk flow. Other surfaces of a fluid element are considered to contribute no net viscous force on a fluid element. Fig. 7 depicts a fluid element from which can be derived the approximate hydraulic diameter of $2b$. Then, a relation is established between f and τ by equating surface forces due to the stagnation pressure gradient to surface forces due to shear stresses at the boundaries of the flow. Hence,

$$pd\theta rb - (p + dp)d\theta rb + 2\tau rd\theta dr = 0$$

or

$$dp = \frac{2\tau dr}{b} \tag{1}$$

By definition,

$$\frac{dp}{\rho} = \frac{f[(v - \Omega r)^2 + u^2]dr}{4b} \tag{2}$$

where the bracketed term is the velocity of the fluid relative to the disks, squared. The factor f is assumed constant throughout the disk system. Combining equations (1) and (2) yields

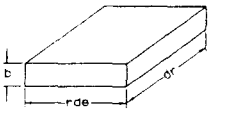

Fig. 7 An element in the flow

$$\tau = \frac{f\rho}{8}[(v - \Omega r)^2 + u^2]$$

as the relation between the shear stress and the factor f.

The force on a fluid element due to shear stress becomes, in the direction opposite that of the relative velocity,

$$\frac{2f\rho}{8}[(v - \Omega r)^2 + u^2]rd\theta dr$$

which has the components

$$r\text{-direction: } -\frac{f\rho}{4}\frac{[(v - \Omega r)^2 + u^2]urd\theta dr}{[(v - \Omega r)^2 + u^2]^{1/2}}$$

$$= -\frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}urd\theta dr$$

$$\theta\text{-direction: } -\frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}(v - \Omega r)rd\theta dr.$$

The developed forces and accelerations are now inserted into Newton's law to obtain the scalar equations

$$-\frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}(v - \Omega r)rd\theta dr$$

$$= \rho brd\theta dr\left[u\frac{dv}{dr} + \frac{uv}{r}\right]$$

and

$$-\frac{dp}{dr}brd\theta dr - \frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}urd\theta dr$$

$$= \rho brd\theta dr\left[u\frac{du}{dr} - \frac{v^2}{r}\right]$$

in the θ and r-directions, respectively. These simplify to

$$\frac{dv}{dr} + \frac{v}{r} + \frac{f(v - \Omega r)}{4bu}[(v - \Omega r)^2 + u^2]^{1/2} = 0 \tag{3}$$

and

$$\frac{1}{\rho}\frac{dp}{dr} + u\frac{du}{dr} - \frac{v^2}{r} + \frac{fu}{4b}[(v - \Omega r)^2 + u^2]^{1/2} = 0 \tag{4}$$

These are the equations of motion for the fluid flow between the disks.

Solution of the Equations of Motion

Equations (3) and (4) were solved through use of an IBM 704 digital computer. In order to retain generality, the equations were first rendered dimensionless through introduction of x and y as the dimensionless distance and dimensionless θ component of velocity, respectively. Further, the continuity statement

$$u = +\frac{Q}{2\pi br_i x}$$

was used to eliminate the radial velocity component from the equations. The resulting equations of motion in dimensionless form are

$$\frac{dy}{dx} + \frac{y}{x} + \frac{fr_i}{4b}\left[1 + \left(\frac{2\pi br_i\Omega}{Q}\right)^2 x^2(y - x)^2\right]^{1/2} \times (y - x) = 0 \tag{5}$$

and

$$\frac{1}{\rho\Omega^2 r_i^2}\frac{dp}{dx} - \left(\frac{Q}{2\pi br_i\Omega}\right)^2\frac{1}{x^3} - \frac{y^2}{x} + \frac{fr_i}{4b} \times \left(\frac{Q}{2\pi br_i\Omega}\right)^2\frac{1}{x^2}\left[1 + \left(\frac{2\pi br_i\Omega}{Q}\right)^2 x^2(y - x)^2\right]^{1/2} = 0 \tag{6}$$

Equations (5) and (6) were presented to the digital computer with the boundary condition $y(0) = 0$. The equations were solved for all possible combinations of the dimensionless parameters given in Table 3. In each solution, x was considered as the

Table 3 Dimensionless variables and values selected for calculations

f	r_i/b	$Q/\Omega r_i^3$
0.01	10	0.002
0.02	50	0.010
0.04	100	0.100
0.06	200	1.00

independent variable and values of y and of $\Delta P_i/\rho\Omega^2 r_i^2$ were tabulated versus x, where ΔP_i is the accumulated pressure change in the flow between the disks as obtained by the solution of equation (6).

Pump Performance Parameters

An expression for the pressure change of the fluid due to reversible acceleration of the fluid from the inlet stagnation state to the necessary radial velocity to enter between the disks is written from conventional one-dimensional hydraulics as

$$\Delta P_a = -\frac{\rho}{2}\left(\frac{Q}{2\pi r_i b}\right)^2$$

since $u_i = Q/2\pi r_i b$ from continuity. In dimensionless form

$$\frac{\Delta P_a}{\rho\Omega^2 r_i^2} = -\frac{1}{2}\left(\frac{Q}{2\pi b r_i^3\Omega}\right)^2.$$

At the rotor discharge the pressure change of the fluid due to reversible diffusion to the outlet stagnation state is written in the same way as

$$\Delta P_d = \frac{\rho}{2}\left[\left(\frac{Q}{2\pi b r_i}\right)^2\frac{1}{x^2} + (\Omega r_i)^2 y^2\right]$$

since $\qquad v_o = \Omega r_i y$ and $u_o = Q/2\pi b r_i x$

In dimensionless form,

$$\frac{\Delta P_d}{\rho\Omega^2 r_i^2} = \frac{1}{2}\left[\left(\frac{Q}{2\pi b r_i^3\Omega}\right)^2\frac{1}{x^2} + y^2\right].$$

The addition of the dimensionless pressure changes through the inlet, the diffuser, and the rotor [the last obtained from the solution of equation (6)], at each value of x, yields a value of $\Delta P_i/\rho\Omega^2 r_i^2$. This is the dimensionless pressure change through the entire pump and was computed at every value of x, for every combination of the variables in Table 3.

In order to form an efficiency expression for the pump, the ideal work per unit mass is first written as

$$W_{ideal} = -\frac{\Delta P_i}{\rho}$$

This ideal work is then rendered dimensionless to get

$$\frac{W_{ideal}}{\Omega^2 r_i^2} = \frac{-\Delta P_i}{\rho\Omega^2 r_i^2}.$$

The actual work required to drive the pump follows from the torque

$$T = -Q\rho\Omega r_i^2 y/x$$

written from considerations of angular momentum. Since the work is the product of torque and angular velocity, it becomes, for a unit mass,

$$W_{actual} = -\Omega^2 r_i^2 x y$$

or in dimensionless form

$$\frac{W_{actual}}{\Omega^2 r_i^2} = -xy.$$

Now the efficiency is defined as

$$\eta = \frac{W_{ideal}}{W_{actual}} = \frac{W_{ideal}/\Omega^2 r_i^2}{W_{actual}/\Omega^2 r_i^2} = \frac{-\Delta P_i/\rho\Omega^2 r_i^2}{-xy} = \frac{\Delta P_i/\rho\Omega^2 r_i^2}{xy}.$$

Values of the torque and of the efficiency were computed at each value of x.

Results of Analysis

The complete results of the computations consist of one data sheet for each possible combination of the variables listed in Table 3. Each sheet presents x as the argument, ranging from $x = 1$ to $x = 5$ with the resulting values of y, $\Delta P_i/\rho\Omega^2 r_i^2$, and the dimensionless torque function. These results are too voluminous to be reproduced here [9].

A single representative case, which is typical of the basic results obtained, is given in Fig. 8, plotted for

$$f = 0.04, \quad r_i/b = 50, \quad \text{and} \quad Q/\Omega r_i^3 = 0.010.$$

In all cases the maximum value of η occurs at the maximum outer radius for which the solution was obtained, namely, at $x = 5$. Therefore, the characteristics of a pump of fixed geometry, $x = 5$, are presented in the curves of Figs. 9 through 24. These curves constitute the useful results of this study and enable the design of a disk pump with any characteristics within the inherent capability of the disk-type rotor. The results, of course, pertain only to a pump with no losses external to the rotor. Available conventional loss information for inlet, diffuser, pump housing, and bearings must be applied to predict actual pump performance.

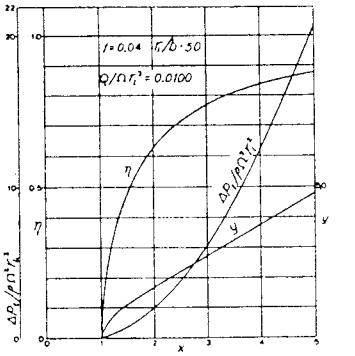

Fig. 8 Typical curves of efficiency, velocity ratio, and pressure change across the pump as a function of x. Plotted for $f = 0.04$, $r_i/b = 50$, and $Q/\Omega r_i^3 = 0.010$.

In an actual pump, many disk spaces must be provided to yield a pump with a prescribed volume flow rate. The necessary number of disks can be computed from the value of $Q/\Omega r_i^3$ for a single disk space, which is the abscissa of the curves presented.

Consideration of the curves shows that the efficiency and the pressure rise through the pump decrease with increasing values of the volume flow rate parameter $Q/\Omega r_i^3$. In fact, there is a value of the flow rate parameter at which the efficiency and pressure rise are zero. For flow rates greater than this there is a pressure drop across the pump which must be supplied externally. This is, of course, not a useful pump condition. In this case, shaft power must still be supplied to the pump to maintain the rotor at the prescribed angular velocity.

In the useful range of the flow rate parameter, both the pressure rise and the efficiency increase as the flow rate decreases. The efficiency approaches 1.00 as the flow rate approaches zero.

Performance curves showing pressure rise as a function of volume flow rate, for a given fluid and a pump operating at constant speed, can be constructed from the information in Figs. 9 through 24. Such curves should compare favorably with the performance curves for conventional pumps with similar inlet and diffuser losses. A very wide range of specific speed can be attained with the single rotor configuration.

Discussion

From the results of the analysis and experimentation, it is possible to predict the probable performance of multiple disk pumps and blowers over a wide range of the use spectrum. However, the possible variety of uses prohibits such considerations here; each prediction would be a study in itself.

The results show that multiple disk pumps, blowers, and compressors are workable, feasible in the engineering sense, and are worthy of further investigation. Eventual competition of multiple disk turbomachinery with conventional turbomachinery over the majority of the possible use spectrum is not indicated by the established results. However, the performance and the efficiency of disk turbomachinery may be expected to remain almost constant as the machines are made physically very small, in contrast to the rapid decrease of efficiency of conventional turbomachinery with decrease in size. This suggests that the multiple disk pump may eventually prove advantageous for very small volume pumping applications.

Further, the results allow the conclusion that disk pumps may have an advantage in the pumping of certain exotic fluids having density and/or viscosity far from that of water. For any such fluid, a combination of geometry and flow parameters exists for a multiple disk pump to satisfactorily handle the fluid; it remains to be seen whether or not the disk pump can handle fluids and powders that are now untenable for conventional pumps, but the possibility suggests itself. Eventual applications seem most likely to be made in missile and space-age systems, and in other special circumstances in which conventional pumps, blowers, and compressors are inadequate.

Experience with the experimental machines indicates that the first cost of multiple disk turbomachinery is low compared with that of conventional devices, and that they can be manufactured in a modest machine shop. This feature may encourage the use of such machines even in cases in which there is no performance or efficiency advantage; for example, automotive, water, fuel, and oil pumps, as the blower in automotive superchargers, and in aircraft and automotive refrigeration systems

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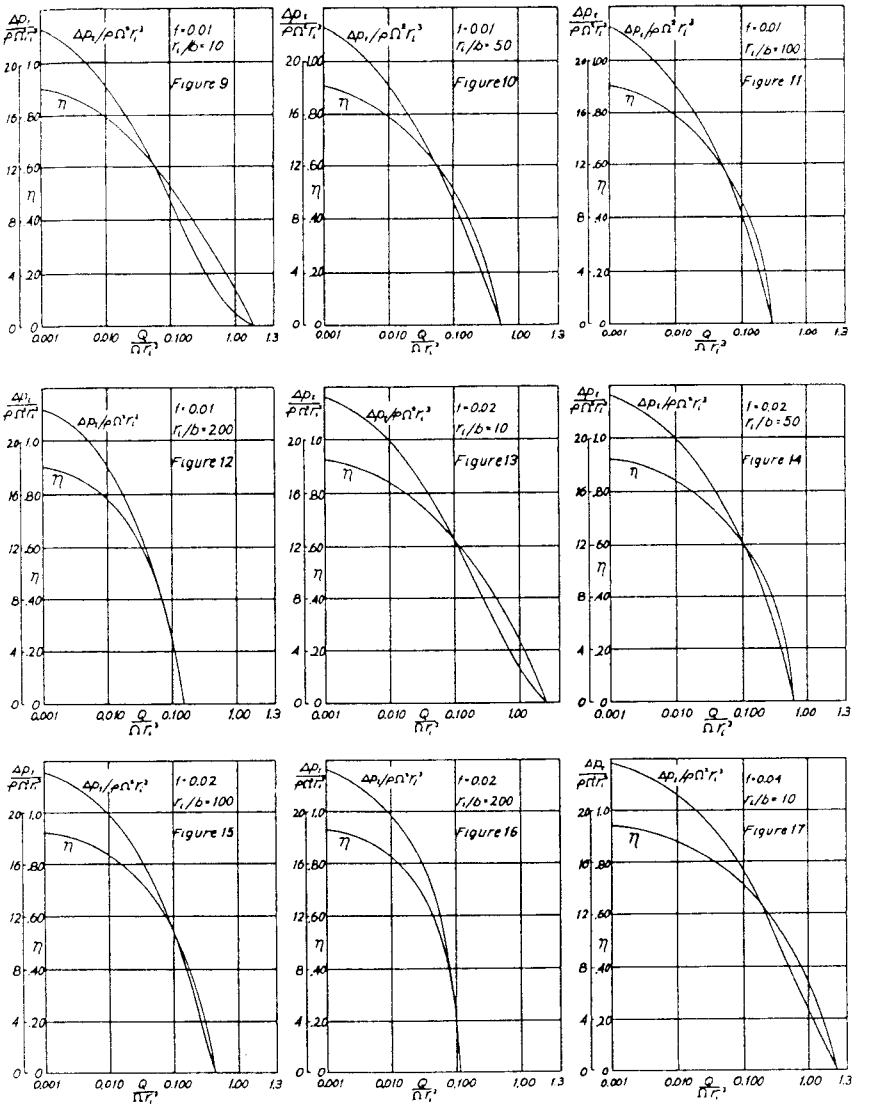
f = 0.01 r_i/b = 10 Figure 9
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f = 0.01 r_i/b = 100 Figure 11
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f = 0.02 r_i/b = 50 Figure 14
f = 0.02 r_i/b = 100 Figure 15
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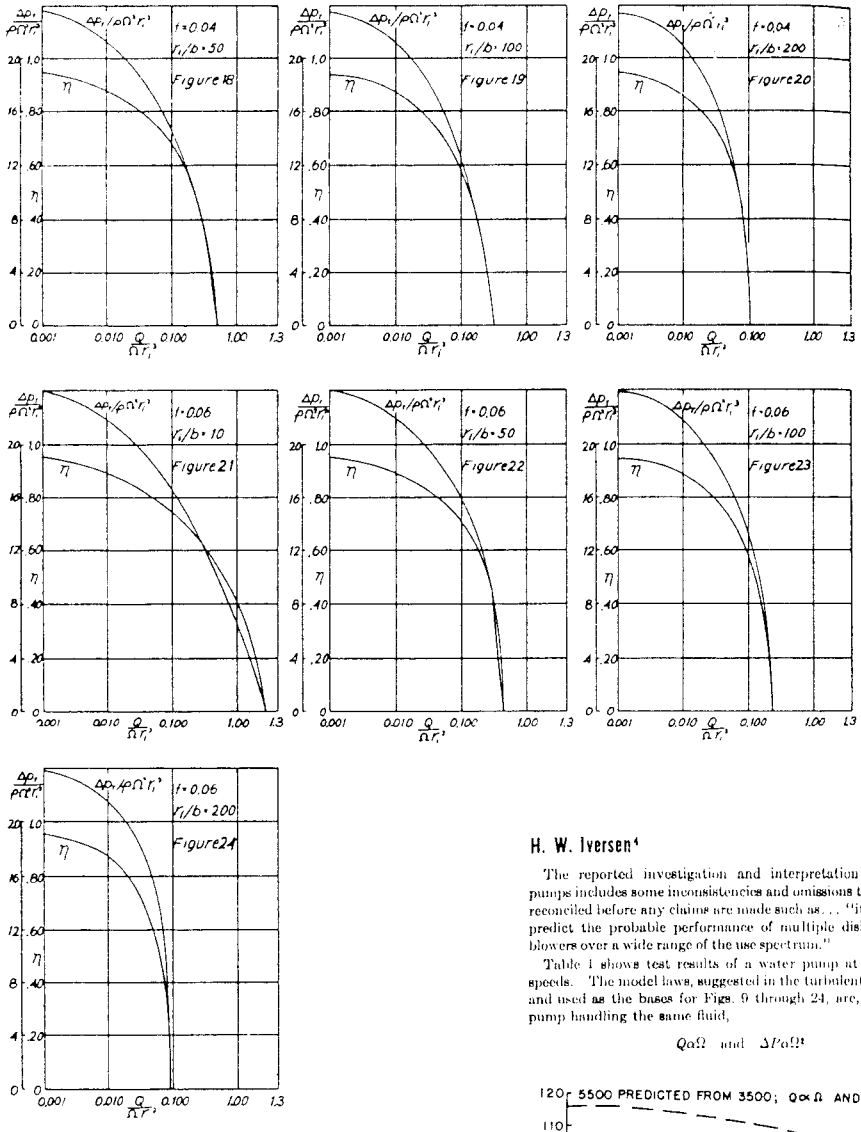
f = 0.04 r_i/b = 10 Figure 17
η
Q/Ωr_i³

Figs. 9 through 17



Figs. 18 through 24

Mr. Tom Lahey of the Center for preparing computer programs and conducting the computations.

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8 Philip Underwood, "Frictional Characteristics of a Radially Outward Air Flow Between Parallel Smooth Discs," 1959, Department of Engineering Science, Arizona State University, Tempe, Ariz. (unpublished).

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DISCUSSION

P. R. Cobb[3]

The author is to be complimented on a fine paper which combines both an analytical and experimental approach to the investigation of the problem. It would be in order for him to discuss further the sensitivity of the analysis of the variability of the friction factor f. The assumption that it is a constant is one of the major assumptions of the analysis. It is hard to determine the effect of changing this parameter from the limited number of curves shown in the paper.

An interesting application of this device has recently appeared in the form of the drive motor for the dentist's drill. The device is 6 in. long and $\frac{3}{4}$ in. in dia and turns at 500,000 rpm for a supply of pressure of 85 psi. In addition to using the air as an energy source for driving the motor, the air supply is also used for air bearings. This solves the bearing problem which can become quite nasty with these high speed devices. It is an interesting application to find a common substance for motive force and bearing material.

H. W. Iversen[4]

The reported investigation and interpretation of the disk pumps includes some inconsistencies and omissions that should be reconciled before any claims are made such as... "it is possible to predict the probable performance of multiple disk pumps and blowers over a wide range of the use spectrum."

Table 1 shows test results of a water pump at two different speeds. The model laws, suggested in the turbulent flow analysis and used as the bases for Figs. 9 through 24, are, for the same pump handling the same fluid,

$$Q \propto \Omega \quad \text{and} \quad \Delta P \propto \Omega^2$$

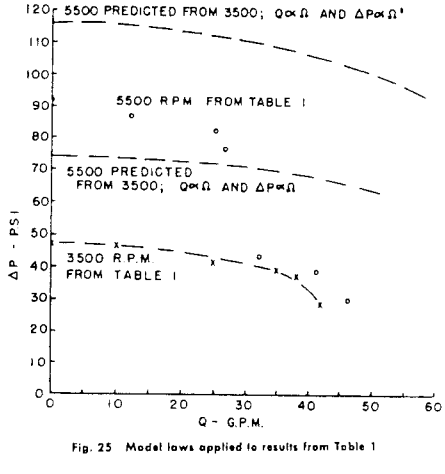
5500 PREDICTED FROM 3500; Q∝Ω AND ΔP∝Ω²
5500 R.P.M. FROM TABLE 1
5500 PREDICTED FROM 3500; Q∝Ω AND ΔP∝Ω
3500 R.P.M. FROM TABLE 1
ΔP - PSI
Q - G.P.M.

Fig. 25 Model laws applied to results from Table 1

When these are applied to the results of Table 1, using the results at 3500 rpm (which appear to show a more consistent trend than those at 5500 rpm) to predict the performance at 5500 rpm the comparison is shown in Fig. 25. Two conclusions may be made: (*a*) The reported test results are not consistent; and (*b*) the author's proposed analysis does not fit the particular combination and range of conditions for the results of Table 1. Two obvious factors may have contributed to this inconsistency. One, as mentioned by the author, is cavitation, the other is a dependence of the friction factor on the flow and geometry—a Reynolds number dependence that is not shown in the simple model laws as previously stated. In fact, some of the test results may have been with partial laminar flow instead of with turbulent flow and model laws which then are

$$Q \propto \Omega \quad \text{and} \quad \Delta P \propto \Omega$$

However, when the turbulent flow model laws for speed changes are applied to the results of Table 2 and of Fig. 5, a much better agreement is found in which the differences between prediction and measured performance at different speeds possibly may be rationalized on the basis of experimental error.

In order to partially substantiate the validity of the numerous "design" curves, Figs. 9 through 24, the author should have made a direct comparison between the actual measured performance of the units tested and the proposed design functions. Without this comparison, and with the inconsistencies previously described, the design curves are of doubtful value.

Figs. 9 through 24 contain two errors. The parameter for pressure rise is shown with an r^2 in the denominator. This should be r^4 according to the proposed analysis. The abscissa is a log scale, an awkward scale to use in the proposed "design" context and a misleading scale with respect to the general shape of the parameter curves as compared to the usual linear scale used for most pump performance graphical presentation. In addition, the abscissa of some of these figures contains a value of 1.3 at the extreme right of each figure. This value should be approximately 4.0 instead of 1.3.

One feature of the analysis needs clarification. The tangential velocity relative to the impeller is used, namely $(v-\Omega r)$, as one component of the total average velocity. While the sign of this term is immaterial in the sign and magnitude of the square of the term, it is of vital importance in the force balance of equation (3), where it appears as the first power. In agreement with a fixed reference frame for the force balance, the tangential velocity relative to the impeller should be $(\Omega r - v)$. If this is the case, then the proposed solutions to equations (3) and (4) are in error.

F. Kreith[5]

The author has presented interesting data on the performance of one small multiple disk pump and has also attempted to predict the performance of an idealized model of such a device analytically. In order to assess the validity of the assumptions made in the analysis, and thereby the significance and reliability of the predicted results, it may be of interest to review some related experiments.

The author assumes in his analysis that the flow between any two adjacent disks is steady and two-dimensional axially symmetric in planes parallel to the disk surface. This would seem to imply that the radial velocity profile is equivalent to slug flow, with the radial velocity vector decreasing with the reciprocal of the radial distance r in accordance with the requirement of continuity. At the same time it is assumed that a surface shear force acts on the fluid, but does not distort the assumed velocity profile.

To assess the credibility of the flow model assumed by the author, Fig. 26 of this discussion shows experimentally measured radial velocity profiles at a mass-flow rate of 0.04 lb/sec of air at normal temperature and pressure between two stationary disks of 8 in. dia, 0.25 in. apart, with a central inlet eye of 1 in. dia in one of the two disks. From these measurements, taken at the University of Colorado by means of a hot wire, it can be seen that

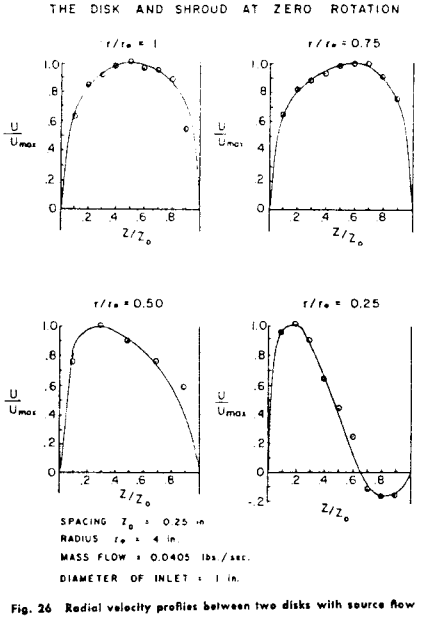
VELOCITY PROFILES OF THE FLOW BETWEEN THE DISK AND SHROUD AT ZERO ROTATION
r/r₀ = 1
r/r₀ = 0.75
r/r₀ = 0.50
r/r₀ = 0.25
U/Umax
Z/Z₀
SPACING Z₀ = 0.25 in
RADIUS r₀ = 4 in
MASS FLOW = 0.0405 lbs/sec
DIAMETER OF INLET = 1 in

Fig. 26 Radial velocity profiles between two disks with source flow

at a radial distance of 1 in. $(r/r = 0.25)$ the radial velocity profile is skewed and there appears to be some reverse flow near the face opposite to the inlet eye. The radial velocity profile near the rim appears to approximate a parabola under the test conditions. It was also observed that there is a qualitative difference in the flow characteristics between the disks as the flow changes from laminar to turbulent flow.

Although the flow conditions under which the velocity profiles described previously were obtained differ from those for which the analytical model is intended, the data suggest that an experimental investigation of the velocity profile between adjacent disks in a disk pump should be undertaken in order to verify the assumptions made in the analysis. Work along this line is presently being planned at the University of Colorado.

In a research program on a related problem, namely the pumping action of a single disk rotating between stationary shrouds with central eye inlets, it has been observed [10, 11][6] that the

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[4] Professor of Mechanical Engineering, University of California, Berkeley, Calif. Mem. ASME.

[5] Visiting Professor of Engineering, University of California, Los Angeles, Calif. Mem. ASME.

[6] Numbers in brackets designate Additional References at the end of this discussion.

flow induced by a rotating disk may not always be steady and that the pressure loss in the inlet section may be appreciable. It would, therefore, be of interest to know whether or not periodic flow as observed in reference [10] with a single disk pump can also occur in a multiple disk pump and what the order of magnitude of the inlet loss is. The assumption made in the analysis that "the flow acceleration from the inlet stagnation state to the radial velocity necessary to enter between the disks is reversible," may be subject to considerable error in most practical designs which will involve turning the flow through ninety deg and passing the fluid through a restriction in the inlet line. This condition may also lead to cavitation. Moreover, experiments reported in reference [12] indicate that in addition to the ratio of the disk radius to the clearance space, also the thickness of the disk rim may influence the flow characteristics of a multiple disk pump. The empirical friction factor f could, of course, be adjusted so as to achieve a satisfactory correlation of test results, but in order to make a rational estimate of the performance of a multiple disk pump or compressor it will be necessary to calculate the actual value of f from the system geometry and operating conditions. Although disk pumps and compressors may have certain practical applications, it appears that an evaluation of the efficiency of an actual device cannot be made on the basis of the material presented in this paper, but must await additional experiments and analyses.

Additional References

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11 F. Kreith, E. Doughman, and H. Kozlowski, "Mass and Heat Transfer From an Enclosed Rotating Disk With and Without Source Flow," *Journal of Heat Transfer*, TRANS. ASME, Series C, vol. 85, 1963, pp. 153–163.

12 P. D. Richardson, "Studies of the Flow and Heat Transfer Associated With a Rotating Disk," PhD thesis, University of London, London, England, 1958.

Author's Closure

The attention given the paper by the discussers is appreciated by the author.

The errors in Figs. 9 through 24 pointed out by Professor Iversen are due to negligent checking of the drawings by the author and are much regretted. The use of a log scale for the abscissa does not seem misleading or awkward to the author. The signs as given in equation (3) and the solutions to equations (3) and (4) are correct. As a check of the matter of signs in equation (3), the first two terms are positive quantities while the third term is a negative quantity, as required for satisfaction of the equation.

One would not expect the model laws applied by Prof. H. W. Iversen to correlate the test data since they apply only to the idealized model. The very large losses at the inlet, in the diffuser, at the rotor edge, and in the bearings of the actual machine cause it to follow much modified laws.

The investigation reported was designed to determine the feasibility of multiple disk turbomachinery; practically no information was available concerning this question. While fully recognizing the weaknesses of the analysis and the meagerness of the experimental data, the author feels that together they provide a considerable aid to the "design" of further experimental devices. In the design of multiple disk turbomachinery, one is not trying to optimize but rather is trying to originate a configuration which will perform the intended function well enough to enable one to learn more about it.

The data and remarks presented by Prof. F. Kreith indicate some of the ways in which future analyses can be made to better approximate real machine performance. The author hopefully expects the appearance of experimental and analytical information concerning multiple disk devices which will enable engineering applications of them. The first sentence of Prof. F. Kreith's discussion corresponds with the spirit in which the paper was presented.

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An Analytical and Experimental Investigation of Multiple-Disk Turbines

Journal of Engineering for Power p. 29–36 JANUARY 1965

An unconventional type of turbomachinery employing a rotor composed of smooth disks is described and the history and motive principle of such devices are briefly discussed. Several experimental multiple-disk air turbines are described and data showing the performance and efficiency of the turbines are presented. A mathematical model of a multiple-disk turbine is described. The equations of motion are written for the flow of an incompressible fluid between the disks, and parameters describing the efficiency and performance of the turbine, in the absence of losses external to the rotor, are derived. The solution for the equation of motion and the performance parameters, obtained by computation with an IBM 704, are summarized, discussed, and referenced. The results indicate that multiple-disk turbines may be attractive in the low-power part of the turbine spectrum, or where first cost is a prime factor, or where the properties of the working fluid cause conventional turbines to be inadequate.

Table 1 Representative performance data from first disk turbine, operated with exhaust to atmosphere, air-inlet temperature of approximately 100 F, and using 15-deg nozzles consisting of drilled holes

Pressure to turbine, psig	Mass rate of flow, lb$_m$/min	Speed, rpm	Power, hp	Efficiency, percent
100	4.91	9400	1.12	16.5
105	5.76	8000	1.36	17.5
105	5.76	10000	1.48	18.8
110	5.86	8000	1.39	17.2
110	5.86	10000	1.31	16.3
125[a]	4.82	10000	1.68	23.2

[a] With flow-limiting inserts in nozzles.

Nomenclature

- b = distance between adjacent disks
- f = friction factor, further defined in context
- p = pressure
- Δp_n = pressure change across nozzles
- Δp_r = pressure change across rotor, from periphery to position x
- Δp_t = $\Delta p_n + \Delta p_r$, total-pressure change across nozzles and rotor
- Q = volume flow rate
- r = radial position coordinate, from center of disks
- T = torque
- u = velocity component in r-direction
- v = velocity component in θ-direction
- W = work per unit mass
- x = r/r_0, dimensionless radial position coordinate
- y = v/v_0, dimensionless velocity component in θ-direction
- η = efficiency, further defined in context
- Ω = angular velocity of disks
- ρ = density of fluid
- τ = shear stress, further defined in context
- θ = angular position coordinate

Subscript 0 denotes value at outer radius

Introduction

In conventional turbomachinery, momentum exchange between the fluid and the rotor is effected by providing blades or shaped passages to deflect and constrain the flow in the rotor. The resulting force is from the pressure distribution over the surface of the rotor. In some turbomachinery, however, such as the conventional drag turbine, force on the rotor is due to shear stress over the rotor surface and no constraining blades or passages are present.

There exists a type of turbomachinery rotor essentially different from conventional bladed or vaned rotors and also very different from the conventional drag-turbine rotor, both in configuration and in effects. This is the multiple-disk rotor, consisting of a number of thin, smooth, flat, parallel disks arranged normal to a shaft and fastened rigidly to it with small spaces between the disks. Holes or slots are provided in the disks near the shaft. This type of rotor, with appropriate housings, can be used to form hydraulic and gas turbines, hydraulic pumps, and gas compressors. A view of such a rotor is presented in Fig. 1. A schematic diagram of a turbine using a disk rotor is given in Fig. 2. The rotor is enclosed in a housing provided with bearings, housing-to-rotor seals, and fluid-injection nozzles. Fluid enters the turbine through the nozzles and is injected into the spaces between the disks in a direction approximately tangential to the rotor periphery. The fluid follows a spiral path between the disks and finally exhausts from the rotor through the holes or slots near the shaft. During passage through the rotor the fluid exerts shear stress on the disk surfaces, giving rise to a force on the rotor resulting in a net torque at the turbine shaft. In so doing, the fluid loses energy and exhausts from the turbine at an energy state lower than that at which it entered the turbine nozzles.

The origins of the idea for such a rotor are somewhat obscure. The first definitive United States patents involving the rotor were issued to N. Tesla [1][1] in 1913, covering both turbine and pump applications of the rotor. The inventor did not seriously attempt to exploit the devices and the principle has received little attention and published engineering investigation. A public demonstration of a disk turbine was given by Tesla prior to the issue of patents and was widely reported in the semitechnical press of that time [2, 3]. Several writers offered explanations of the motive principle and brief analyses of the flow process within the rotor [4, 5]. More recently, several investigators have reported experimental data from multiple-disk turbines [6, 7].

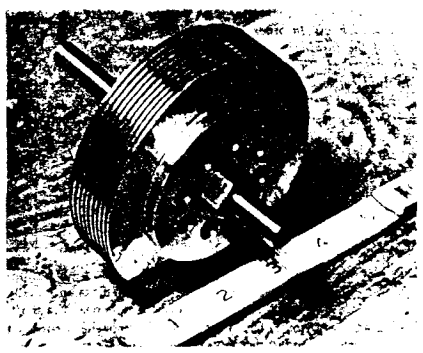

Fig. 1 Typical rotor of a disk-type turbomachine

A continuing analytical and experimental investigation of multiple-disk turbomachinery was begun in 1958 by the author with the objectives of determining the feasibility of such machinery. The investigation resulted in the design, construction, and testing of a variety of experimental air turbines and air compressors, and a liquid pump using multiple-disk rotors.[2]

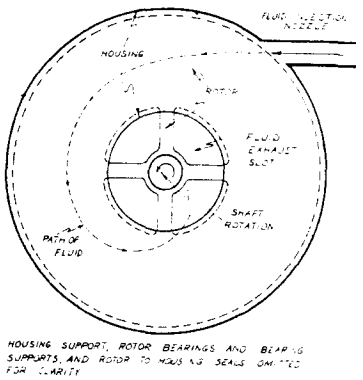
FLUID INJECTION NOZZLE
HOUSING
ROTOR
FLUID EXHAUST SLOT
SHAFT ROTATION
PATH OF FLUID
HOUSING SUPPORT, ROTOR BEARINGS AND BEARING SUPPORTS, AND ROTOR TO HOUSING SEALS OMITTED FOR CLARITY

Fig. 2 Schematic diagram of turbine using a disk rotor

Analyses of disk turbomachinery were also performed in the course of the investigation. The equations of motion of the fluid while between the disks were written and solved for the case of an incompressible fluid in the turbine mode of application. The results describe the performance and efficiency of the idealized multiple-disk turbine for any incompressible fluid, speed, flow, rate, rotor geometry, and size in terms of dimensionless performance parameters.

All of the machines constructed performed the functions for which they were intended. The test results and the analyses indicate that multiple-disk turbomachines are feasible, may offer attractive performance in certain applications, and are worthy of further investigation. This paper presents the results of testing the several multiple-disk turbines and the most informative and useful of the analyses.

Experimental Multiple-Disk Turbines

Six disk turbines were constructed, of which the three most interesting are described here.

The first turbine constructed was designed from the inventor's descriptions, before the results of the analytical portion of the investigation were available. The turbine operated on compressed air, exhausting to the atmosphere. The rotor was 7 in. dia, composed of 9 disks each $^3/_{32}$ in. thick with $^1/_{16}$-in. spacing between the disks. Each space between disks was supplied individually with air through a single nozzle directed at an angle of 15 deg from the tangential direction. The nozzles were placed in two groups at opposite sides of the rotor to produce the effect of a pure couple on the rotor, with no axial or radial load imposed on the bearings due to the flow.

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[1] Numbers in brackets designate References at end of paper.

[2] The investigation reported herein was sponsored by Texas Instruments Incorporated, Dallas, Texas.

The turbine was built entirely of mild steel, using ball bearings with no provision for continuous lubrication. The air exhausting from the rotor was collected from the bearing housings into a single exhaust pipe.

Some of the results obtained from the testing of this turbine are given in Table 1, in which the test conditions are included. In the table and throughout the later reported results, the efficiency of a turbine is defined as

$$\eta = \frac{\text{actual shaft work per unit mass}}{\text{ideal work per unit mass}}$$

in which the ideal work is that of an isentropic expansion from the actual turbine-inlet temperature and pressure to the actual turbine-exhaust pressure, with zero velocity assigned the flow at the beginning and end of the isentropic expansion. The data reported in Table 1 pertain to the first turbine constructed, operated with nozzles consisting of drilled holes.

The same turbine was later operated with nozzles especially designed to produce supersonic nozzle exit velocities with reasonably uniform flow fields. Further, various nozzle angles were used in subsequent testing of this turbine. In no case did these modifications improve the turbine performance although the performance indicated in Table 1 was equalled with several of the modified nozzle arrangements [8, 9].

The same turbine was further modified by increasing the number of disks to 11 and reducing the spaces between the disks to 0.040 in., holding the overall rotor width essentially the same as in the original turbine. Table 2 presents some of the results of testing of the turbine with the modified rotor.

The analyses performed as a part of this investigation indicate that disk turbines may be designed and operated over a range extending from an almost pure impulse type of operation to an almost wholly reaction type of operation. A comparison of the operational parameters for the first turbine with the corresponding portions of the analyses revealed that the turbine was an almost pure impulse turbine, with a large pressure drop in the nozzles and a very small pressure drop in the rotor. The comparison also revealed that the geometry, flow rate, and speed combination used in the turbine were not near those indicated by the analyses for optimum turbine efficiency.

Consequently, four other air turbines were constructed, progressively incorporating the experience gained in earlier versions and the quantitative information provided by the analyses. The last and most successful of these turbines is referred to here as the second turbine (for which performance is reported). Figs. 3 and 4 present views of this turbine.

The second turbine used a rotor of 8 in. dia composed of 21 disks each 0.020 in. thick and spaced 0.020 in. apart. The outer-edge diameter of the exhaust slots was 1.32 in. The turbine was supplied with air through two slots extending the width of the rotor and arranged at opposite sides of the rotor to maintain a pure couple loading. Thus each space between the disks was supplied with air from two slots simultaneously.

The slots directed the air into the rotor at an angle of 20 deg from the tangential direction. The turbine was provided with ball bearings and with an oil-mist lubrication system. It was further provided with carbon-ring seals between the housing and the rotor to reduce air leakage around the rotor. Four very small through-bolts were used in the rotor at about midradius to maintain correct disk spacing and prevent disk flutter. The turbine was constructed of mild steel except that the disks were of stainless steel.

The turbine was tested exhaustively over a period of about 200 hr of running time. In testing, compressed air was supplied to the turbine with exhaust to the atmosphere. Figs. 5 and 6 present some of the test results obtained with the pertinent test conditions indicated.

The turbine was mostly impulsive but partly reactive in operation as indicated both by comparison with the analyses and by direct measurement of the pressure distribution within the turbine. Limitations of test facilities prevented performance measurements in some of the most interesting areas of the performance map, but the results presented establish the current performance level of multiple-disk turbines. It is a certainty that more efficient multiple-disk turbines, of the same power, can be built of smaller size, to operate at much higher speeds.

The power output of the turbines described was purposely limited by the extremely small nozzles and airflow rates, since the multiple-disk turbine is likely to find its first use in the low power output-high energy input applications. As defined in the nomenclature of conventional turbines, the disk turbines reported herein had an admission of a fraction of 1 percent. If desired, the power output of any of the turbines reported could be increased manyfold simply by increasing the total airflow rate.

The third disk turbine described here was a special purpose turbine designed specifically to demonstrate one of the possible variations of this and other turbomachinery, not so far exploited extensively. The effect desired, and obtained, was the separation of an air supply at high pressure and room temperature into two streams, each at lower pressure, but at two temperatures, one above and one below the temperature of the air supply.

The third disk turbine was essentially two separate disk-turbine rotors on a single shaft, in a single housing. In this case the shaft was hollow, as an additional experimental variation of practical disk-turbomachinery geometry. One of the rotors was provided with nozzles tending to rotate the shaft in one direction, while the other rotor had nozzles tending to cause rotation in the other direction. Separate air-supply controls and separate exhaust systems for the rotors were provided (the latter being the hollow shafts extending from the machine). In operation, the rotors were supplied with air at different pressures (or the same pressures, with different flow rates, alternatively). The entire system of rotors then rotated in a preferred direction, with one rotor producing and the other absorbing work. The speed and exhaust temperatures were determined by the increase or decrease of air to the work-absorbing rotor. Speeds to 40,000 rpm were obtained in testing and the temperature difference earlier mentioned was obtained.

Analytical Development

The experimental machines described being air turbines, a proper analysis of the flow within the rotor would require the solving of the equations descriptive of the flow of a viscous compressible fluid within the geometry of the space between the disks. While many approaches to such an analysis can be made, a rather crude idealization of the real flow problem was chosen as the mathematical model, in keeping with the "feasibility" nature of the investigation. The mathematical model on which the analysis is based is a compromise between the need for accurate description of the problem and the need for manageable, interpretable results.

Consequently, the real multiple-disk turbine is replaced in the analysis with a mental model consisting of a single pair of disks, supplied with fluid through reversible nozzles, operating with no friction between the rotor and housing or at the bearings and seals, and exhausting to a stagnant atmosphere through a reversible diffuser. Fluid friction is accounted for only within the disk spaces, where it gives rise to the motive mechanism. Between the disks, the frictional characteristics of the system are described through the use of a "friction factor" assumed to be constant throughout the passage. Lastly, the driving fluid is assumed to be incompressible.

Such an analysis is not expected to correspond closely with the real air-turbine performance earlier presented. However, the analysis indicates the approximate upper limits of performance and efficiency possible in multiple-disk turbines and displays the manner in which the performance and efficiency vary with turbine geometry and operational parameters. Further, the analysis gives some insight into the nature of the flow processes within the rotor and the range of phenomena encountered over the operational spectrum of multiple-disk turbines.

It has been demonstrated [8] that the analysis can be used to enable the prediction of the performance and efficiency of multiple-disk turbines when used with conventional "loss" data for the components of a turbine other than the rotor, provided that compressibility effects are not large. Such procedure is lengthy, as with conventional turbomachinery, and is not discussed further.

Analytical Problem Statement

The steady flow of an incompressible fluid is considered, supplied from stagnation conditions through one-dimensional frictionless nozzles to the spaces between rotating disks. The fluid is supplied uniformly at the outer radius of the disks, approximately tangentially (a radial component is required to introduce the flow between the disks). The fluid occupies the entire space between the disks and is considered as two-dimensional in the plane of the disks and axially symmetric. The disks are smooth, plane, parallel, and rotating with constant angular velocity. The entire rotor is contained within a housing which is considered to offer no restraining torque and to allow no leakage around the rotor. The fluid exhausts uniformly at the inner exhaust radius.

It is only necessary to consider a single space between two ad-

Table 2 Representative performance data from first disk turbine modified to increase number of disks, operated with exhaust to atmosphere and using 10-deg nozzles consisting of drilled holes

Pressure to turbine, psig	Temp to turbine, deg F	Mass rate of flow, lb_m/min	Speed, rpm	Power, hp	Efficiency, percent
40	203	2.82	6300	0.68	21.7
40	203	2.82	8500	0.81	25.4
40	203	2.82	9200	0.85	25.8
60	175	4.05	8000	1.13	24.2
60	175	4.05	10000	1.29	23.8
60	175	4.05	11000	1.32	24.4
60	175	4.05	11500	1.07	20.2
80	182	3.48	8000	1.12	24.7
80	182	3.48	10000	1.24	23.8
80	182	3.48	11000	1.24	23.8
80	182	3.48	11500	1.10	21.1
100	175	4.38	9000	1.49	24.6
100	175	4.38	11000	1.67	24.3
100	175	4.38	12000	1.32	19.0
120	165	5.30	9300	1.88	24.2
120	165	5.30	11000	2.02	22.9
120	165	5.30	12200	1.16	13.5
140	165	6.00	11800	2.41	23.2
140	165	6.00	12500	1.18	11.9

[a] This and following data with flow-limiting inserts in nozzles.

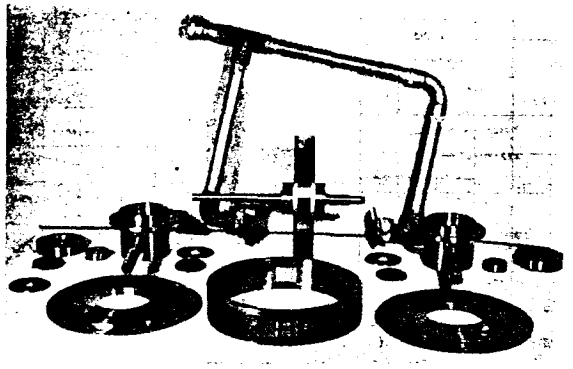

Fig. 4 Disassembled view of turbine pictured in Fig. 3

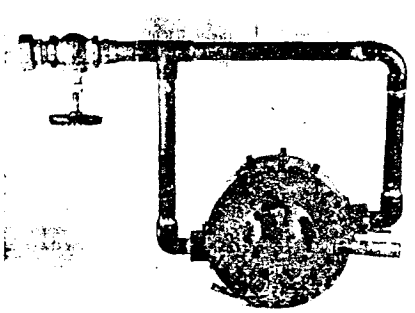

Fig. 3 Air turbine using a disk rotor

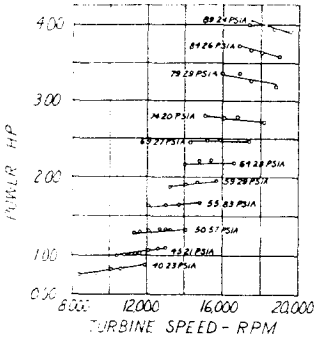

Fig. 5 Power-speed characteristics of second air turbine, at selected input pressures

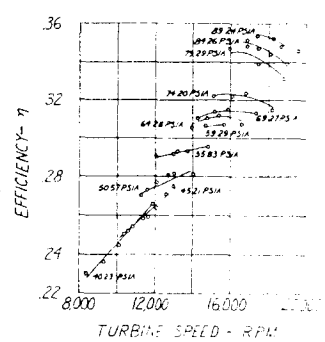

Fig. 6 Efficiency-speed characteristics of second air turbine at selected input pressures

...ent disks, and the nozzle supplying the fluid to this space, since the turbine is merely a number of such elements in parallel.

In the following, the process occurring in the space between ... disks is first analyzed. Using the results, the overall turbine performance is then considered.

Equations of Motion of Fluid Between Disks

Under the idealizations made the equations of motion for the fluid between the disks can be written. Fig. 7 depicts the plan form of the disks and a fixed coordinate system.

Since all derivatives with respect to time and θ are zero, the components of acceleration for a general element are

$$r\text{-direction: } u\frac{du}{dr} - \frac{v^2}{r}$$

$$\theta\text{-direction: } u\frac{dv}{dr} + \frac{uv}{r}$$

The acceleration occurs under the action of surface and body forces on the fluid element. Body forces are neglected herein and the forces considered are due entirely to pressure and shear stress acting on the surface of an element. These forces can be expressed in rigorous fashion but to do so leads to equations of great mathematical complexity, inconsistent with the aim of a first analysis. Therefore, following a suggestion made to the writer by A. Shapiro, a fluid element is chosen bounded by the solid disk surfaces (an element of finite thickness b). The shear forces on this element are accounted for by expressing the shear stress at the surfaces in contact between the fluid and the disks in terms of a factor f according to the conventional hydraulic treatment of bulk flow. Other surfaces of the fluid element are considered to contribute no net shear force on the fluid element.

To arrive at a statement of the shear stress, the fluid element in polar coordinates) is considered, and the hydraulic diameter of the flow passage is easily shown to be approximated by $2b$.

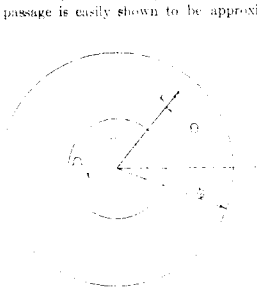

Fig. 7 Disk geometry and coordinate system

It then follows, in the usual manner of hydraulic analysis, that f and τ are related by

$$\tau = \frac{f\rho}{8}[(v - \Omega r)^2 + u^2]$$

since the bracketed term is the square of the fluid velocity relative to the walls of the flow passage; that is, the bracketed term is the square of the relative velocity of the fluid with respect to the disks. It follows that the force on a fluid element due to the shear stress is, in the direction opposite that of the relative velocity,

$$\frac{2f\rho}{8}[(v - \Omega r)^2 + u^2]rd\theta dr$$

since the stress acts on the two faces of the element in contact with the disks. This shear force has the components:

r-direction:

$$-\frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}urd\theta dr = -\frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}rd\theta dr$$

θ-direction:

$$-\frac{f\rho(v - \Omega r)}{4}[(v - \Omega r)^2 + u^2]^{1/2}rd\theta dr$$

The remaining surface forces, due to pressure, are written in the usual way from the pressure gradient. In the θ-direction, the pressure gradient is zero because of axial symmetry. Thus the forces due to pressure become

$$r\text{-direction: } -\frac{dp}{dr}brd\theta dr$$

$$\theta\text{-direction: } 0$$

The developed forces and accelerations are now inserted into Newton's law to obtain the scalar equations

$$-\frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}(v - \Omega r)drd\theta = \rho brdrd\theta\left[u\frac{dv}{dr} + \frac{uv}{r}\right]$$

and

$$-\frac{dp}{dr}brd\theta dr - \frac{f\rho}{4}[(v - \Omega r)^2 + u^2]^{1/2}urdrd\theta = \rho brdrd\theta\left[u\frac{du}{dr} - \frac{v^2}{r}\right]$$

in the θ and r directions, respectively. These simplify to

$$\frac{dv}{dr} + \frac{v}{r} = -\frac{f(v - \Omega r)}{4b}[(v - \Omega r)^2 + u^2]^{1/2} \quad (1)$$

and

$$\frac{1}{\rho}\frac{dp}{dr} = -u\frac{du}{dr} - \frac{v^2}{r} + \frac{f\rho}{4b}[(v - \Omega r)^2 + u^2]^{1/2}u = 0 \quad (2)$$

These are the equations of motion.

Solution of Equations of Motion

Equations (1) and (2) were solved through use of an IBM 704 digital computer. In order to retain generality, the equations were first rendered dimensionless through introduction of x and y as the dimensionless distance and dimensionless θ-component of velocity, respectively. Further, the continuity statement

$$u = -\frac{Q}{2\pi brx}$$

was used to eliminate the radial-velocity component from the equations. The resulting equations of motion in dimensionless form are

$$\frac{dy}{dx} + \frac{y}{x} = -\frac{fr_0\Omega}{4bv_0}\left(\frac{v_0}{\Omega r_0}y - x\right) \times\left[1 + \left(\frac{2\pi br_0^2\Omega}{Q}\right)^2 x^2\left(\frac{v_0}{\Omega r_0}y - x\right)^2\right]^{1/2} = 0 \quad (3)$$

$$\frac{1}{\rho\Omega^2r_0^2}\frac{dp}{dx} = \left(\frac{Q}{2\pi r_0^2b\Omega}\right)^2\left(\frac{1}{x^3}\right) - \left(\frac{v_0}{\Omega r_0}\right)^2\left(\frac{y^2}{x}\right) - \frac{fr_0}{4b}\left(\frac{Q}{2\pi r_0^2b\Omega}\right)^2\left(\frac{1}{x^2}\right)\left[1 + \left(\frac{2\pi br_0^2\Omega}{Q}\right)^2 x^2\left(\frac{v_0}{\Omega r_0}y - x\right)^2\right]^{1/2} = 0 \quad (4)$$

The dimensionless parameters which appear in equations (3) and (4) are legitimate turbine-operation parameters and become the variables in terms of which the analytical results can be related to actual turbine conditions.

Equation (3) describes the kinematic behavior of the fluid in the flow through the turbine while equation (4) describes the pressure change resulting along radial lines in the fluid flow.

Equations (3) and (4) were presented to the digital computer with the boundary condition $y(1) = 1$. The equations were solved for all possible combinations of the parameters given in Table 3. In each solution, corresponding values of x and y were tabulated, together with a value of $\Delta p_r/\rho\Omega^2r_0^2$ at each value of x where Δp_r is the accumulated pressure change in the flow between the disks.

Table 3 Dimensionless parameters and values selected for calculations

f	r_0/b	$v_0/\Omega r_0$	$Q/\Omega r_0^3$
0.01	50	0.3	0.0000
0.02	100	0.6	0.0001
0.03	200	0.8	0.001
0.05	...	1.0	0.01
...	...	1.2	0.10
...	...	1.5	1.0

Development of Turbine-Performance Parameters

An expression for the pressure change of the fluid in the nozzles was written from conventional one-dimensional hydraulics as

$$\Delta p_n = -\frac{\rho}{2}\left[v_0^2 + \left(\frac{Q}{2\pi r_0b}\right)^2\right]$$

since

$$u_0 = \frac{Q}{2\pi r_0b}$$

from continuity. In dimensionless form

$$\frac{\Delta p_n}{\rho\Omega^2r_0^2} = -\frac{1}{2}\left[\left(\frac{v_0}{\Omega r_0}\right)^2 + \left(\frac{Q}{2\pi br_0^2\Omega}\right)^2\right]$$

The addition of this function to $\Delta p_r/\rho\Omega^2r_0^2$ obtained in the numerical solution of equation (4) at each value of x yielded a value of $\Delta p_t/\rho\Omega^2r_0^2$, the dimensionless pressure change of the flow through the entire turbine at each value of x, for every combination of parameters in Table 3. No diffuser pressure recovery in the exhaust process was assumed; the kinetic energy of the fluid exhausting from between the disks was considered to be dissipated irreversibly.

The torque exerted by the fluid on the disks is expressed as

$$T = -(rv - r_0v_0)Q\rho$$

In dimensionless form, this becomes

$$\frac{T}{Q\rho\Omega r_0^2} = \left(\frac{v_0}{\Omega r_0}\right)(1 - xy) \quad (5)$$

From equation (5), since power is given by the product of torque and angular velocity, the work done by the fluid per unit mass on the disks, in dimensionless form, is

$$\frac{W}{\Omega^2r_0^2} = \left(\frac{v_0}{\Omega r_0}\right)(1 - xy) \quad (6)$$

The function $\Delta p_t/\rho\ \Omega^2r_0^2$ was earlier derived. This function represents the dimensionless available energy of a unit mass of fluid supplied to the turbine. The turbine efficiency is defined as

$$\eta = \frac{W}{\Delta p_t/\rho} = \frac{W/\Omega^2r_0^2}{\Delta p_t/\rho\Omega^2r_0^2} = \frac{(v_0/\Omega r_0)(1 - xy)}{\Delta p_t/\rho\Omega^2r_0^2} \quad (7)$$

Values of the torque function from equation (5) and of the efficiency η, were computed at each value of x for each combination of the parameters listed in Table 3.

Results of Analysis

The complete results of the computations consist of one data sheet for each possible combination of the dimensionless variables listed in Table 3. Each sheet presents x as the argument with the resulting values of y, $\Delta p_t/\rho\Omega^2r_0^2$, η and dimensionless torque and power functions. These results are too voluminous to be reproduced here [10].

A single representative case, which is typical of the basic results obtained, is given in Fig. 8, plotted for $f = 0.02$, $r_0/b = 50$, $\Omega r_0/v_0 = 1.00$, and $Q/\Omega r_0^3 = 0.00001$. A maximum value of η occurs at a definite value of x, with a corresponding value of $\Delta p_t/\rho\Omega^2r_0^2$. The value of x at which the maximum value of η occurs is the dimensionless inner radius of the disks resulting in greatest efficiency. In general, this value of x is different for each combination of the basic parameters in Table 3. The corresponding value of $\Delta p_t/\rho\Omega^2r_0^2$ gives the dimensionless pressure which must be supplied to drive the flow through the turbine.

Using the maximum values of η and the corresponding values of $\Delta p_t/\rho\Omega^2r_0^2$, curves such as presented in Figs. 9 and 10 result. These curves are typical and are plotted for $f = 0.05$ and $r_0/b = 50$.

In every case, efficiency decreases as the volume flow-rate parameter $Q/\Omega r_0^3$ increases. The dimensionless velocity ratio $\Omega r_0/v_0$ resulting in the highest efficiency increases with decreased $Q/\Omega r_0^3$ to a limiting value of 1 as $Q/\Omega r_0^3$ approaches zero. For values of $Q/\Omega r_0^3 \leq 0.01$, η is sensibly independent of r_0/b, but the pressure-change parameter $\Delta p_t/\rho\Omega^2r_0^2$ is not and increases significantly with increasing r_0/b for $Q/\Omega r_0^3 > 0.001$.

In general, η increases with increased friction parameter f. For $Q/\Omega r_0^3 \leq 0.01$, the pressure-change parameter $\Delta p_t/\rho\Omega^2r_0^2$ decreases with increasing f, but increases with f for larger values of the flow-rate parameter.

The complete results constitute a general quantitative correlation of the dimensionless parameters which determine disk-turbine performance, if losses external to the rotor are not considered. In a specific case, the results enable the proper choice of inner and outer disk radii, disk spacing, angular velocity, nozzle velocity, fluid flow rate, and operating pressure to produce any chosen performance within the capabilities of a disk rotor. The number of disks which must be incorporated in a complete tur-

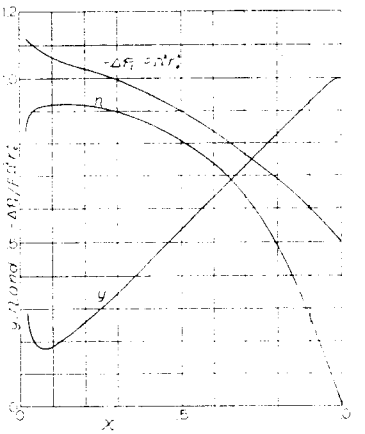

Fig. 8 Typical solution of equations describing turbine performance. Plotted for $f = 0.02$, $r_0/b = 50$, $\Omega r_0/v_0 = 1.00$, and $Q/\Omega r_0^3 = 0.00001$

bine rotor to provide desired rotor power may of course be computed since the power from one disk space is known.

The results show that the flow between the disks as analyzed here does not always produce a "turbine" effect; that is, the effect of work done by the fluid on the disks. Instead, for some combinations of the parameters in Table 3, energy is absorbed from the disks, resulting in a "radial inflow compressor" of the disk type. The efficiency as a compressor is very low in this case. Further, in some cases, compression occurs in the outer elements of the disk space while the remaining elements produce a turbine effect, the overall rotor producing a net turbine effect. In the desirable cases, however, every part of the flow performs as a turbine.

Discussion

The experimental results together with the analysis clearly show the feasibility of multiple-disk turbines. The "losses" in a disk turbine, exclusive of the rotor, can be categorized as:

1 Energy dissipated due to shear forces on the outer surface of the rotor, both on the sides of the outer disks and the edges of all the disks.

2 Losses of available energy due to irreversibility of the nozzles supplying fluid to the rotor.

3 Losses of available energy in the exhaust process due to uncontrolled diffusion.

4 Bearing losses and losses from seals.

5 Losses due to partial admission and admission blockage caused by finite thickness of the disks.

Most of these losses can be estimated by methods available in the engineering literature. When used in conjunction with the results of the disk-turbine analysis, the loss estimates result in the prediction of the approximate performance of an actual disk turbine corresponding with any chosen combination of the parameters used in the analysis, for turbines in which com-

possibility effects are st

It is here pointed up that the sum of the losses enumerated in the foregoing is a slowly increasing fraction of the available energy as the power output and physical size of the turbine is decreased. Therefore disk turbines have a promise of retaining reasonable efficiency as the turbine size is decreased. This is in contrast to the conventional bladed machine in which total losses, as a

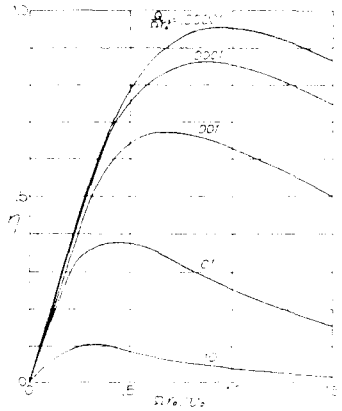

Fig. 9 Typical results for maximum efficiency as a function of flow rate and speed parameters. Plotted for $f = 0.05$, $r/b = 50$

Fig. 10 Typical results for pressure-change parameter as a function of flow rate and speed parameters. Plotted for $f = 0.05$, $r/b = 50$

fraction of the available energy, increase rapidly as turbine size is decreased, because of the increased ratio of wetted shear area to fluid through-flow area within the rotor and housing turbine-blade passages.

Primarily for this reason, the multiple-disk turbine is clearly not competitive with conventional turbines over the major portion of the power-size spectrum, but may become competitive in the high energy input-low power output part of the spectrum. It is suggested that it may find use also with fluids with density and or viscosity far from that of air, such as certain exotic fluids considered for missile auxiliary drives and for working fluids in space and satellite power cycles.

It is conceivable that the multiple-disk turbine will find application in cases in which it is uncompetitive in performance and efficiency, because of advantages in first cost, ease of manufacture, low noise level, and ease of balancing. Significant advantages in each of these areas were apparent in the experimental work herein reported. It is probable that such turbines could be used at a cost advantage as drive units in high-speed testing of other machinery in laboratories and production testing, as "prime-approach" turbines in ground testing of "exotic-fluid" engines, and as drive units in air tools including dental drills and in automotive and aircraft accessories.

It may be possible to give short cut boundary-layer-type analysis of the flow within the rotor of a multiple-disk turbine. Such an analysis has been performed for a multiple-disk pump [11]. Experimental investigations of such pumps have also been reported [12, 13].

Acknowledgment

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The author expresses his appreciation to the General Electric Computer Center on the Arizona State University campus and to Mr. Tom Lahey of the Center for preparing computer programs and conducting the computations.

Acknowledgment is also made of the several individuals who participated in the construction and testing of the several devices. Among these are E. L. Gruber, F. J. Smith, E. Y. Harper, and Wm. S. Gordon.

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DISCUSSION

S. H. Hasinger[5] and L. G. Kehrt[5]

Since Professor Rice treats the flow in his analysis as turbulent, he apparently does not fully appreciate the basic requirements of efficient shear-force (multidisk) turbomachines, namely, very close disk spacing and small velocity differences between the fluid and the shear surfaces. These are conditions which make the flow between the disk, in general, laminar.

From the fundamental momentum considerations it follows that in any device which transfers energy by shear forces the relative velocity between the driving medium and the driven medium must be small, if the transfer should be efficient. Thus the necessary forces must be derived from large shear surfaces rather than large velocity differences (for power concentration at the expense of efficiency, of course, the reverse is true). The conditions for high efficiencies are, however, inducive to laminar flow. For this reason, our analytical treatment of a shear-force pump [12] was based on laminar flow conditions. In the course of these investigations we were able to show that turbulent flow conditions may exist only in relatively large pumps. Our theoretical investigations show clearly the need for very close disk spacing. Characteristic data for our test rotor are: OD 4 in.; ID 1 in.; 174 disks spaced at 0.005 in.; rotor efficiency 54 percent at 4500 rpm. A recent test with modified rotor inlet gave 60 percent rotor efficiency.

The conventional friction factor applied in Professor Rice's analysis is very inconvenient to use in the laminar case since it becomes strongly influenced by the Reynolds number. The basic approach used in our pump analysis [12] should be equally suitable for the turbine. This engineering type analysis, which is based upon angular momentum considerations, avoids the involved mathematics of a boundary layer calculation by assuming parabolic velocity profiles. The mathematical simplicity of the analysis allows by slide rule calculations a concise survey of the performance potential of a disk pump. The generality of this analysis is insured by incorporating a similarity parameter which resulted from the exact boundary-layer type investigations of the flow between rotating disks by Breiter and Pohlhausen [11].

Author's Closure

The analysis presented in no way assumes the flow in the multiple-disk turbine as turbulent; rather the friction factor f is retained as a parameter in the solution. It is true that the analysis would be less realistic for laminar than for turbulent flow because the value of f is assumed constant over the disk radius. Since the local Reynolds number varies over the disk radius, the friction factor in a real disk turbine varies much more over the disk radius for laminar than for turbulent flow. In either case, one must regard the f factor used in the analysis as a representative or mean value for the entire disk flow system.

As shown in the example case presented in Fig. 9, the maximum efficiency for a multiple-disk turbine occurs for low relative velocity of the fluid with respect to the disks, at very low power. But in order to produce a turbomachine of reasonably small size, larger flow rates per disk space must be used, corresponding to curves such as those labeled 0.001 and 0.01 in Fig. 9. Then the maximum efficiency is lower and the relative velocity is higher. The flow regime for such practically interesting machines is not always laminar. The flow in the tests reported in the paper was mostly turbulent, corresponding with local Reynolds numbers as high as approximately 600,000.

The analysis was undertaken to discover facts about disk turbomachines; it was widely believed that such machines were limited in efficiency to very low values and were not feasible. The results show the reverse of this and that categoric statements concerning the "best" flow conditions and rotor geometry are unwise since a number of parameters are necessary to describe the results. The performance of a multiple-disk turbomachine is not strongly dependent on f, and the performance is independent of whether the flow which produces a given friction factor is laminar or turbulent.

The writer agrees that one can purposely reduce the disk spacing to produce laminar flow at all operating conditions and achieve very good multiple-disk turbomachines. He further agrees that the analysis presented is not the best simple analysis for that condition. Investigations already are under way to develop boundary-layer-type descriptions and integral-type descriptions of laminar flow in multiple-disk turbines in the same general manner as has already been done for liquid pumps by the discussers [12] and Breiter and Pohlhausen [11].

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Investigation into the Performance Characteristics of a Friction Turbine

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A friction or Tesla turbine was theoretically investigated by using the differential form of the equation of motion. A partial closed-form solution was obtained for the case of incompressible laminar flow, and a method of solution is indicated for other types of flow. The performance of a 6-in. air turbine was calculated, and it was tested over ranges of angular velocity (4000–18,000 rpm), supply pressure (10–40 psig), and disk spacing (0.026–0.5 in.). Turbine efficiencies ranged from 7 to 25%. The qualitative agreement between calculated and experimental performance was satisfactory (correct trends), but the quantitative agreement was less than satisfactory.

Nomenclature

C_t = coefficient of torque $= M/\rho\omega_i r_i$
c = relative bulk tangential velocity $= u - \omega r$
c_p = specific heat at constant pressure
M = torque
N = radial Reynolds number $= \rho v r_i/\mu$
P = power
p = pressure
k = radius ratio $= r/r_i$
R_t = tangential Reynolds number $= \rho s u/\mu$
r = radius
s = half the distance between the disks
T = temperature
u = bulk tangential velocity
v = bulk radial velocity
w = flow rate
z = dimension across the spacing distance
β = friction factor $= 6/\sigma^2 N$
μ = coefficient of viscosity
ρ = density
σ = spacing ratio $= 2s/r_i$
τ_c = wall fluid shear stress in tangential direction
τ^r = wall fluid shear stress in radial direction
ϕ = angular dimension
Ω = angular velocity ratio $= r\omega/u$
ω = angular velocity

Introduction

THE type of friction turbine invented by Nikola Tesla† in 1906 is noted for its simple rotor, which consists of a series of parallel disks‡ that may be either flat or tapered and either smooth or rough. Such a turbine might be used with fuels that produce solid particles in their combustion products with less difficulty due to blade clogging than would be encountered with conventional turbines. The nature of the flowfield over the rotor and the centrifugal force should make the disks self-cleaning. However, a thin layer of particles can provide thermal insulation and increase surface roughness and hence friction. No previous investigation of the friction turbine was sufficient to permit a valid comparison between its capabilities and those of conventional turbines. The present investigation accomplishes this through a theoretical analysis and an experimental investigation of a friction turbine with smooth flat disks.

Received November 9, 1964; revision received May 27, 1965. The major portion of the investigation was conducted at the Pennsylvania State University for a Ph.D. Thesis, under the direction of M. S. Gissdahl.

* Group Leader, Power Systems, Columbus Division.

† It was the desire to obtain a simple rotor which led Tesla, who is better known for his inventions of the alternating current motor and Tesla coil, to the invention of his turbine.

The theoretical analysis is based upon the equations of momentum, continuity, and energy, applied to a "near" infinitesimal control volume adjacent to the surface of the rotor. The set of equations is reduced to three ordinary differential equations (two momentum and the energy equation) with the radius of the rotor as the independent variable. A partial analytical solution of this set is obtained for the case of incompressible laminar flow. Solutions for other cases of interest, such as compressible turbulent flow, are obtained by numerical methods. Results of experiments on a 5-in. air turbine are in qualitative agreement with the calculated performance, but quantitative agreement is less than satisfactory. However, the quantitative agreement is sufficient to make the theoretical analysis useful in designing and developing suitable similarity rules for this type of friction turbine.

Theoretical Investigation

The friction turbine produces an exchange of momentum by the use of fluid friction (rather than by impulse or reaction) by passing the fluid between disks (Fig. 1). The relative velocity of the fluid across the disks creates a frictional force that will accelerate the rotors. As the rotational velocity increases, the frictional force decreases. The rotational velocity will stabilize when the frictional force equals the externally applied load and windage losses of the turbine.

The dimensions (in cylindrical coordinates) of the elementary control volume are shown in Fig. 2, where s is half the distance between adjacent disks. This choice of the control volume simplifies the analysis, because the shear stress on the "top" of the element is zero due to the symmetry of the velocity profile. Since the height of the control volume is not the infinitesimal dz, the parameters shown in Fig. 2 are average or bulk parameters. A complete admission turbine is assumed, so that there is no variation in pressure and velocity in the ϕ direction. Assuming that steady flow conditions prevail, the rotor radius r becomes the only independent variable. From any text on fluid dynamics, e.g., Ref. 10, the continuity equation and momentum equations can be obtained in cylindrical coordinates for both conditions described previously,

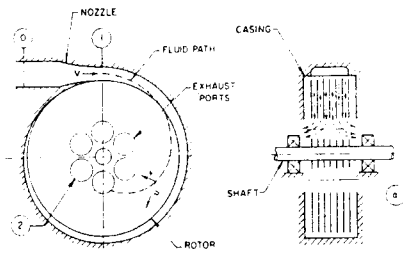

Fig. 1 Schematic of Telsa turbine.

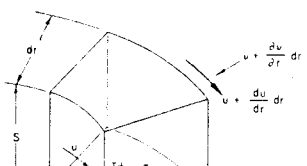

Fig. 2 Elementary control volume.

$$d(\rho r v)/(dr) = 0 \quad \text{or} \quad \rho r v = \text{const} \tag{1}$$

$$\rho r[(dv_r/dr) + (v_r/r)] - (\tau_\phi/s) = 0 \tag{2}$$

and

$$\rho[r(dv_r/dr) - (v_\phi^2/r)] = -(dp/dr) + (\tau_r/s) \tag{3}$$

where v and v_r and τ_ϕ and τ_r are the fluid velocities and fluid shear stresses in the ϕ and r directions, respectively, ρ is the density, and p is the pressure. For the case of adiabatic flow of a perfect gas, the expression for the change in energy across the control volume is

$$w(d/dr)[c_p T + (v_r^2 + v_\phi^2)/2] = -(dP/dr) = -\omega(dM/dr) \tag{4}$$

where T is the temperature of the fluid, w is the flow rate, c_p is the specific heat, ω is the angular velocity, P is the power, and M_t is the torque. The moment of momentum equation from station 1 (at the rotor entrance) to any radius r is

$$M = w(v_1 r_1 - v r) \tag{5}$$

which, when differentiated, becomes

$$dM/dr = -wd(vr)/dr \tag{6}$$

Equations (1–6) form a set of nonlinear differential equations that can be solved on an electronic computer to obtain the variations in v, r, p, and T across the turbine rotor, which can then be used to determine torque, power, and efficiency.

Incompressible Laminar Flow

For the case of incompressible laminar flow, the expression for the fluid shear stress at the disk in the ϕ direction is

$$\tau_\phi = \mu(dv/dz)_w \tag{7}$$

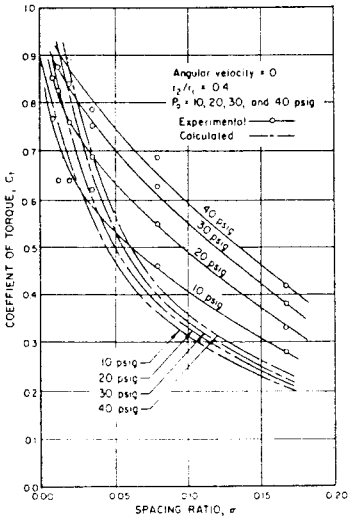

Fig. 3 Coefficient of torque C_t as a function of spacing ratio σ.

where μ is the coefficient of viscosity of the fluid, and v is the relative velocity of the fluid across the disk. The expression for $(dv/dz)_w$ is obtained by assuming a parabolic velocity profile between the disks. By substituting the preceding expression into Eq. (2), one obtains for the momentum equation

$$(dv/dr) + [(1/r) - (2\beta r/r_1^2)]v + 2\omega = 0 \tag{8}$$

where $v = u - \omega r$, $\beta = 6/N\sigma^2$, $\sigma = 2s/r_1$, and $N = \rho_1 v_1 r_1/\mu_1$. Equation (8) is solvable by the use of an integrating factor

$$u/u_1 = (1 - \Omega)e^{-\beta(1-R^2)} + (\Omega/R)[1 - e^{-\beta(1-R^2)}] + \Omega R \tag{9}$$

where R is r/r_1 and Ω is $\omega r_1/u_1$.

The torque M is obtained by substituting Eq. (9) into Eq. (5):

$$M = \omega v_1 r_1 \{[1 - e^{-\beta(1-R^2)}] - \Omega[(1 - e^{-\beta(1-R^2)})(1 + 1/\beta + R^2)]\} \tag{10}$$

Hence, Eq. (10) shows that the relationship between torque-angular velocity is linear for incompressible laminar flow. Computed results[11] give linear relationships for incompressible and compressible turbulent flow. Experimental results[11,13] indicate also a linear relationship between torque and angular velocity.

If the velocity profile is assumed to be parabolic in the radial direction, an expression for τ_r can be obtained by replacing v with r in Eq. (7). By substituting the proper expression for τ_r and the continuity equation into Eq. (3), one obtains the following form for the radial momentum equation:

$$d_p/r\, d = \rho[(v_1 r_1)^2/r^3 + u^2 r + 2\beta v_1^2/r] \tag{11}$$

which, when integrated, becomes

$$\frac{p_1 - p_2}{\rho} = \frac{v_1^2}{2}\left[1 - \left(\frac{r_2}{r_1}\right)^2\right] + v_1^2\beta \ln\left(\frac{r_1}{r_2}\right)^2 + \int \frac{u^2 dr}{r} \tag{12}$$

Unfortunately, the solution to Eq. (11) cannot be integrated in a closed form because of the complex nature of the expression for u, [Eq. (9)]. However, the integral in Eq. (12) can be evaluated easily by numerical methods. From solutions to Eqs. (9, 10, and 12), the variation of turbine efficiency and power can be determined as a function of the important turbine parameters (β, Ω, u_1, and R).

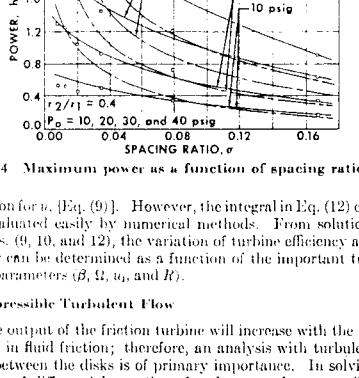

Fig. 4 Maximum power as a function of spacing ratio σ.

Compressible Turbulent Flow

The output of the friction turbine will increase with the increase in fluid friction; therefore, an analysis with turbulent flow between the disks is of primary importance. In solving the set of differential equations for the case of compressible turbulent flow, the following relationship was used for evaluating the fluid shear stress at the disk:

$$\tau/(\rho u^2) = 0.0217(Re)^{-1/4} \tag{13}$$

where $Re = \rho s u/\mu$.

Equation (13) was shown to be correct from experimental data[10] and was developed[11] from the current theory of turbulence. In order to compare experimental and theoretical results, the theoretical analysis presented in this paper for compressible turbulent flow is modeled after the design of the test model, and is, therefore, limited. Since the test model incorporates convergent nozzles, the analysis is limited to subsonic flow. The maximum mass flow through the turbine is obtained, therefore, when the relative Mach number leaving the rotor is 1. A detailed investigation[10] of this limiting flow condition shows that the pressure ratios used in the experimental investigation are not sufficient to reach this condition in the test model. Therefore, limiting flow was not obtained in the experimental investigations.

The test model also incorporated nearly tangential admission. Because of this, it was concluded from an order of magnitude analysis that the term $v^2/2$ is small as compared to $u^2/2$ and was, therefore, neglected in the energy equation (4). As a result of this conclusion, Eq. (4) becomes

$$(d/dr)[c_p T + (u^2/2)] = \omega[d(ur)/dr] \tag{14}$$

or

$$c_p(T - T_1) + (u^2 - u_1^2)/2 = \omega(ur - u_1 r_1) \tag{15}$$

between station 1 and any point on the rotor. It is not necessary for the theoretical analysis to neglect the term $v^2/2$, but by doing so, the calculation procedure for solving the set of differential equations is greatly simplified. An estimate of the magnitude of the term $v^2/2$ can be obtained by using the limiting case, which is isentropic flow. For the theoretical analysis in which friction effects were considered, the value of $v^2/2$ will be less than the isentropic case. For isentropic flow, the continuity equation [Eq. (1)] becomes

$$v_2/v_1 = (p_1/p_2)^{1/k}(r_1/r_2) \tag{16}$$

For the test model the maximum values for v_1/u_1, p_2/p_1 and r_2/r_1 are 0.038, 0.42, and 0.4, respectively. When the preceding values are inserted into Eq. (16), the resulting value for v_2/u_1 is 0.18. The term $(v_1^2 - v_2^2)$ is approximately 3% of u_1^2. It must be cautioned that the term $v^2/2$ is small only for this type of design and cannot be neglected in any general analysis of compressible flow. The analysis presented here is, therefore, limited to tangential admission turbines. The numerical solutions of Eqs. (1–3 and 14), using Eq. (13) to determine the fluid shear stress, are presented in Figs. 3–6 for given set of inlet conditions. Figures 3–6 contain the experimental results for the same set of inlet conditions also. Solutions to the set of differential equations for the cases of compressible laminar flow and incompressible turbulent flow are discussed in detail in Ref. 11.

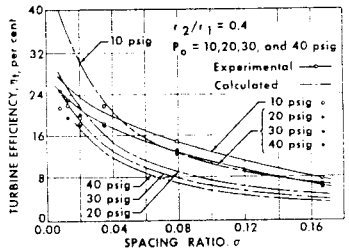

Fig. 5 Turbine efficiency at maximum power as a function of spacing ratio σ.

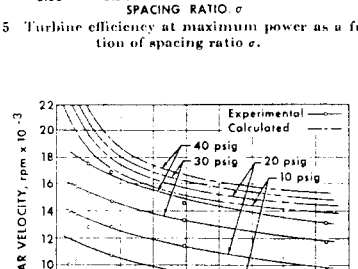

Fig. 6 Angular velocity for maximum power as a function of spacing ratio σ.

Experimental Investigation

The major parts of the experimental turbine are the casing, which contains two flow nozzles, and the removable rotor (see Fig. 7). The two flow nozzles are simple convergent nozzles and are integral parts of the casing. Each nozzle exits tangentially into a scroll that covers half the circumference of the rotor. The scrolls were used to balance the flow over the circumference and to make the turbine a full-admission turbine. The flow exhausted to the atmosphere from both sides of the casing. The 6-in.-diam rotor (Fig. 8) has $r_2/r_1 = 0.4$. It comprises two $\frac{1}{4}$-in. end disks and a series of 24-gage, 0.026-in., intermediate disks. The rotor shown in Fig. 8 has five intermediate disks with a spacing between disks of 0.061 in. The disk spacing was changed by inserting more or less intermediate disks. The output shaft of the turbine was coupled directly to the dynamometer. The major mechanical problem encountered was disk vibration. It was solved by placing between each pair of disks three thin plastic pads that were held in place by three small bolts.

The turbine was driven by compressed air. The independent variables were angular velocity, supply pressure, and disk spacing. Figure 9 shows an example of the variation in turbine performance with angular velocity for a given supply pressure and disk spacing. The torque-angular velocity relationship is linear as was predicted in the theoretical inves-

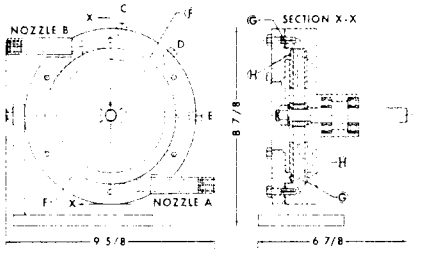

Fig. 7 Test model of friction turbine.

Fig. 8 Turbine rotor.

tigation. Similar tests were carried out for supply pressures of 10, 20, 30, and 40 psig and disk spacings of 0.500, 0.237, 0.105, 0.061, 0.037, and 0.026 in.

The test results are compared with calculated performance in Figs. 3–6. In Fig. 3, the torque coefficient $C_t \equiv M/\pi\omega\rho r_1^5$ is presented as a function of spacing ratio for constant supply pressures at zero angular velocity. In Figs. 4 and 5, the variation in maximum power and maximum efficiency with spacing ratio is illustrated. In Fig. 6, the angular velocity of the turbine for maximum power as a function of spacing ratio is presented for the various supply pressures.

The qualitative agreement between experimental and calculated results is good. The torque and power of the turbine increase with an increase in supply pressure or inlet velocity and with a decrease in spacing ratio. Both parameters increase the frictional effects of the turbine. The turbine efficiency, however, decreases with an increase in supply pressure; i.e., with increasing pressure, the power output of the turbine does not increase as rapidly as does the available energy or the enthalpy drop across the turbine. In general, the shapes of the experimental and calculated performance curves are the same.

The quantitative agreement between experimental and calculated results was less than satisfactory. The calculated results in most cases were less than the experimental results. This discrepancy is probably the result of the difference in pressure ratio across the rotor for the two cases.

In the theoretical analysis, it was assumed that all the flow passed between the disks. The pressure ratio across the rotor calculated on this basis was greater than that which was measured. Since the pressure ratio across the turbine was constant for both cases, the pressure drop across the nozzle for the theoretical analysis will be less; this will result in a decrease in the inlet velocity, torque per unit flow rate, etc. It was noticed during the experimental investigation that a significant amount of the flow bypassed the rotor. This leakage would reduce the pressure ratio across the rotor and could produce an effect opposite that described previously. The fluid path in the test model is, therefore, different from the one assumed in the theoretical analysis.

Analysis of Experimental Results

From the experimental results, it appears that the loss of power due to a decrease in mass flow through the rotor is more than compensated by the increase in frictional effects, which is mainly due to the increased inlet velocity. Obtaining a large mass flow through the rotor with a small pressure drop across the rotor appears to be one of the major design problems.

From the correlation between experimental and calculated results presented in Figs. 6–9, it can be concluded that the theoretical investigation presents an inadequate description of the performance of friction turbines. The theoretical results, therefore, can be used in designing and in developing scaling laws for this type of turbine.

The maximum turbine efficiency achieved by the test turbine was 24%, which is low for a single-stage turbine of its specific speed, 0.1 rpm. A small, single-stage steam turbine in this specific speed range will have an efficiency of 40 to 45%. The test turbine, however, was not in a high state of development and with improvement in the design and with better scaling of the rotor, increased efficiency should be obtained. Since the maximum theoretical turbine efficiency of a radial exhaust turbine is 66.7%, it is not likely that a turbine efficiency of much greater than 40 to 45% can be obtained for the friction turbine. Although the friction turbine does not have a distinct performance advantage over that of conventional turbines, its simple rotor may offer an advantage when design conditions require a simplified and inexpensive unit.

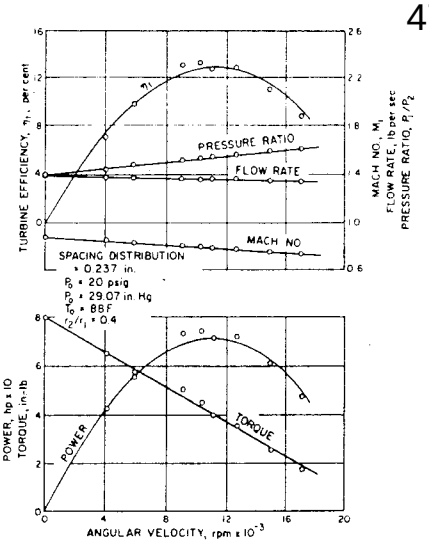

Fig. 9 Friction turbine performance.

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Potential Flow Between Two Parallel Circular Disks With Partial Admission

LEE MATSCH[1] and WARREN RICE[2]

Introduction

In a study of the fluid flow in unconventional turbomachinery, and also in other analytical investigations in engineering, it has been found necessary to have a solution for the potential flow of an incompressible fluid between two circular disks. Of main interest is the case for which the disks are closed at the outer periphery except for one or more slots through which fluid is injected with both normal and tangential components. The disks are considered parallel with the fluid exhausting through a circular hole at smaller radius than the outer radius of the disks. The problem is formulated under the assumption of an arbitrary distribution of radial velocity at the outer radius, with the average value of the tangential component as a parameter. A special case in which the fluid is injected through a single slot with uniform radial velocity across the slot is treated in detail.

Analysis

Under the idealizations stated and in r, θ polar coordinates the equations of motion are

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$
$$u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial \theta} \quad (1)$$

where u is the radial velocity component, v the tangential component, p the pressure, and ρ the fluid density. The equations of continuity and irrotationality are

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \theta} = 0 \quad (2)$$

$$\frac{\partial v}{\partial r} + \frac{v}{r} - \frac{1}{r}\frac{\partial u}{\partial \theta} = 0 \quad (3)$$

Lastly, the flow must satisfy the Bernoulli equation, where p_0 is the stagnation pressure:

$$\frac{p}{\rho} + \frac{u^2 + v^2}{2} = \text{const} = p_0.$$

Equations (1), (2), and (3) together with the definitions $u = \partial\phi/\partial r$ and $v = \frac{1}{r}\partial\phi/\partial\theta$ are summarized in the equation $\nabla^2\phi = 0$ with $\phi = \phi(r, \theta)$; that is, the flow is potential flow, where ϕ is the velocity potential function. The problem statement becomes

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0 \quad (4)$$

$$\frac{\partial\phi}{\partial r}(r_0, \theta) = u(r_0, \theta) \qquad 0 \le \theta \le 2\pi \quad (5a)$$

$$\frac{1}{r}\frac{\partial\phi}{\partial\theta}(r_0, \theta) = v(r_0, \theta) \qquad 0 \le \theta \le 2\pi \quad (5b)$$

where r_0 is the outer disk radius, $u(r_0, \theta)$ an arbitrary function, and $v(r_0, \theta)$ a function which can be partly arbitrary, as detailed later herein. In addition, the solution is required to be bounded everywhere in the region of flow and everywhere on the boundaries.

Solution of the differential equation is made by the method of separation of variables to yield

$$\phi(r, \theta) = a_0 + a_1 \ln r + a_2\theta + a_3\theta \ln r + \sum_{n=1}^{\infty} r^n(a_{1n}\cos n\theta + a_{2n}\sin n\theta) + \sum_{n=1}^{\infty} r^{-n}(a_{3n}\cos n\theta + a_{4n}\sin n\theta)$$

To produce periodicity of the velocity components, a_3 is chosen equal to zero, and to correspond with a finite flux at the origin, the coefficients of terms involving r^{-n+1} in the velocity component expressions are set to zero. The velocity components become

$$u(r, \theta) = \frac{a_1}{r} + \sum_{n=1}^{\infty} nr^{n-1}(a_{1n}\cos n\theta + a_{2n}\sin n\theta)$$
$$v(r, \theta) = \frac{a_2}{r} + \sum_{n=1}^{\infty} nr^{n-1}(a_{2n}\cos n\theta - a_{1n}\sin n\theta) \quad (6)$$

The constants are determined by satisfaction of the boundary conditions. Expanding equation (5a) in a Fourier series and equating the result to $u(r_0, \theta)$ formed from equation (6) yields

$$a_1 = \frac{r_0}{2\pi}\int_{-\pi}^{\pi} u(r_0, \alpha)\,d\alpha$$
$$a_{1n} = \frac{1}{n\pi r_0^{n-1}}\int_{-\pi}^{\pi} u(r_0, \alpha)\cos n\alpha\,d\alpha$$
$$a_{2n} = \frac{1}{n\pi r_0^{n-1}}\int_{-\pi}^{\pi} u(r_0, \alpha)\sin n\alpha\,d\alpha$$

Now the tangential velocity is

$$v(r, \theta) = \frac{a_2}{r} + \sum_{n=1}^{\infty}\frac{1}{\pi}\left(\frac{r}{r_0}\right)^{n-1}\left\{\int_{-\pi}^{\pi} u(r_0, \alpha)\sin\alpha\,d\alpha\cos n\theta - \int_{-\pi}^{\pi} u(r_0, \alpha)\cos n\alpha\,d\alpha\sin n\theta\right\} \quad (7)$$

Clearly, $v(r_0, \theta)$ and $u(r_0, \theta)$ are not independent; that is, $v(r_0, \theta)$ cannot be prescribed arbitrarily. From the form of equation (7) it is seen that the average value of $v(r_0, \theta)$ may be prescribed; it is denoted by $\bar{v}(r_0)$ and is defined according to

$$\bar{v}(r_0) = \frac{1}{2\pi}\int_0^{2\pi} v(r_0, \theta)\,d\theta \quad (8)$$

Integrating equation (7) with respect to θ, dividing by 2π, and equating the result to the prescribed $\bar{v}(r_0)$ yields

$$a_2 = r_0\bar{v}(r_0)$$

Finally,

$$u(r, \theta) = \frac{1}{2\pi}\left(\frac{r_0}{r}\right)\int_{-\pi}^{\pi} u(r_0, \alpha)\,d\alpha + \sum_{n=1}^{\infty}\frac{1}{\pi}\left(\frac{r}{r_0}\right)^{n-1}\left\{\int_{-\pi}^{\pi} u(r_0, \alpha)\cos n\alpha\,d\alpha\cos n\theta + \int_{-\pi}^{\pi} u(r_0, \alpha)\sin n\alpha\,d\alpha\sin n\theta\right\} \quad (9)$$

$$v(r, \theta) = \bar{v}(r_0)\left(\frac{r_0}{r}\right) + \sum_{n=1}^{\infty}\frac{1}{\pi}\left(\frac{r}{r_0}\right)^{n-1}\left\{\int_{-\pi}^{\pi} u(r_0, \alpha)\sin\alpha\,d\alpha\cos n\theta - \int_{-\pi}^{\pi} u(r_0, \alpha)\cos n\alpha\,d\alpha\sin n\theta\right\} \quad (10)$$

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where $u(r_0, \theta)$ and $\bar{v}(r_0)$ are prescribed.

Special Case

In particular, it is of interest to specialize the results to apply to the case for which

$$u(r_0, \theta) = \begin{cases} -u_0, \text{ const} & -\beta/2 \le \theta \le \beta/2 \\ 0 & \beta/2 < \theta \le 2\pi - \beta/2 \end{cases}$$

where β is the angle over which fluid is admitted with constant radial velocity through a single slot, and with an average tangential velocity around the entire disk periphery of $\bar{v}(r_0)$ prescribed. If the dimensionless variables are defined according to

$$u^* = \frac{u}{u_0}, \qquad v^* = \frac{v}{u_0}, \qquad r^* = \frac{r}{r_0},$$

$$P = \frac{\bar{v}(r_0)}{u_0}, \qquad p^* = 2\frac{(p_0 - p)}{\rho u_0^2}$$

where p_0 is the stagnation pressure for the flow, then equations (9) and (10) specialize to

$$u^* = -\frac{\beta}{2\pi r^*} - \sum_{n=1}^{\infty} \frac{2r^{*n-1}}{n\pi} \sin\frac{n\beta}{2} \cos n\theta \tag{11}$$

$$v^* = \frac{P}{r^*} + \sum_{n=1}^{\infty} \frac{2}{n\pi} r^{*n-1} \sin\frac{n\beta}{2} \sin n\theta \tag{12}$$

Defining a stream function according to $u = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ and $v = -\frac{\partial \psi}{\partial r}$ results in

$$\psi^* = \frac{\psi}{u_0 r_0} = \frac{\beta\theta}{2\pi} - P \ln r^* - \sum_{n=1}^{\infty} \frac{2}{n^2\pi} r^{*n} \sin\frac{n\beta}{2} \sin n\theta \tag{13}$$

From the energy equation, $p^* = u^{*2} + v^{*2}$.

Fig. 1 presents a plot of three streamlines for the case in which $\bar{v}(r_0)/u_0 = 0.1$ and $\beta = 20.5$ deg. It is noted that $v^*(r_0)$ is negative over part of the inlet slot. Fig. 2 presents a plot of a single streamline for the case in which $\bar{v}(r_0)/u_0 = 1.0$ and $\beta = 20.5$ deg.

As a further special case, for full admission, the admission angle β becomes 360 deg and the results reduce to

$$u^* = \frac{1}{r^*}, \qquad v^* = \frac{1}{r^*}, \qquad \psi^* = \theta - P \ln r^*, \qquad p^* = \frac{2}{r^{*2}}$$

All series appearing in the solution are rapidly convergent. It is relatively easy to evaluate equations (9) and (10) for any number of flow inlet slots or for a continuously distributed inlet flow around the outer radius of the disk. Because there is no momentum exchange between the fluid and the disks for potential flow, the solution applies for rotating disks, which is a case of main interest.

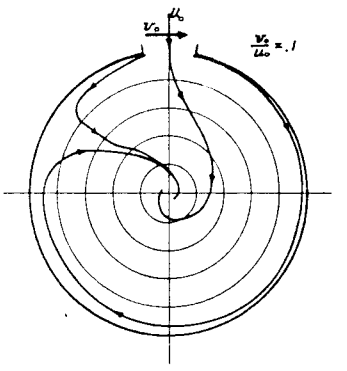

Fig. 1 Three streamlines for case in which $\beta = 20.5$ deg and $\bar{v}_0(r_0)$ is small compared with u_0

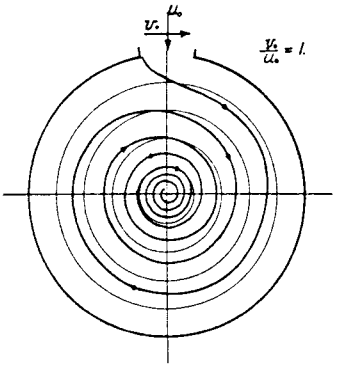

Fig. 2 A streamline for case in which $\beta = 20.5$ deg and $\bar{v}_0(r_0)$ is equal to u_0

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Flow at Low Reynolds Number With Partial Admission Between Rotating Disks

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Introduction

In the analysis of the flow in unconventional turbomachinery, and in certain other engineering situations, it has been found necessary to have a solution for the laminar flow field between rotating disks when the fluid is admitted only over part of the disk circumference and when the flow is at very low Reynolds number. In particular, the solution is a beginning solution to be perturbed by the addition of inertia forces in order to obtain an approximate solution for partial admission flows between rotating disks at higher Reynolds numbers.

Analysis

The Navier-Stokes equations in cylindrical polar coordinates are applied with R, θ, Z as the radial, tangential, and axial space coordinates and U, V, W as the corresponding velocity components. Dimensionless varibles are defined according to $u = U/r_0\Omega, v = V/r_0\Omega, w = W/h\Omega, r = R/r_0, z = Z/h, p = P/\rho r_0^2\Omega^2$, and $N_{Re} = \rho h^2\Omega/\mu$, where Ω is the angular velocity, P is the pressure, h is the spacing between disks, r_0 is the outer radius, and ρ and μ are the fluid density and viscosity. The flow is assumed steady with low Reynolds number and low volume flow rate; then $u \ll v$ and $v = r$. Further, it is assumed that $h \ll r_i$, where r_i is the inner radius of the disks. Following order-of-magnitude arguments and linearization of the equations, allowed by the foregoing assumptions, the governing equations become

$$-r = -\frac{\partial p}{\partial r} + \frac{1}{N_{Re}}\frac{\partial^2 u}{\partial z^2} \tag{1}$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{N_{Re}}\frac{\partial^2 v}{\partial z^2} \tag{2}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} \tag{3}$$

The boundary conditions are $u(r, \theta, \pm 1/2) = 0$, $v(r, \theta, \pm 1/2) = r$, $w(r, \theta, \pm 1/2) = 0$, and $p(r_i, \theta) = 0$, corresponding with no fluid slip at the disk surfaces and with fluid exhaust into a stagnant reservoir at the inner radius r_i. The equations describe creeping flow with the centrifugal force retained.

Equations (1) and (2) are integrated from 0 to z and again from $1/2$ to z, using the appropriate boundary conditions, to yield

$$u = N_{Re}\left(\frac{z^2}{2} - \frac{1}{8}\right)\left(\frac{\partial p}{\partial r} - r\right) \tag{4}$$

$$v = N_{Re}\left(\frac{z^2}{2} - \frac{1}{8}\right)\frac{1}{r}\frac{\partial p}{\partial \theta} + r \tag{5}$$

Integrating (3) over $-1/2 \le z \le 1/2$ and using the third boundary condition gives

$$0 = \int_{-1/2}^{1/2} \left[\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \theta}\right] dz \tag{6}$$

and substituting u and v from (4) and (5) finally yields the governing equation for the pressure field,

$$\nabla^2 p = r^2 \tag{7}$$

Introducing $\bar{u}$ to indicate the mean velocity, integration of (4) over z gives

$$\bar{u}(r, \theta) = N_{Re}\left[\frac{\partial p}{\partial r}(r, \theta) - 1\right](-1.12)$$

from which

$$\frac{\partial p}{\partial r}(1, \theta) = 1 - \frac{12\bar{u}(1, \theta)}{N_{Re}}$$

results as a boundary condition for (7). The solution for (7) is found by the method of separation of variables to be

$$p(r, \theta) = \frac{r^2 - r_i^2}{2} - \frac{6}{\pi N_{Re}}\int_{-\pi}^{\pi} \bar{u}(1, \alpha)d\alpha \ln\frac{r}{r_i} - \frac{12}{\pi N_{Re}}\sum_{n=1}^{\infty} \frac{1}{n}\left[\frac{r^{2n} - r_i^{2n}}{r^n(1 + r_i^{2n})}\right]\left\{\int_{-\pi}^{\pi} \bar{u}(1, \alpha)\cos n\alpha d\alpha \times \cos n\theta + \int_{-\pi}^{\pi} \bar{u}(1, \alpha)\sin n\alpha d\alpha \sin n\theta\right\} \tag{8}$$

According to (4) and (5), the velocity components are given by

$$u(r, \theta, z) = -\left(\frac{z^2}{2} - \frac{1}{8}\right)\left\{\frac{6}{\pi r}\int_{-\pi}^{\pi} \bar{u}(1, \alpha)d\alpha + \frac{12}{\pi}\sum_{n=1}^{\infty}\left[\frac{r^{n-1} + r_i^{2n}r^{-n-1}}{1 + r_i^{2n}}\right]\left[\int_{-\pi}^{\pi} \bar{u}(1, \alpha)\cos n\alpha d\alpha \cos n\theta + \int_{-\pi}^{\pi} \bar{u}(1, \alpha)\sin n\alpha d\alpha \sin n\theta\right]\right\} \tag{9}$$

and

$$v(r, \theta, z) = -\left(\frac{z^2}{2} - \frac{1}{8}\right)\left(\frac{12}{\pi}\right)\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\frac{r^{2n} - r_i^{2n}}{r^{n+1}(1 + r_i^{2n})}\right] \times\left\{\int_{-\pi}^{\pi} \bar{u}(1, \alpha)\cos n\alpha d\alpha \sin n\theta - \int_{-\pi}^{\pi} \bar{u}(1, \alpha)\sin n\alpha d\alpha \times \cos n\theta\right\} + r \tag{10}$$

The mean velocity components may now be consolidated into a single stream function ψ defined by $u_m = \frac{\partial \psi}{r\partial \theta}$ and $v_m = -\frac{\partial \psi}{\partial r}$, where

$$u_m = \int_{-1/2}^{1/2} u dz \quad \text{and} \quad v_m = \int_{-1/2}^{1/2} v dz$$

Thus ψ becomes

$$\psi = -\frac{r^2}{2} + \frac{\theta}{2\pi}\int_{-\pi}^{\pi} \bar{u}(1, \alpha)d\alpha - \sum_{n=1}^{\infty}\left[\frac{(r^{2n} + r_i^{2n})}{r^n n\pi(1 + r_i^{2n})}\right] \times\left[\int_{-\pi}^{\pi} \bar{u}(1, \alpha)\sin n\alpha d\alpha \cos n\theta - \int_{-\pi}^{\pi} \bar{u}(1, \alpha)\cos n\alpha d\alpha \sin n\theta\right] \tag{11}$$

Special Case—Admission Through a Single Slot

In this case, let $\bar{u}(1, \theta)$ be defined as

$$\bar{u}(1, \theta) = \begin{cases} -\bar{u}_0 & -\beta/2 \le \theta \le \beta/2 \\ 0 & \beta/2 < \theta < 2\pi - \beta/2 \end{cases}$$

where β is the central angle of the admission slot. The mean velocity components and the pressure become

$$u(r, \theta) = -\frac{\bar{u}_0}{2\pi}\left\{\beta + 4\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{r^{n-1} + r_i^{2n}r^{-n-1}}{1 + r_i^{2n}}\right] \times \sin\frac{n\beta}{2}\cos n\theta\right\} \tag{12}$$

$$v(r, \theta) = r + \frac{2\bar{u}_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\frac{r^{2n} - r_i^{2n}}{r^{n+1}(1 + r_i^{2n})}\right]\sin\frac{n\beta}{2}\sin n\theta \tag{13}$$

$$p(r, \theta) = \frac{r^2 - r_i^2}{2} + \frac{6\bar{u}_0\beta}{\pi N_{Re}}\ln\frac{r}{r_i} + \frac{24\bar{u}_0}{\pi N_{Re}}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\frac{r^{2n} - r_i^{2n}}{r^n(1 + r_i^{2n})}\right] \times \sin\frac{n\beta}{2}\cos n\theta \tag{14}$$

It is noted that the mean velocity components depend on $\bar{u}_0$ as a parameter while the pressure distribution depends in addition on Reynolds number as a parameter. The stream function is

$$\psi = -\frac{r^2}{2} + \frac{\bar{u}_0}{\pi}\left\{\frac{\beta\theta}{2} + 2\sum_{n=1}^{\infty}\frac{(r^{2n} + r_i^{2n})\sin(n\beta/2)}{n^2(1 + r_i^{2n})r^n}\sin n\theta\right\} \tag{15}$$

As examples, numerical computations were made for several flow cases. For the case for which $\beta = 20.5$ deg, $r_i = 0.1$, $\bar{u}_0 = 0.1$, and $N_{Re} = 0.5$, the idealizations are valid except very near the slot inlet. Figs. 1 and 2 present the radial velocity component and the pressure, respectively, as a function of the radial and tangential space coordinates. The streamlines are tight spirals for this case; the flow makes approximately 20 revolutions in traveling from the inlet to the exhaust circle.

For the case for which $\beta = 20.5$ deg, $r_i = 0.3$, $\bar{u}_0 = 0.5$, and $N_{Re} = 0.5$, the idealizations are somewhat compromised by the relatively large radial velocity. Figs. 3 and 4 present $\bar{u}$ and p for this case.

Conclusion

In the case of full admission, $\beta = 2\pi$ and the solution reduces to

$$u(r, \theta) = -\frac{12\bar{u}_0}{r}\left(\frac{z^2}{2} - \frac{1}{8}\right)$$

$$v(r, \theta) = r$$

$$w(r, \theta) = 0$$

$$\bar{u}(r, \theta) = -\frac{\bar{u}_0}{r}$$

$$\bar{v}(r, \theta) = r$$

$$p(r, \theta) = \frac{r^2 - r_i^2}{2} + \frac{12\bar{u}_0}{N_{Re}}\ln\frac{r}{r_i}$$

Here, the nature of the solution is clearly displayed and the effect of the idealizations is pointed up. The quadratic term in the pressure expression corresponds to that required to sustain the fluid between the disks with no relative flow.

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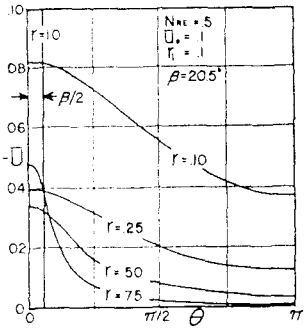

Fig. 1 Distribution of radial velocity component as function of space coordinates, $\bar{u}_0 = 0.10$, $r_i = 0.10$

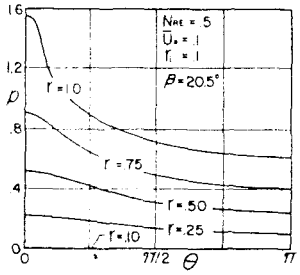

Fig. 2 Distribution of pressure as function of space coordinates, $\bar{u}_0 = 0.10$, $r_i = 0.10$

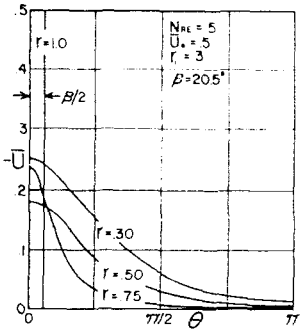

Fig. 3 Distribution of radial velocity component as function of space coordinates, $\bar{u}_0 = 0.50$, $r_i = 0.30$

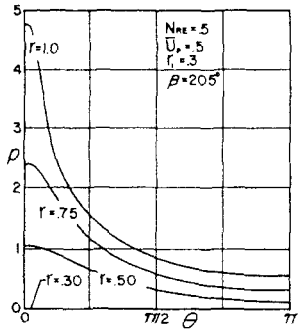

Fig. 4 Distribution of pressure as function of space coordinates, $\bar{u}_0 = 0.50$, $r_i = 0.30$

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Laminar Inward Flow of an Incompressible Fluid Between Rotating Disks, With Full Peripheral Admission

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The laminar flow of an incompressible Newtonian fluid, radially inward between parallel co-rotating disks is considered. The through-flow is supported by an externally applied pressure difference between the outer periphery and a circular fluid exhaust hole at an inner radius. The fluid supplied at the outer periphery is considered with arbitrary velocity components, such that the tangential component may be greater or less than the disk peripheral velocity. A sufficiently complete problem statement is formulated from the Navier-Stokes' equations. The problem has three parameters: a Reynolds number, a flow-rate parameter, and a peripheral tangential velocity component parameter. A numerical method of solution is detailed and typical numerical results are given illustrating the phenomena that occur in the inlet region for various inlet conditions. It is shown that the solution becomes the asymptotic solution given by previous investigators at interior radii following the inlet. Correspondence between the complete solution given herein and the earlier asymptotic solutions is established as dependent on corresponding values of Reynolds number and flow rate only. The results are discussed from the point of view of application of the solution in the development of multiple-disk turbines.

Introduction

THE CONCEPTION and early development of multiple-disk turbomachinery were reported in the semitechnical press in the first part of this century [1-6].[2] Subsequently, numerous experimental and analytical investigations have confirmed the feasibility of such turbomachinery [7-13]. Several analyses have appeared using a hydraulic friction factor or other empirical information to obtain an approximate description of the flow between the disks and others have assumed velocity distributions to estimate performance [14-17]. These analyses have been useful in further confirming feasibility and for estimation of the torque and work delivered to or absorbed by the disks, but have given only crude descriptions of the pressure field and hence of the efficiency of the multiple-disk turbomachine. Furthermore, the analyses are mainly limited to the flow of an incompressible fluid although both laminar and turbulent flow are considered.

The fluid flow fields about rotating disks have of course received intensive investigation, but most results pertain to disks rotating in free space and to housed disks with no through-flow of fluid and no externally imposed pressure difference. Schlichting [18] has summarized such results and a treatise by Dorfman [19] has further organized the methods of analysis and the results for rotating disk problems. Dorfman [19] also indicates an integral-type approach for calculation of the flow between rotating disks, for radially outward flow. Savage [20] has given a solution for laminar flow between stationary circular disks for inward flow; the solution is iterative and is carried through one iterative improvement of a creeping flow solution. Other approaches to the problem of flow between rotating disks have been made and are of interest but not of direct usefulness for determining the flow field in multiple-disk turbomachinery [21-24].

For the particular case of outward flow between rotating disks, corresponding with the multiple-disk pump, Breiter and Pohlhausen [25] obtained a numerical solution of sufficiently complete flow governing equations with realistic inlet conditions. The solution is for laminar flow of an incompressible fluid and is definitively useful for the outward flow case. Breiter and Pohlhausen [25] also obtained a solution for the problem when the governing equations are linearized by assuming very small flow rate. The solution is then a similarity solution valid for small flow rate and at radial stations well removed from the entrance region; that is, it describes the asymptotic flow field for small through-flow rate. Experimental work following the analysis was reported by Hasinger and Kehrt [26]. Poube and Kreith [27] have given a truncated series-type solution for the asymptotic flow field, away from the inlet, for outward flow of an incompressible fluid between rotating disks.

For the case of inward flow between rotating disks, corresponding with the multiple-disk turbine, analyses and results were given by Matsch and Rice for potential flow between the disks [28] and for creeping flow between the disks [29] with centrifugal effects accounted for. These analyses considered both partial admission and full admission of the fluid at the outer periphery. The results are of little interest with regard to the multiple-disk turbine except that the characteristics of the flow field are displayed for the limiting cases of large and small Reynolds number at all volume flow rates corresponding with laminar flow. Matsch and Rice [30] further have given an iterative solution for the asymptotic flow field reached by an incompressible flow at all radii interior to a certain entrance region. The asymptotic flow depends only on the fluid flow rate and the Reynolds number as parameters and is independent of the magnitude of the tangential component of velocity at the entrance and of the disk-to-disk distribution of all velocity components at the entrance. The solution is for a problem statement composed of sufficiently descriptive governing equation and boundary conditions; thus the results for both the velocity and pressure fields are useful for calculating the performance of an actual multiple-disk turbine in the flow region away from the entrance. However, important departures from asymptotic flow usually occur near the entrance and are reflected in the torque and the pressure field in the turbine in that region; hence the asymptotic solution is not sufficient alone to enable calculation of actual turbine performance with desirable entrance conditions.

Definitive analyses for the inward flow between rotating disks for turbulent incompressible flow and for both laminar and turbulent compressible flow are ultimately of main interest but have not yet appeared.

Herein, an analysis and a numerical solution for the flow of an incompressible Newtonian fluid between parallel rotating disks are given, valid over the flow rate and Reynolds number ranges for which the flow is laminar. General entrance conditions are considered and the governing equations, boundary conditions, and solution method are of sufficient accuracy to make the results definitively useful. The solution requires the flow rate, Reynolds number, and mean tangential velocity component at the entrance as parameters; hence it describes the flow in the entrance and approaches the asymptotic flow [30] at interior radii.

The geometry considered and a space-fixed coordinate system are shown in Fig. 1. Full admission of fluid around the outer periphery is assumed. It is realistically assumed that the fluid fills the entire space between the disks and that the flow is steady with constant angular velocity of the parallel circular disks about the z-axis. The fluid is considered to exhaust at an inner radius into a region of constant sink pressure.

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[2] Numbers in brackets designate References at end of paper.

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Nomenclature

ρ = density of fluid, dimensional, M/L^3

Ω = angular velocity of disks, dimensional, $1/T$

μ = viscosity of fluid, dimensional, FT/L^2

h = spacing between disks, dimensional, L

r_o = outer radius of disks, dimensional, L

r_i = inner radius of disks, dimensional, L

$\hat{u}$ = radial component of velocity, dimensional, L/T

$z = \hat{z}/h$ = dimensionless axial space coordinate

$N_{RE} = \rho\Omega r_o^2/\mu$ = Reynolds number

$u_{(1,z)}$ = distribution of dimensionless radial component of velocity at $r = 1$

$v_{(1,z)}$ = distribution of dimensionless tangential component of velocity at $r = 1$

$\bar{v}$ = tangential component of velocity, dimensional, L/T

$\hat{w}$ = axial component of velocity, dimensional, L/T

$\hat{r}$ = radial space coordinate, dimensional, L

$\hat{z}$ = axial space coordinate, dimensional, L

$\hat{p}$ = pressure, dimensional, F/L^2

$u = \hat{u}/\Omega\hat{r}_o$ = dimensionless radial component of velocity

$r_i = \hat{r}_i/\hat{r}_o$ = dimensionless inner radius of disks

i, j = integers denoting location of grid point in grid space

JM = maximum value of j

Δr = increment of dimensionless radial space coordinate

Δz = increment of dimensionless axial space coordinate

$\Delta u, \Delta v, \Delta w$ = variables after transformation defined by equation (27)

$U_0 = \int_{-1/2}^{1/2} u(1, z)dz$ = mean value of $u_{(1,z)}$

$V_0 = \int_{-1/2}^{1/2} v(1, z)dz$ = mean value of $v_{(1,z)}$

$v = \hat{v}/\Omega\hat{r}_o$ = dimensionless tangential component of velocity

$w = \hat{w}/\Omega h$ = dimensionless axial component of velocity

$p = \hat{p}/\rho\Omega^2\hat{r}_o^2$ = dimensionless pressure

$r = \hat{r}/\hat{r}_o$ = dimensionless radial space coordinate

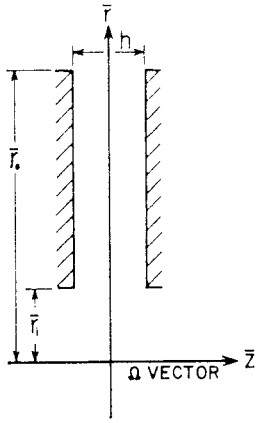

Fig. 1 Geometry and coordinate system

Analysis

Under the stated idealizations the Navier-Stokes' equations apply with all derivatives in the tangential direction omitted due to axial symmetry of the flow [19]:

$$\rho\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{r}} - \frac{\bar{v}^2}{\bar{r}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}\right) = -\frac{\partial\bar{p}}{\partial\bar{r}} + \mu\left(\frac{\partial^2\bar{u}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{u}}{\partial\bar{r}} - \frac{\bar{u}}{\bar{r}^2} + \frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right) \quad (1)$$

$$\rho\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{r}} + \frac{\bar{u}\bar{v}}{\bar{r}} + \bar{w}\frac{\partial\bar{v}}{\partial\bar{z}}\right) = \mu\left(\frac{\partial^2\bar{v}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{v}}{\partial\bar{r}} - \frac{\bar{v}}{\bar{r}^2} + \frac{\partial^2\bar{v}}{\partial\bar{z}^2}\right) \quad (2)$$

$$\rho\left(\bar{u}\frac{\partial\bar{w}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}\right) = -\frac{\partial\bar{p}}{\partial\bar{z}} + \mu\left(\frac{\partial^2\bar{w}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{w}}{\partial\bar{r}} + \frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right) \quad (3)$$

$$\frac{\partial\bar{u}}{\partial\bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \quad (4)$$

With the assumptions that the velocity and pressure fields are well behaved and that $h \ll \bar{r}_0$, which is true for geometries of interest, order-of-magnitude arguments show that the following equations are sufficiently accurate to describe the flow throughout the region between the disks and for any Reynolds number for which the flow is laminar. In terms of dimensionless variables,

$$u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{N_{RE}}\frac{\partial^2 u}{\partial z^2} \quad (5)$$

$$u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = \frac{1}{N_{RE}}\frac{\partial^2 v}{\partial z^2} \quad (6)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (7)$$

Entrance and exhaust conditions are

$$u(1, z) = u_0(z) \qquad -1/2 \le z \le 1/2 \quad (8)$$

$$v(1, z) = v_0(z) \qquad -1/2 \le z \le 1/2 \quad (9)$$

$$p(r_i, z) = 0 \qquad -1/2 \le z \le 1/2 \quad (10)$$

and appropriate boundary conditions are

$$u(r, \pm 1/2) = 0 \qquad r_i \le r \le 1 \quad (11)$$

$$v(r, \pm 1/2) = r \qquad r_i \le r \le 1 \qquad a \le r_i \le 1 \quad (12)$$

$$w(r, \pm 1/2) = 0 \qquad r_i \le r \le 1 \quad (13)$$

where $a \sim h/\bar{r}_0$; that is, where r_i is not so small as to invalidate certain of the order of magnitude arguments.

It is convenient to consider only entrance conditions that are symmetrical about the $r - \theta$ plane midway between the disks; then the region of interest is reduced to that corresponding with $0 \le z \le 1/2$ and $r_i \le r \le 1$. The entrance, exhaust, and boundary conditions become

$$\left.\begin{aligned} u(1, z) &= u_0(z) \\ v(1, z) &= v_0(z) \\ p(r_i, z) &= 0 \end{aligned}\right\} \quad 0 \le z \le 1/2 \quad (14)$$

$$\left.\begin{aligned} u(r, 1/2) &= w(r, 1/2) = w(r, 0) = 0 \\ v(r, 1/2) &= r \\ \frac{\partial u}{\partial z}(r, 0) &= \frac{\partial v}{\partial z}(r, 0) = 0 \end{aligned}\right\} \quad r_i \le r \le 1 \quad (15)$$

Here the exhaust pressure is written as zero merely for convenience. Equations (5), (6), (7), (14), and (15) constitute the problem statement. It is noted that the problem is well posed and has N_{RE}, $u_0(z)$, and $v_0(z)$ as parameters. $u_0(z)$ and $v_0(z)$ are arbitrary but specified functions and are considered to be even functions in the region $-1/2 \le z \le 1/2$. It is easy to show that $u(r, z)$ is an odd function in that region and that

$$\frac{\partial w}{\partial z}(r, 1/2) = 0 \quad (16)$$

Equation (16) is required for the solution as an auxiliary relationship.

An extensive investigation by one of the authors [31], using methods given by Hansen [32], has shown that no similarity solution exists for the problem. Furthermore, no non-trivial similarity solution exists for the reduced asymptotic problem statement due to Matsch and Rice [30]. As previously stated a similarity solution exists for the problem in which equations (5), (6), and (7) are linearized, and that solution was found by Breiter and Pohlhausen [25] but is not generally useful; it is thought to be the only similarity solution pertinent to flows between rotating disks.

It is very unlikely that a functional solution can be found for the problem statement composed of equations (5), (6), (7), (14), (15), and (16). For that reason, a numerical solution with sufficient accuracy and requiring reasonable execution time using a digital computer was used as described in the following section.

Solution

A numerical solution employing a direct difference scheme was

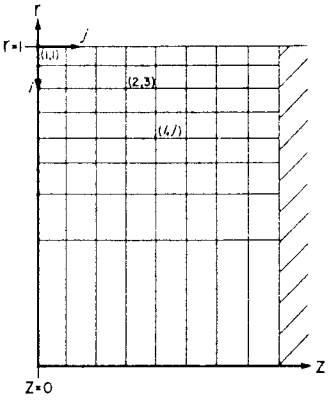

Fig. 2 Indexing of field grid points in relation to r, z-coordinate system

exhaustively pursued and failed to yield satisfactory results. In that approach, and in the successful approach subsequently used, the field was provided with a grid with uniform grid spacing in the z-direction and nonuniform grid spacing in the r-direction. Grid points were designated by i, j subscripting as shown in Fig. 2.

It is clear from the form of equations (5), (6), and (7) that a forward-stepping solution can be used to solve for field values at interior radii starting from known conditions at $r = 1$. In the direct difference approach, equations (5), (6), and (7) were written at the grid points not located on a boundary using first-order difference approximations for the derivatives and equations (14) and (15) were similarly written at grid points located on the boundaries. By a suitable logical scheme, values were found which caused the difference equations evaluated at $r = 1$ to be satisfied and with the solution thus started, a direct forward-stepping solution was carried out. The results indicated that unreasonably small radial and axial grid spacing would be necessary to achieve sufficient accuracy since stepping errors were cumulative. Computational stability problems were also encountered. Numerous modifications and improvements within the direct approach were investigated without success, including the incorporation of numerous derived boundary compatibility relationships in the system of difference equations.

Application of an indirect, or inverse, difference scheme proved successful; this approach was also used by Breiter and Pohlhausen [25] for the solution for outward flow. In the indirect approach, the grid displayed in Fig. 2 was used. Difference equations were established by approximately representing the derivatives by first-order difference forms in equations (4), (5), and (6) at grid points not located on the boundaries. Equations (14), (15), and (16) were similarly replaced by first-order difference equations at grid points located on the boundaries. The resulting equations are, for $1 < j < JW$,

$$-u_{i+1,j}\left(\frac{u_{i+1,j} - u_{i,j}}{(\Delta r)_i}\right) - \frac{v_{i+1,j}^2}{r_{i+1}} + w_{i+1,j}\left(\frac{u_{i+1,j+1} - u_{i+1,j-1}}{2(\Delta z)_j}\right) = \frac{dp}{dr}_{i+1} + \frac{1}{N_{RE}}\left(\frac{u_{i+1,j+1} - 2u_{i+1,j} + u_{i+1,j-1}}{(\Delta z)_j^2}\right) \quad (17)$$

$$-u_{i+1,j}\left(\frac{v_{i+1,j} - v_{ij}}{(\Delta r)_i}\right) + \left(\frac{u_{i+1,j} \times v_{i+1,j}}{r_{i+1}}\right) + w_{i+1,j}\left(\frac{v_{i+1,j+1} - v_{i+1,j-1}}{2(\Delta z)_j}\right) = \frac{1}{N_{RE}}\left(\frac{v_{i+1,j+1} - 2v_{i+1,j} + v_{i+1,j-1}}{(\Delta z)_j^2}\right) \quad (18)$$

$$-\left(\frac{u_{i+1,j} - u_{i,j}}{(\Delta r)_i}\right) + \frac{u_{i+1,j}}{r_{i+1}} + \left(\frac{w_{i+1,j+1} - w_{i+1,j-1}}{2(\Delta z)_j}\right) = 0 \quad (19)$$

and, representing the boundary conditions and the auxiliary condition,

$$u_{i+1,JW} = 0 \quad (20)$$

$$v_{i+1,JW} = r_{i+1} \quad (21)$$

$$w_{i+1,JW} = 0 \quad (22)$$

$$-3u_{i+1,1} + 4u_{i+1,2} - u_{i+1,3} = 0 \quad (23)$$

$$-3v_{i+1,1} + 4v_{i+1,2} - v_{i+1,3} = 0 \quad (24)$$

$$w_{i+1,1} = 0 \quad (25)$$

$$w_{i+1,JW-2} - 4w_{i+1,JW-1} + 3w_{i+1,JW} = 0 \quad (26)$$

Equations (17–26) constitute $(3JW + 1)$ linearly independent algebraic equations in $(3JW + 1)$ unknowns; in principle, a solution can proceed from known values of u, v, w, and $\frac{dp}{dr}$ at $r = 1$, where $i = 1$, by solving the set of equations at $i + 1$. Subsequent forward progress through the field would proceed in the same manner. However, the set of equations is nonlinear and therefore such solution is not feasible. For this reason, a transformation was applied to equations (17–26) according to

$$\left.\begin{aligned} \Delta u_j &= u_{i+1,j} - u_{i,j} \\ \Delta v_j &= v_{i+1,j} - v_{i,j} \\ \Delta w_j &= w_{i+1,j} - w_{i,j} \end{aligned}\right\} \quad (27)$$

After transformation, the equations constitute $(3JW + 1)$ equations in $(3JW + 1)$ unknowns, the unknowns being $\Delta u_{i+1,j}$, $\Delta v_{i+1,j}$, $\Delta w_{i+1,j}$, and $\frac{dp}{dr}_{i+1}$. The equations contain terms having products of the Δ quantities. These terms are negligibly small compared with terms not containing Δ quantity products and are consequently dropped with the restriction that (Δr) be kept very small. After thus linearizing the difference equations, they are

$$\Delta u_{j-1}\left(-\frac{w_{i,j}}{2(\Delta z)_j} - \frac{1}{N_{RE}(\Delta z)_j^2}\right) + \Delta u_j\left(-\frac{u_{i,j}}{(\Delta r)_i} + \frac{2}{N_{RE}(\Delta z)_j^2}\right) + \Delta u_{j+1}\left(\frac{w_{i,j}}{2(\Delta z)_j} - \frac{1}{N_{RE}(\Delta z)_j^2}\right) + \Delta v_j\left(-\frac{2v_{i,j}}{r_{i+1}}\right) + \Delta w_j\left(\frac{u_{i,j+1} - u_{i,j-1}}{2(\Delta z)_j}\right) + \frac{dp}{dr}\Big|_{i+1} = \frac{v_{i,j}^2}{r_{i+1}} - \frac{w_{i,j}(u_{i,j+1} - u_{i,j-1})}{2(\Delta z)_j} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{N_{RE}(\Delta z)_j^2} \quad (28)$$

$$\frac{\Delta u_j r_{i,j}}{r_{i+1}} + \Delta v_{j-1}\left(-\frac{w_{i,j}}{2(\Delta z)_j} - \frac{1}{N_{RE}(\Delta z)_j^2}\right) + \Delta v_j\left(-\frac{u_{i,j}}{(\Delta r)_i} + \frac{u_{i,j}}{r_{i+1}} + \frac{2}{N_{RE}(\Delta z)_j^2}\right) + \Delta v_{j+1}\left(\frac{w_{i,j}}{2(\Delta z)_j} - \frac{1}{N_{RE}(\Delta z)_j^2}\right) + \Delta w_j\left(\frac{v_{i,j+1}}{2(\Delta z)_j} - \frac{v_{i,j-1}}{2(\Delta z)_j}\right) = -\frac{u_{i,j}v_{i,j}}{r_{i+1}} - \frac{w_{i,j}(v_{i,j+1} - v_{i,j-1})}{2(\Delta z)_j} + \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{N_{RE}(\Delta z)_j^2} \quad (29)$$

$$\Delta v_j\left(-\frac{1}{(\Delta r)_i} + \frac{1}{r_{i+1}}\right) + \frac{\Delta u_{j+1}}{2(\Delta z)_j} - \frac{\Delta u_{j-1}}{2(\Delta z)_j} = -\frac{u_{i,j}}{r_{i+1}} - \frac{w_{i,j+1}}{2(\Delta z)_j} + \frac{w_{i,j-1}}{2(\Delta z)_j} \quad (30)$$

$$\Delta u_{JW} = -u_{i,JW} \quad (31)$$

$$\Delta v_{JW} = -(\Delta r)_i \quad (32)$$

$$\Delta w_{JW} = -w_{i,JW} \quad (33)$$

$$-3\Delta u_1 + 4\Delta u_2 - \Delta u_3 = 3u_{i,1} - 4u_{i,2} + u_{i,3} \quad (34)$$

$$-3\Delta v_1 + 4\Delta v_2 - \Delta v_3 = 3v_{i,1} - 4v_{i,2} + v_{i,3} \quad (35)$$

$$\Delta w_1 = -w_{i,1} \quad (36)$$

$$\Delta w_{JW-2} - 4\Delta w_{JW-1} + 3\Delta w_{JW} = -w_{i,JW-2} + 4w_{i,JW-1} - 3w_{i,JW} \quad (37)$$

Equations (28–37) are a set of $(3JW + 1)$ linear, independent equations in the unknowns Δu_j, Δv_j, Δw_j, and $\frac{dp}{dr}$ at the $i + 1$ radial station. The equations may be solved at each forward step by matrix inversion. However, the matrix involved is "sparse"; the particular technique known as Gaussian elimination is ideally suited for the solution [33].

The indirect solution was carried out, starting from prescribed $u_0(z)$ and $v_0(z)$ at $r = 1$, by first assuming $w(1, z) = 0$ and calculating $\frac{dp}{dr}$ (1) from equations (17) and (19). While the $w(1, z)$-value assumed is not correct, $w(1, z)$ is actually very small and the use of an extremely small radial step size near $r = 1$ caused a correct value of $w(r, z)$ to be computed after approximately 20 radial steps, corresponding with approximately $r = 0.9999$. It is not worthwhile to use alternative procedures to calculate a correct value of $w(1, z)$. Then, equations (28–37) were solved at $i = 2$ after choosing small values for Δr and Δz. The Δ quantities obtained were used in equations (27) to obtain $u_{i,j}$, $v_{i,j}$, and $w_{i,j}$. The same procedure was then applied repeatedly to get field values successively at inner radii.

The accuracy of the solution was determined by calculating the lack of satisfaction of certain wall compatibility relationships and of the conservation of total volume flow rate. Specifically, six compatibility relationships were developed by evaluating equations (5) and (6), and those equations differentiated with respect to z once and twice, at the disk surface and at $z = 0$. An expression for the total volume flow rate was formed by integrating $u(z)$ with respect to z at each radial station, using numerical integration according to Simpson's rule. With the lack of satisfaction of these relationships displayed, the r step size was varied in the solution so as to cause satisfaction of all of the compatibility relations within a bound insuring sufficient accuracy of the solution. It was found that the solution was very insensitive to the size of Δz and that a radial step small enough to insure accuracy also assured computational stability.

A computational program [34] was written and the program was executed using a CDC 3400 digital computer having word length, and consequently digital accuracy, greater than routinely found in large-scale computers. The program was executed preliminarily using both double and single precision and single precision was found to be sufficient. For a single case prescribed by values of N_{RE}, U_0, and V_0, execution time was approximately 45 min using single precision.

The computational program finally was used with $JW = 21$ and with approximately 1500 radial steps between $r = 1$ and $r = 0.03$, the radial step size being scheduled to satisfy the accuracy criteria previously described. The program was also arranged to accept as part of the input information any $u_0(z)$ and $v_0(z)$ represented by JW data points. As special cases, the

program was arranged to compute JIC-values of $u_0(z)$ and $v_0(z)$ internally from given values of U_0 and V_0, where these are the mean radial and tangential velocity components at the entrance, when instructions were also given to assume either a parabolic distribution or a uniform distribution of velocity over the space between the disks. In all cases of different shapes assumed for the entering velocity distribution, U_0 and V_0 were used as the necessary parameters for a solution rather than $u_0(z)$ and $v_0(z)$, together with a statement of the shape assumed for the computation of a particular case.

Results

As partial verification of the consistency of the present analysis with earlier results, computations were conducted for a number of cases, each defined by specifying U_0, V_0, and N_{RE}, and with the further specification that $u_0(z)$ and $v_0(z)$ were parabolic. These results were compared at inner radial stations with the results of Matsch and Rice [30] for asymptotic flow. In all cases agreement was found for both the velocity and pressure fields, as is necessary for a valid solution.

As partial verification of the internal consistency and reasonableness of the present analysis, several computations were conducted for cases having the same values of U_0 and N_{RE} but having different values of V_0, with the specification that $u_0(z)$ and $v_0(z)$ were parabolic. At inner radii, away from the inlet, these cases all showed the same asymptotic field, as is necessary for a valid solution. As further partial verification, computations were conducted for cases having the same values of U_0, V_0, and N_{RE} but with various $u_0(z)$ and $v_0(z)$ distributions; cases examined included parabolic, uniform, and discontinuous entrance distributions. It was found that the fields computed at inner radii were independent of the distributions $u_0(z)$ and $v_0(z)$, as is necessary for a valid solution.

Fig. 3 is a reproduction of a figure from the report of the asymptotic flow solution by Matsch and Rice [30]. The figure shows that, for every set of values of U_0 and N_{RE}, there is an inner radius at which $u(z)$ becomes inflected; it has been hypothesized that an experimental flow will undergo transition to turbulent flow near that radius. For additional verification of the accuracy of the present analysis, computations were conducted using parabolic distributions of $u_0(z)$ and $v_0(z)$, to obtain the radii at which $u(z)$ becomes inflected; excellent agreement with the results shown in Fig. 3 was obtained over the entire domain of U_0 and N_{RE}.

The comparisons described allow high confidence in the analysis and in the computational accuracy obtained in subsequent results for cases which were not included in the results of earlier investigators. Only example cases of the computed results can be presented here, chosen to illustrate the general features of the flow between rotating disks with inward through-flow.

Typical results are shown in Figs. 4, 5, 6, and 7 for the velocity components and for pressure; these results are for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$ and for parabolic distributions of $u_0(z)$ and $v_0(z)$. From Fig. 4 it is noted that the maximum value of u increases as r decreases, as expected from continuity requirements. The profiles are not similar, and become less "full" progressively toward inner radii; an inner radius is finally reached at which the u profile becomes inflected. At all smaller radii the u profiles as calculated are increasingly inflected but are not believed of interest because an actual flow would prove to be turbulent at such radii. From Fig. 5 it is noted that rapid adjustment of the v profile takes place over a relatively small region of radial distance near the entrance, after which the v profiles are quite "flat," becoming less flat at inner radii. Behavior of the axial velocity w is shown in Fig. 6 while Fig. 7 depicts the pressure static and total pressure difference between a fluid supply reservoir and the flow field at radial station r. The stagnation pressure shown at $r = 1$ is the static pressure drop necessary to accelerate the flow reversibly from a stagnant supply reservoir to the velocities u, v, and $v_0(z)$ at $r = 1$; the static pressure drop between two radial stations within the rotor is due to the combination of a reversible pressure drop caused by the changing velocity field and an irreversible pressure drop caused by viscous action.

The scale to which Figs. 4, 5, and 6 are drawn obscures details of the entrance region. Fig. 8 presents details of the behavior of the radial velocity component in the entrance region; it is noted that $u_0(z)$ is parabolic but that u becomes inflected in a very small radial distance. With further radial progress, the inflection disappears although the monotonic distribution resulting in the region $0 \leq z \leq 1/2$ is not parabolic. At still smaller radius, the u profile becomes the distribution corresponding with the asymptotic flow for $U_0 = -0.10$ and $N_{RE} = 3.0$. Fig. 9 shows the behavior of the tangential velocity component in the entrance region. Here it is evident that the entrance behavior of both the radial and tangential components is due to primarily the adjustment that $v(r, 1/2)$ must make in moving from a stationary wall just outside of the disks to the wall moving with tangential velocity Ωr_0 just inside of the disks.

It is believed that the inflected u profiles near $r = 1$ would not lead to transition to turbulence in actual flow, since these occur

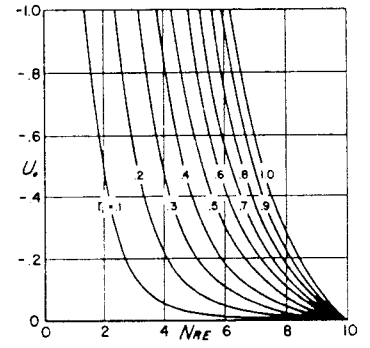

Fig. 3 Dimensionless inner radius r_i at which the radial velocity component profile becomes inflected, as a function of U_0 and N_{RE}.

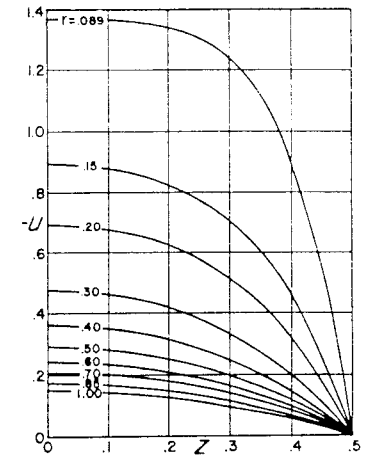

Fig. 4 Axial profiles of component u with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

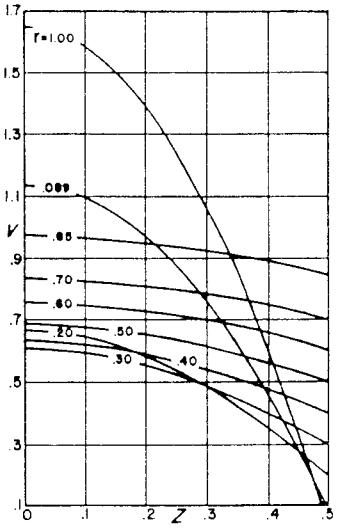

Fig. 5 Axial profiles of component v with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

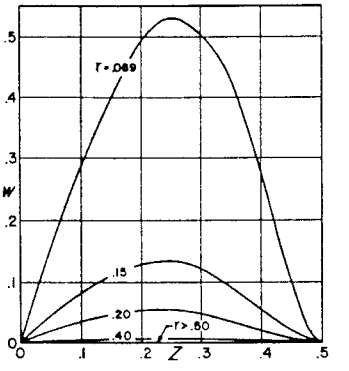

Fig. 6 Axial profiles of component of w with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

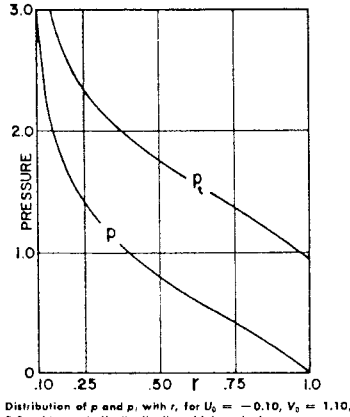

Fig. 7 Distribution of p and p_t with r, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

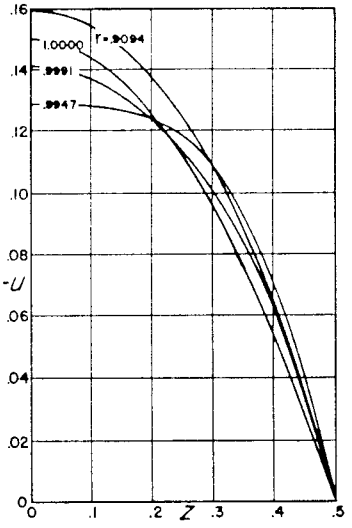

Fig. 8 Detail in inlet region of axial distribution of u with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

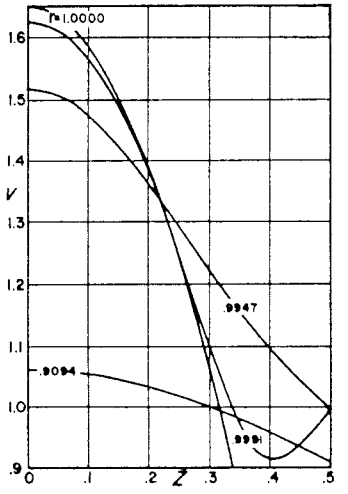

Fig. 9 Details in inlet region of axial distribution of v with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

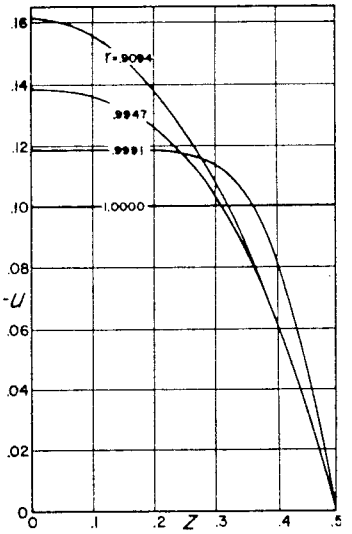

Fig. 10 Detail in inlet region of axial distribution of u with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with uniformly distributed inlet velocity

only to allow accommodation of the flow fields inside and outside of the disk space at $r = 1$. This is in contrast with inflection of the u profiles which occurs at radii at which the flow is asymptotic; inflection there occurs as the end result of a succession of profiles which become less full and hence less stable as r decreases and is believed to correspond with transition to turbulent flow.

Details of the inlet region for a case for which $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, and for which $u_0(z)$ and $v_0(z)$ are uniform, are shown in Figs. 10 and 11. The flow at inner radii is the same as given in Figs. 4 and 5 since the flow in that region is asymptotic and the two cases have the same values of U_0 and N_{RE}. It is noted that the pressure drop occurring in the inlet region is dependent on the $u_0(z)$ and $v_0(z)$ distributions; $p(0.9) = 0.168$ for parabolic inlet profiles and $p(0.9) = 0.145$ for uniform inlet profiles for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$.

In an actual multiple-disk turbine the flow can be admitted in several ways but most commonly the inlet distribution is either parabolic or uniform for most disk spaces. Because a turbine uses a number of disk spaces in parallel, both types of entrance conditions normally occur in a single turbine.

The results of a large number of cases computed show that a disk turbine is most efficient when V_0 is near one; this would be expected intuitively and from experience with conventional turbines. However, the torque and hence the work delivered per disk space increase with U_0 at the expense of efficiency; therefore, a range of U_0-values is of interest. Figs. 12 and 13 show inlet detail results for a typical case having large V_0; in these figures $U_0 = -0.10$, $V_0 = 2.00$, and $N_{RE} = 3.0$. The flow at inner radii is again the same as given in Figs. 4 and 5 because the U_0 and N_{RE}-values are the same. The pressure drop is much larger in the inlet region with increased tangential velocity V_0.

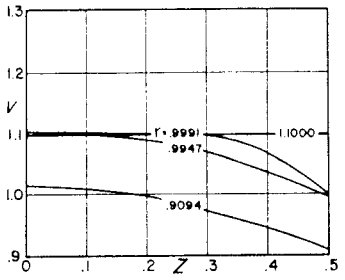

Fig. 11 Detail in inlet region of axial distribution of v with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.10$, and $N_{RE} = 3.0$, with uniformly distributed inlet velocity

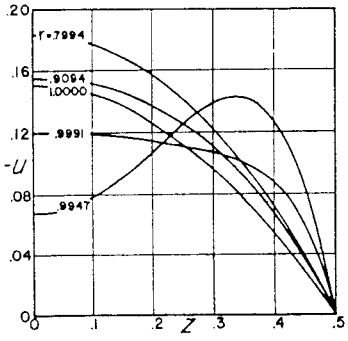

Fig. 12 Detail in inlet region of axial distribution of v with r as a parameter, for $U_0 = -0.10$, $V_0 = 2.00$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

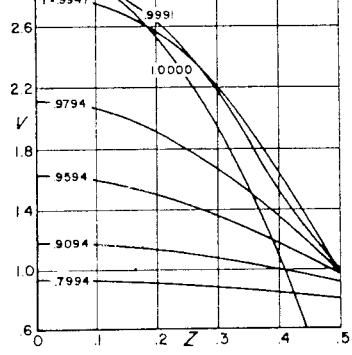

Fig. 13 Detail in inlet region of axial distribution of v with r as a parameter, for $U_0 = -0.10$, $V_0 = 2.00$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

Inlet details for a case with $V_0 < 1$ is given in Figs. 14 and 15; in these figures, $U_0 = -0.10$, $V_0 = 0.80$, and $N_{RE} = 3.0$. It is unlikely that a disk turbine would use $V_0 < 1$ as a design value since efficiency is low and torque is small compared with flow for which $V_0 \geq 1$.

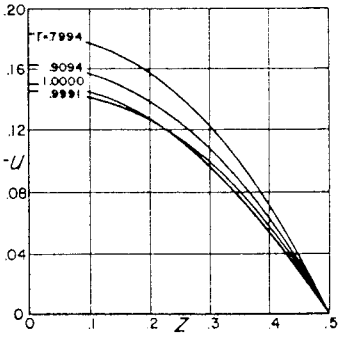

Fig. 14 Detail in inlet region of axial distribution of v with r as a parameter, for $U_0 = -0.10$, $V_0 = 0.80$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

The effect of increased U_0 and of increased N_{RE} on the radial component of velocity is shown in Figs. 16 and 17, respectively. The upper limit of combinations of U_0 and V_0 for which laminar flow is believed to occur was given in Fig. 3; however, this figure presumes that $u_0(z)$ and $v_0(z)$ are distributed according to the asymptotic solution. For nonasymptotic entrance conditions, such as the parabolic and uniform entrance conditions, it was earlier pointed up that inflected u profiles occur near the entrance, reverting to noninflected profiles at smaller radii. Now, for large N_{RE} and/or U_0-values asymptotic flow is not reached; that is, the inflections at the entrance persist and no uninflected profiles are reached. N_{RE}, U_0, and V_0-values are not sufficient to define this occurrence; it is dependent also on the distribution of $u_0(z)$ and $v_0(z)$. It is believed that the entire flow between disks would be turbulent when the laminar calculations result in inflected u profiles throughout the disk space.

Conclusion

The results give new insight into the phenomena occurring in the flow between rotating disks when radially inward flow occurs with a pressure difference provided over the disk radius. Because the problem description and the methods of solution yield accurate calculation of the velocity field, the pressure field is correspondingly accurate and the efficiency, torque, and power can be accurately determined for a turbine application of the flow.

It does not appear feasible to present the complete calculated results over the entire range of interest in any form. It is however feasible to make a summary of the torque, power, efficiency, and pressure drop as a function of U_0, V_0, N_{RE}, and r_i for specified distributions of $u_0(z)$ and $v_0(z)$. The results would enable rational design of multiple-disk turbines for laminar flow. The question of the regions of occurrence of laminar and of turbulent flow should first be settled through experimental research before the summary described would be meaningful and reliable. Experimental verification of transition from turbulent to laminar flow in outward radial flow between stationary parallel disks was provided by Kreith [35]. Furthermore, Kreith presented quantitative criteria for the radial station at which such transition occurs and suggested that laminar flow will exist only where the $u(z)$ profiles are uninflected. The present investigators have suggested that similar phenomenon occurs in radial inflow between rotating disks and that transition from laminar to turbulent flow will occur near the radii depicted in Fig. 3. Experimental verification of this and of the results of the present analysis is needed; no experimental data adequate for this purpose are presently available.

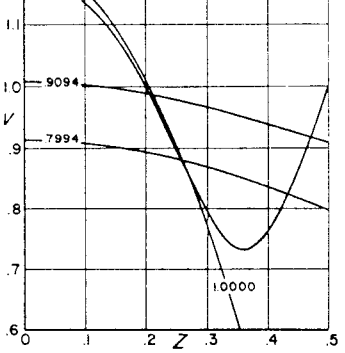

Fig. 15 Detail in inlet region of axial distribution of v with r as a parameter, for $U_0 = -0.10$, $V_0 = 0.80$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

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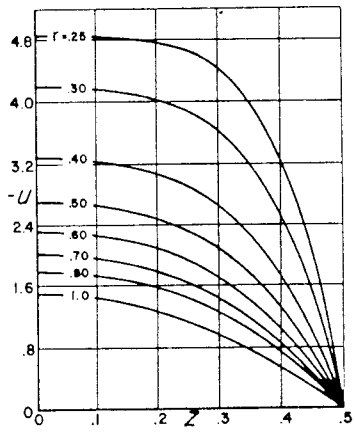

Fig. 16 Axial profiles of component u with r as a parameter, for $U_0 = -1.00$, $V_0 = 1.1$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

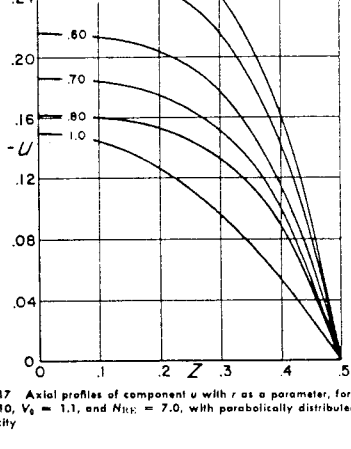

Fig. 17 Axial profiles of component u with r as a parameter, for $U_0 = -0.10$, $V_0 = 1.1$, and $N_{RE} = 7.0$, with parabolically distributed inlet velocity

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Laminar Inward Flow of an Incompressible Fluid Between Rotating Disks, With Full Peripheral Admission[1]

DUŠAN NENDL.[2] The author has presented a fine analytical approach to the investigation of the problem. The solution yields an accurate calculation of the velocity field for all radius $r < 1$. But for the immediate vicinity of the inlet, i.e., $r = 1 + \epsilon$, where $\epsilon \ll 1$, the boundary-layer assumption is too rough. Assuming the presented numerical solution as a first approximation for this region and plotting the results of Fig. 13 of the paper as a function of r with z as a parameter (Fig. 1 here), we see that in the immediate vicinity of $r = 1$ the derivative $\partial v/\partial r$ is very great and that in the region where $\partial v/\partial r$ changes its sign

[1] By K. E. Boyd and W. Rice, published in the June, 1968, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 35, TRANS. ASME, Vol. 90, Series E, pp. 229–237.

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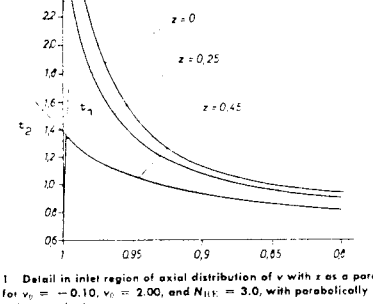

Fig. 1 Detail in inlet region of axial distribution of v with z as a parameter for $v_0 = -0.10$, $v_c = 2.00$, and $N_{RE} = 3.0$, with parabolically distributed inlet velocity

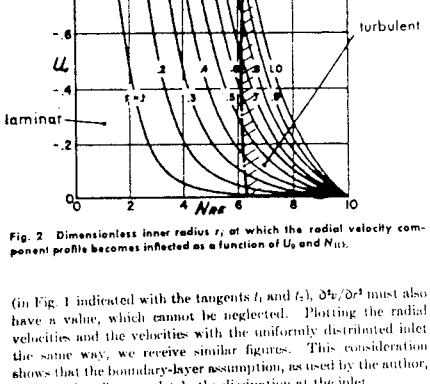

Fig. 2 Dimensionless inner radius r_i at which the radial velocity component profile becomes inflected as a function of U_0 and N_{RE}

(in Fig. 1 indicated with the tangents t_1 and t_2), $\partial^2 v/\partial r^2$ must also have a value, which cannot be neglected. Plotting the radial velocities and the velocities with the uniformly distributed inlet the same way, we receive similar figures. This consideration shows that the boundary-layer assumption, as used by the author, does not describe completely the dissipation at the inlet.

Further considerations show that the inflection of the $u(z)$ profiles does not need to correspond with the transition from laminar to turbulent flow. Rearranging Kreith's transition criterion, with $Q = 2\pi h \bar{v}_c \bar{u}_c$ (in the Nomenclature of the paper),

$$\frac{1}{K^2} = \pi\rho\frac{(h^2\bar{r}_c\bar{u}_c}{\mu} = \pi\left(\frac{h}{\bar{r}_c}\right)\left(\frac{\rho h\bar{v}_c}{\mu}\right) \tag{1}$$

we see, that the factor $1/K^2$ has the same form as the viscogeometrical coefficient α used in the analytical analyses of Fiebig[3] and Nendl [17] of the paper. The Reynolds number $\mathrm{Re} = \rho h\bar{v}_c/\mu$ (= $\sim\rho Q/\mu\bar{r}$) alone (see (1)), as used by Moller, does not seem to be a satisfactory criterion for a radial flow between two planes, as the geometrical form expressed in $h/\bar{r}_c$ is of essential importance. Assuming, that the radial inflow between two rotating disks has the same value as the coefficient of transition $K = 0.22$ as an outward radial flow between stationary parallel disks and rearranging equation (1) for the outer inlet conditions, we receive

$$\frac{1}{K^2} = \pi N_{RE}\, U_0\frac{1}{r_c} \tag{2}$$

Using $r_c = r_i$ as a solution parameter, we can plot equation (2) in Fig. 2. The shaded area is the turbulent region with $K > 0.22$, the unshaded left area the laminar region with $K < 0.22$. This consideration shows:

Given a critical value of K_r = const or α_c = const for a radial inflow between two rotating disks, the transition from laminar to turbulent flow does not have to correspond with the inflection in the radial velocity distribution. These can be two quite independent phenomena.

J. L. DUSSOURD.[4] This discusser is pleased, indeed, to see that the work initiated at Arizona State University on the subject of rotating disks turbomachinery has been carried to the point where exact solution of the flow has been sought. This type of device should be finding applications in many processes where unusual fluids are handled and where efficiency is secondary.

Study of the mathematics of the authors' flow model did, however, pose some problems to this discusser and help or clarification is hereby sought.

1 Following reduction of equations (1)–(4), to equations (5)–(7), care must be taken in the selection of the boundary condition which, now must be reduced in number. The following points are made:

(*a*) Inlet condition at $r = 1$ should specify the distribution of u. In the present case either $u(1, z) = 0$ or some other condition must be introduced such as a given profile in u and v at $r \geq 1.0$. This in turn defines w at $r = 1.0$.

(*b*) The condition $p(r_i, z) = 0$ (equation (14) of the paper) is not really a boundary condition since the nature of the solution is unchanged by the chosen value of p at r_i. Note that the condition of consistency of pressure at $r = r_i$ is already implicit in (5)–(7).

(*c*) Condition (16) of the paper

$$\frac{\partial w}{\partial z}(r, \tfrac{1}{2}) = 0$$

is really artificial and unrealistic. Should not a more useful one be

$$\frac{\partial^2 w}{\partial z^2}(r, 0) = 0$$

because of the inflection point at $z = 0$?

2 In an effort to study the importance of equation (3), which is later dropped by order-of-magnitude arguments, this discusser attempted to calculate the magnitude of some of the terms in equations (1)–(7) using the results of Figs. 4–7 of the paper. A selected point was $z = 0.4$, $r = 0.3$. Equation (5), and to a lesser extent, equation (6) could not be satisfied. Have the authors verified the results and satisfied themselves that Figs. 4–7 are indeed solutions of equations (5)–(7)?

3 Considerable interest exists in the resultant calculated shear stresses and torque for application to rotary disks turbines. However, nowhere in the paper is this vital information presented. This detracts considerably as to the usefulness of the results since verification of the accuracy of the solution can only be made by torque and pressure measurements and not at all by velocity profile measurements. The authors have considerable earlier results from experiments and less refined analysis. Do they agree with the present results?

This discusser hopes these points of clarification can be quickly disposed of by the authors and that they do not detract from the merits of the paper. Again, very sincere congratulations are extended for attacking a difficult and interesting problem.

Authors' Closure

The remarks of the discussers are considered by the authors to be thoughtful and informative.

In the entrance region (and in the region where r is small), the viscous terms dropped by the authors are indeed appreciable. However, omission of the terms is not so serious as suggested by Nendl; the entrance region is a very small part of the total flow of interest, and the terms are not appreciable in the asymptotic flow. Furthermore, the pressure drop is the main quantity of interest in the entrance region, and omission of the terms does not strongly affect the pressure calculations. Lastly, of the three viscous terms omitted, two of the same order of magnitude are of opposite sign and tend to cancel. The authors are confident that the solution is definitively useful even though the terms are omitted.

The justification of the foregoing remarks is brief computer studies of the size of the neglected terms and the effect of the terms on the pressure gradient in the entrance region, and the results of experimental work completed since publication of the paper under discussion. Details of the experimental study await publication, but it can be reported here that the analytical results are completely substantiated even in the entrance region, insofar as pressure distribution is concerned.

The experimental results also show that the authors' hypothesis of transition near the radius at which inflection of the u profile occurs is incorrect. The suggestions of Nendl concerning transition are appreciated, and may be correct. However, the assumptions that rotation of the disks does not change the nature of the phenomenon of transition, or the value of K_r seem speculative. The experimental results referred to are expected to definitively answer the question of the criteria for transition for the flow between rotating disks.

The correctness of the problem statement used in the solution is of concern, as indicated by Dussourd. It is true that the condition $p(r_i, z) = 0$ is not a boundary condition and should have been introduced only as a convenience by the authors. The authors used $w(1, z) = 0$ for convenience; this matter is discussed briefly in the paper. The specification of $w(1, z)$ does not appear to be an inlet condition independent of $u(1, z)$ and $v(1, z)$; given u and v, w is uniquely determined, but it is inconvenient to use it "correctly," and a very small error is introduced by using $w(1, z) = 0$.

The authors agree that the use of $\dfrac{\partial w}{\partial z}(r, 1/2) = 0$ as an auxiliary condition is artificial. A more "correct" condition would be that suggested by Dussourd. A still more correct procedure would be to retain some form of the differential equation expressing forces in the axial direction, even though all terms of the equation are of negligible size compared with terms kept in the other equations. All that is necessary, however, is to introduce some restraint on the w solution which will cause $w(r, 0) = 0$ and $w(r, 1/2) \neq 0$ in the forward-stepping procedure and provide the additional equation needed for unique solution for u, v, w and dp/dr. The condition used is sufficient although not palatable.

The problem statement is satisfied by the results presented at $z = 0.4$, $r = 0.3$ to acceptable accuracy, for the U_0 and N_{RE} values used in Figs. 4–7 of the paper. It is difficult to obtain an accurate check using the figures in the paper; the authors conducted accuracy checks during computations which assured that all results were within acceptable error bounds with regard to satisfaction of continuity, equations of motion, and several "compatibility" functions. Again, recent experimental results confirm the correctness of the analytical results.

The shear stresses and the torque, and also the turbine efficiency, were computed originally but were not included in the paper because very extensive graphical presentation will be necessary. Also, these turbine performance characteristics will be more suitably discussed in a separate publication and with additional considerations which would be out of context in the paper under discussion. The authors agree that the information is vital for use of the results for disk turbine purposes and will make the information available as rapidly as possible.

An earlier paper by one of the authors[5] gave an unrefined analysis of the flow, using bulk flow parameters including a "friction factor," referred to by Dussourd. In the computations discussed here, the friction factor was computed as an afterthought, and was compared with the results of the earlier paper. It was found that the friction factor is actually far from constant, and the authors believe that the results of the earlier paper should not be used since better analyses are now available.

Earlier experimental results obtained by one of the authors were obtained using air as the fluid, with pressure ratios high enough to grossly violate the incompressible idealization of the present paper. Therefore, no comparison of results was attempted by the authors.

The authors hope that these remarks indicate the concern that they feel for the points raised by the discussers and the intent to continue improvement of knowledge in the field and reliable documentation of results.

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Experimental Investigation of the Flow Between Corotating Disks

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Experimental results are presented for the flow of an incompressible Newtonian fluid, with full admission, between closely spaced corotating disks. The radial pressure distribution is shown to coincide very closely with analytical results for laminar flow earlier presented by other investigators, over a wide range of the parameters pertinent to the flow description. A referenced analytical model for laminar flow between corotating disks is thus given credence. The experimental arrangement and procedure are described and a supporting uncertainty analysis is presented.

Introduction

The laminar throughflow of an incompressible Newtonian fluid between closely spaced parallel corotating disks has been considered analytically by numerous investigators recently [1–10].[2] This problem is distinguished from other flows in the presence of rotating disks by a throughflow of fluid between the disks. Breiter and Pohlhausen [1] and Puebe and Kreith [2] gave solutions for the case in which the fluid progresses radially outward between the disks; other investigators [3–9] have considered flow which progresses radially inward due to an externally applied pressure difference between the outer and inner peripheries.

The analytical interest stems from the occurrence of such flows in multiple-disk turbomachinery and in certain elements useful in fluidics. The analyses of references [1, 2, 10] relate to multiple-disk pumps; references [3–9] are concerned with the flow in multiple-disk turbines and in fluidic momentum rate sensors. The flow is also of interest in a variety of applications including water desalinization, acoustically quiet rotating machinery, and certain heat and mass transfer apparatus.

Several experimental investigations have been reported [11–15] in which multiple-disk pumps, blowers, and turbines were constructed and tested. These have shown the feasibility of multiple-disk turbomachinery and have demonstrated the uniqueness of certain performance characteristics, but the experimental results have not enabled confirmation of analytical results because of the losses in the turbomachines external to the disk rotors.

Each laminar flow analysis earlier referred to has incorporated one or more idealizations and/or compromises in the assumed geometry or in the terms kept in the governing differential equations, in order to facilitate a solution for the equations. In particular, Puebe and Kreith [2] used truncated series to represent the velocity components and the pressure and incurred errors of unknown magnitude due to the truncation. Investigators in references [1, 3–9] omitted certain terms from the Navier-Stokes equations to render them parabolic in form, following order-of-magnitude arguments. References [1, 7] found solutions numerically by methods subject to computational errors; these errors were shown to be minimal but undetermined. Matsch and Rice [5] used an iterative technique subject to undetermined error accumulation and a limited region of convergence.

The sources of error in the published analyses, and in particular, the reduction of the governing equations to parabolic form, have been critically reviewed by the discussers of earlier published analytical results [16, 17]. For further progress in application of flows between rotating disks, a large amount of parametric computation and presentation is required, based on the analytical results; resolution of the questions of correctness and accuracy of the analytical results has been needed.

Progress in application (and in further analytical study) of the flow between rotating disks has also been impeded by the lack of knowledge concerning criteria for transition from laminar to turbulent flow in a radially inward flow, and from turbulent to laminar flow in a radially outward flow. All analyses of laminar flow have shown a radial region in which the radial velocity component is inflected, and a region at larger radius in which it is uninflected. For radially inward flow, Matsch and Rice [5] correlated the radius at which inflection occurs with the two parameters for asymptotic flow, and suggested that the laminar flow for which the inflected profiles were calculated would be unstable and experimentally would mark the beginning of transition to turbulent flow. This possibility was restated and amplified by Boyd and Rice [7]. Nendl [16], in a discussion of reference [7], discounted these hypotheses, and suggested instead that transition is not directly related to the calculated inflected velocity profiles, but rather is described by an empirical correlation adapted from experimental results of Kreith [18] on transition in radially outward flow between stationary disks. Resolution of the question of the correctness of any of the foregoing hypotheses regarding transition is needed.

For assumed laminar flow, the analysis due to Boyd and Rice [7] is the most definitive, and the most practical with which to compare experimental data. It includes and agrees with all other analyses known to the authors, for flow in the asymptotic flow region. It is therefore used in the comparisons discussed later herein.

Much less has been accomplished analytically for turbulent flow than for laminar flow. Rice [12] and Beans [14] introduced different lumped-parameter governing equations and turbulent shear stress assumptions and calculated the resulting performance of turbine elements. Both approaches involved approximations that make the results speculative.

The experimental investigation herein reported was designed and conducted to provide experimental data with which previous analytical results for laminar analyses can be compared.

Design of Experiment and Apparatus

As shown by Matsch [6], the most general flow between disks is that for partial admission at the outer periphery; in this general laminar flow, the parameters are the Reynolds number N_{Re}, a volume flow rate parameter U_0, a tangential velocity parameter V_0, the peripheral distribution of the pressure and of the velocity components, and the disk-to-disk distribution of the velocity components. However, at a distance sufficiently inward from the periphery "asymptotic" flow is reached; the asymptotic flow region has only N_{Re} and U_0 as parameters. In the case of full admission, which all references except [3, 4, 6] treat exclusively, the flow has N_{Re}, U_0, V_0 and the disk-to-disk distribution of the velocity components as parameters; the flow becomes asymptotic at a larger radius than for partial admission, and has only the two parameters, N_{Re} and U_0, in the asymptotic flow region.

Following these realizations, the experiment was designed to provide full admission as nearly as possible, and to provide data at a wide range of combinations of N_{Re} and U_0, with no specific control of V_0 values or of the disk-to-disk distribution of the velocity components. This was in anticipation of at least a limited experimental range of laminar flow. Thus valid comparison of experimental and analytical data can be made only in the region of asymptotic flow. The experimental data also provide qualitative insight in the nonasymptotic entrance region.

The various analyses provide the three velocity components as functions of (r, z) and the pressure as a function of radius, as results. However, the analytical results are predicated for a configuration for which $h << r$; thus, in an experiment, the spacing between disks must be very small and measurement of the velocity profiles seems to be precluded. From examination of the calculated pressure as a function of radius, in the asymptotic flow region, for a wide range of combinations of the parameters N_{Re} and U_0, it is clear that the radial distribution of pressure is a rapidly changing and unique function of the parameters when the flow is laminar. It was concluded that the measurement of the radial pressure distribution is sufficient to verify the correctness (or incorrectness) of calculated results when the flow is laminar.

Experimentally determined pressure distribution data that agree with the calculated pressure distribution based on the laminar model in a radial region are taken to indicate that the flow is laminar in that region. A wide lack of agreement would indicate turbulent flow. It is pointed up that here, as elsewhere in fluid mechanics, the terms are somewhat relative; flow called "laminar" probably has intermittent turbulent regions and/or a thin locally turbulent core; flow called "turbulent" is merely mostly turbulent in time and space.

An extended study was conducted of the effect on uncertainty in the experimental results, of experimental errors in determining ρ, μ, Ω, h, R_0, Q, and p. The study resulted in the choice of disks with $R_0 = 8.000$ in., $R_i = 1.000$ in., $h = 0.060$ in., and in the decision to design and manufacture the experimental apparatus in a very substantial version using precision manufacturing techniques and massive structural parts. In particular, the disks were extremely rigid and were surface ground to achieve excellent dimensional tolerance with respect to flatness and smoothness. Large carefully aligned bearings were used to assure that the disks were parallel and concentric; direct measurement confirmed that these were attained after assembly. The flow was supplied by a nozzle ring which received flow from a plenum chamber and directed the flow through 30 discrete nozzles at a fixed small angle between the flow and a tangent to the disk circumference. The flow exhausted from the disks at radius R_0 through a hollow bearing shaft, into an exhaust plenum which was maintained full of fluid. The foregoing features are shown schematically in Fig. 1.

Small clearances were provided between the rotating disk assembly and the housing; carbon seals effectively prevented leakage between the stationary housing and the rotating assembly.

The choice of rotor dimensions, and the desired ranges of N_{Re} and U_0 to be achieved, governed the choice of angular velocity and of fluid properties. Errors were found to be minimized for $\nu \sim 5$ centistokes. Accordingly, the experimental fluid was chosen to be a water-ethylene glycol mixture having the desired value of ν. It was determined that this fluid also has a reasonably small coefficient of viscosity change with respect to temperature, which is required to minimize errors due to viscosity change with the small temperature rise through the flow caused by viscous dissipation.

The fluid flowing between the disks produces a turbine effect; however, for the single pair of disks, insufficient torque is produced to maintain rotation in the presence of restraining torques incident to the experimental apparatus. Therefore, the disk assembly was rotated at any desired speed by a variable speed gearmotor, indicated in Fig. 1.

The design study also resulted in the choice of an electronic turbometer for measurement of the volume flow rate, Q, and of electronic means of measuring the angular velocity, Ω.

Immediately before operation to obtain data, the density and viscosity of the water-ethylene glycol mixture were measured. Density was determined by use of a hydrometer; viscosity was determined using a Ubbelohde viscosimeter in a constant temperature bath.

The upper one of the two rotating disks was provided with 60 holes arranged in two diametrical patterns, as pressure taps at known radial positions. The holes were each 0.015 in. in diameter, made by drilling normal to the disk surface. It was determined by experiment during the design phase of the investigation that larger static pressure taps would introduce error in the pressure measurement due to the small (0.060 in.) disk spacing. The pressure measurement becomes precise (with respect to the effect of static pressure tap size) as the diameter of the pressure tap approaches zero; it was experimentally determined that the diameter chosen for the pressure taps for the main experimental apparatus yielded the asymptotically correct pressure well within other sources of experimental error in the pressure measurement.

Each pressure tap was connected hermetically to one of two rotary electrical selection valves, each of which connected to a pressure transducer, and all of which rotated with the disk assembly. In this way, at any time during the collection of data a selected pressure tap could be connected to a pressure transducer, and specifically, each pressure transducer was stepped from one pressure tap to the next in sequence in the normal collection of data.

Precautions were taken to assure that all connection tubing and selection valve parts and transducer volumes were filled with liquid at all times. In subsequent data reduction, due accounting was made for the pressure difference between each pressure

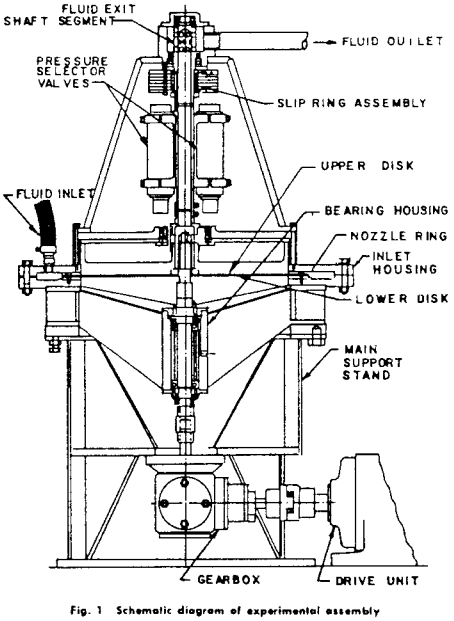

Fig. 1 Schematic diagram of experimental assembly

Nomenclature

h = distance between disks (L)

N_{Re} = dimensionless Reynolds number, $\Omega h^2/\nu$

p = pressure difference (F/L^2)

p = dimensionless pressure difference, $p/\rho\Omega^2R_0^2$

Q = volume flow rate (L^3/T)

r = radial distance (L)

r = dimensionless radial distance, r/R_0

R_0 = outer radius (L)

R_i = inner radius (L)

U_0 = dimensionless volume flow rate parameter, $Q/2\pi R_0^2 h\Omega$

V_0 = dimensionless velocity parameter, $\int_{-h/2}^{h/2} V_T(z)\,dz/h\Omega R_0$

V_T = tangential velocity at radius R_0, $V_T(z)$, (L/T)

ν = kinematic viscosity of fluid (L^2/T)

Ω = angular velocity of disks (rad/T)

ρ = density of fluid (M/L^3)

μ = viscosity of fluid (M/LT)

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[2] Numbers in brackets designate References at end of paper.

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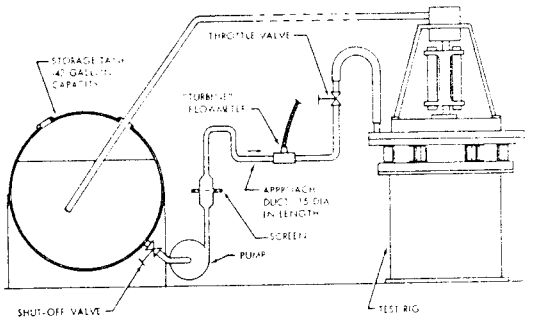

Fig. 2 Schematic diagram of fluid-flow circuit

tap and the transducer due to centrifugal and gravitational effects on the liquid in the connecting spaces. In the course of data collection, selection stepping was done sufficiently slowly to accord with the slow response caused by the liquid spaces and by the very small pressure taps. The entire experimental apparatus was designed and constructed to allow steady conditions over a long period of time.

Each pressure transducer was of the unbonded strain-gage type, with compensation elements of the bridge integrally constructed in the transducer. The a-c excitation was provided from a source located on the rotating assembly. The effective output from the pressure transducer was telemetered from the rotating assembly using an FM telemetry transmitter and an FM receiver; no slip rings were used. The output of the receiver was in the form of a d-c signal related to the measured pressure, and the signal was finally recorded by X-Y recorder having "pressure" as the vertical input and "radius" as the horizontal input. The use of telemetry and the avoidance of slip rings allowed excellent resolution and repeatability of all pressure data.

The overall pressure measurement system, consisting of tap, selector, transducer, telemetry, and recorder, was calibrated at two points on the calibrated linear operation curve, immediately before collection of each group of data. The two pressure transducers were provided with a common, constant reference pressure, which was the pressure at a tap at a radius of 5.648 in. from the center of the disk assembly.

A storage reservoir, a centrifugal pump, and appropriate valves were provided to circulate and control the fluid at desired rates. Fig. 2 presents a schematic diagram of the fluid-flow loop and certain instrumentation elements.

All data reduction was conducted by digital computer using a program written for that purpose. The design study mentioned earlier also provided the basis for estimation of uncertainty in the final results, as detailed later.

Results

Data were reduced to dimensionless form using nominal (measured) values for all computations. Thus each experimental case was characterized by nominal values of N_{Re} and U_0. The dimensionless pressure drop, p, is the static pressure difference between the disk outer periphery and a general radial station, r; this nomenclature was also used in references [3-9].

Analytical results from the Boyd and Rice [7] laminar flow analysis were then computed for the nominal values of N_{Re} and U_0 corresponding with the experimental case. The value of U_0 was that indicated by the mean direction of a small vane placed experimentally in the nozzle efflux and was used as the nominal value of V_0. In addition, the asymptotic flow field was calculated for nominal values of N_{Re} and U_0 for each case, and was used to determine the calculated point at which the asymptotic flow region began. All nonasymptotic analytical computations were made for full admission and for parabolic disk-to-disk velocity distribution of the outer periphery.

In each graphical presentation of results, the experimental data were first plotted so that a curve representing the data passed through ($r = 1$, $p = 0$); calculated data from the Boyd and Rice analysis were then superposed on the experimental plot, with the point at which the flow becomes asymptotic being common, and without regard for whether or not the calculated curve passed through ($r = 1$, $p = 0$). This corresponds with the observation made in a foregoing section that comparison is only valid in the asymptotic flow region, and that only the slopes of the pressure distribution curves should be compared.

Typical results are presented herein as demonstrative combinations of N_{Re} and U_0, chosen from the large volume of results obtained by the investigators.

Fig. 3 depicts results for moderate N_{Re} and moderate U_0. The asymptotic flow region consists of the region in which r is smaller than the value at which it begins; the calculated curve and the experimental data merge in the asymptotic region; in each figure the beginning of the asymptotic flow region is indicated by "A". Excellent agreement between the experimental data and the laminar analytical results is evident in this region. For large r, the experimental data are seen to follow the analytical curve corresponding with the experimental value of U_0, until local near the outer periphery are reached. Very near the outer periphery, the experimental data do not lie on the analytical curve primarily because of partial admission effects due to the finite nozzle configuration in the experiment, and secondarily because the disk-to-disk velocity distribution actually attained experimentally was not truly parabolic. It is in this region that a comparison of the analytical and experimental data gives some insight into phenomena in the entrance region. The large number of original cases, only excerpted herein, enable the observation that the pressure distribution in the entry region is only weakly affected by changes in V_0 but is strongly affected by partial admission effects due to blanking in the third supply nozzle ring and by details of the disk-to-disk inlet velocity distributions. Furthermore, it is observed that all departures from the asymptotic flow in the entrance region increase the pressure gradient in that region.

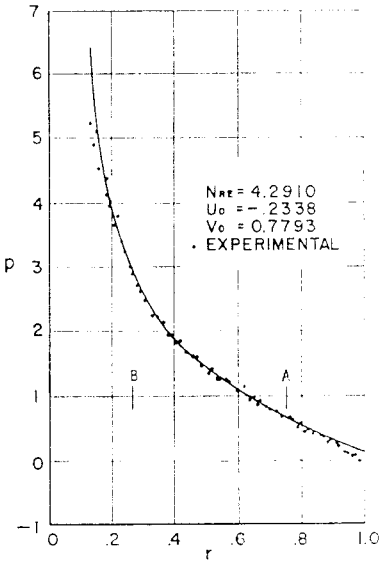

Fig. 3 Typical results at moderate N_{Re} and U_0

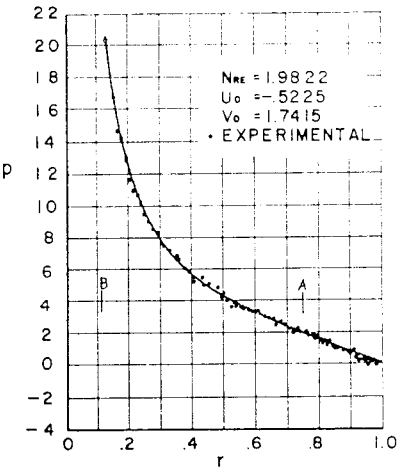

Fig. 4 Typical results at low N_{Re}, high U_0

Fig. 4 presents results for low N_{Re} and high U_0. Fig. 5, for low N_{Re} and low U_0; Fig. 6, for high N_{Re} and low U_0. Results are not presented for high N_{Re} and high U_0, or for $N_{Re} > 10$, since all calculated radial velocity component profiles are then inflected, and those regions were beyond the operational range of the experiment.

Consideration of Figs. 3-6 gives credence to the correctness and sufficient completeness of the analysis of Boyd and Rice [7], for both the asymptotic flow region and the entrance region and over the entire useful range of combinations of N_{Re} and U_0.

In each figure, the radius at which the radial velocity component profile (calculated) becomes inflected is indicated by "B"; the profiles are inflected for all radii less than at which they become inflected. It is seen that the analytical results based on the assumption of laminar flow, continue to agree with the experimental data in the region in which the calculated velocity profiles are inflected.

Each figure displays data points obtained using two different pressure transducers and two sets of pressure taps on diametrically opposed patterns. The close agreement of all data indicates that conditions were steady with time, that axial symmetry was attained, and that the two pressure measurement systems were in excellent agreement.

Error Analysis

Experimental data and calculations from analysis, presented in Figs. 3-6, are based on nominal values of N_{Re} and U_0 determined from values of ρ, μ, Q, R_0, $\Omega^2 R_0$ as measured. Errors are inherent in each measurement; there is a degree of uncertainty in the value of N_{Re} and U_0 that should be used for comparison of analytical results with which the experimental data are compared. Furthermore, there is an experimental uncertainty in p and r. To establish the conclusions reported, it is necessary to show that the uncertainty is small enough to make the conclusions reasonable.

By differentiating the defining relations for N_{Re} and U_0,

$$\frac{dN_{Re}}{N_{Re}} = \frac{d\Omega}{\Omega} + \frac{dR_0}{R_0} + 2\frac{d\rho}{\rho} \qquad (1)$$

$$\frac{dU_0}{U_0} = \frac{dQ}{Q} + 2\frac{dR_0}{R_0} + \frac{d\Omega}{\Omega} + \frac{dk}{k} \qquad (2)$$

Each of the terms in eqs. (1) and (2) was carefully estimated, as functions of the experimental nominal conditions, using manufacturers' data for test equipment, locally conducted tests and calibrations, and brief analytical investigations. As a result, the uncertainty in the N_{Re} and U_0 values corresponding with each set of experimental data were obtained from equations (1) and (2).

For each case denoted by nominal values of N_{Re} and U_0, additional analytical results were calculated with the extreme variations in N_{Re} and U_0 indicated from equations (1) and (2). A typical result is given in Fig. 7, in which the curves given as solid lines are the extreme analytical results for the N_{Re}, U_0 combinations, if all errors combined in such a way as to result in maximum experimental error.

Sources of error considered include those from all electronic processing and recording equipment, errors in measurements of dimensions and parallelism of the disks, errors in force measurements, errors due to calibration and repeatability variations for the fluid-flow meters and, finally, errors in specifying the value of the kinematic viscosity ν. Of these quantities, the viscosity value resulted in the greatest uncertainty; errors inherent in the viscosimeter, errors in the measurement of the fluid temperature during calibration and testing, data collection, and the errors due to the temperature rise caused by energy dissipation in the flow and inadvertent heat transfer were all assessed in the estimation of $d\nu$.

If the measured r and pressure and radial position were perfect, all experimental data would be expected to fall between the two solid curves in Fig. 7. However, an additional error estimate is necessary because of uncertainty in the pressure and position measurements. An expression for dp, p was formed in terms of calibration constants for the pressure measurement system; these included radial and vertical lengths of connecting tubes, calibration constants for the transducer-telemetry-recorder system, fluid density, and the angular velocity of the rotor. Each term of the dp, p equation was evaluated using the manufacturer's information and tests and studies of repeatability and deviations. Finally, dr, r was estimated. The result was the determination of the uncertainty of p and r for each data point. Typical results are indicated for representative data in Fig. 7. The results show that most data scatter was accounted for, and the reasonability of the conclusions earlier stated was established.

Approximately 2 percent of the collected data were not compatible with the uncertainty analysis. These data were "erratic," that is, remote from the bulk of the data, in a direction that is assignable to temporary plugging of a pressure tap by foreign matter. These data were discarded in the process of arriving at conclusions.

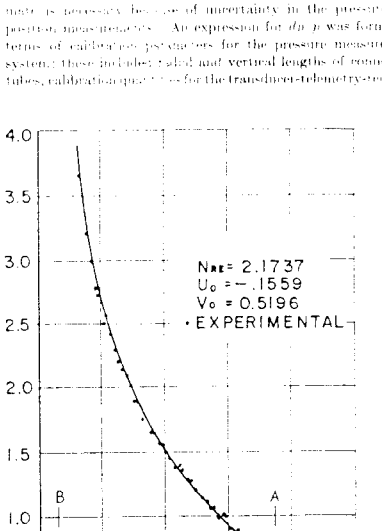

Fig. 5 Typical results at low N_{Re}, low U_0

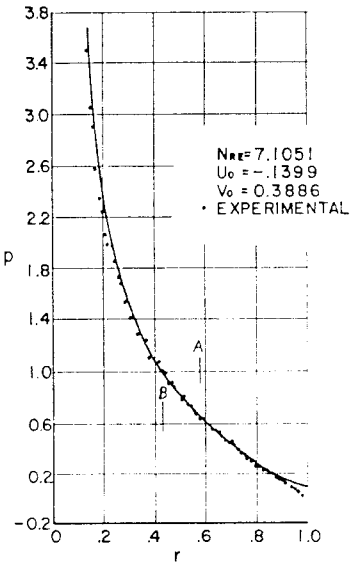

Fig. 6 Typical results at high N_{RE}, low U_0

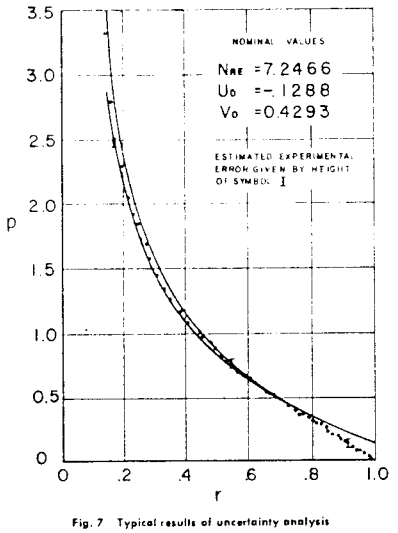

Fig. 7 Typical results of uncertainty analysis

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Integral Method for Flow Between Corotating Disks

An integral method is developed for the three-dimensional, nonboundary-layer flow which occurs for laminar, radially inward through-flow of an incompressible Newtonian fluid between parallel corotating disks. The method is a forward-stepping procedure which forces satisfaction of integrals of the governing differential equations, plus boundary conditions, plus the governing differential equations at every radius. The velocity components are represented by polynomials of order N; the method is extendable with extraordinary ease to any value of N. It is reported that, with N = 8, the results agree very closely with results earlier obtained by a conventional finite-difference method and which agree with experiment. It is pointed out that the method presented is extremely conservative of computational time and might be adapted to many other problems.

Nomenclature

$a = a(r)$ = coefficient in polynomial (dimensionless)
$b = b(r)$ = coefficient in polynomial (dimensionless)
h = distance between disks (length)
i, j, n = indices (integers)
N = order of polynomial (integer)
$N_{RE} = \frac{\rho\Omega h^2}{\mu}$, Reynolds number (dimensionless)
p = pressure (force/area)
$p = p/\rho\Omega^2 r_0^2$, dimensionless pressure
r = radial space coordinate (length)
$r = r/r_0$, dimensionless radial space coordinate
$\bar{u}$ = radial component of velocity (length/time)
$u = \bar{u}/\Omega r_0$, dimensionless radial component of velocity
$U_0 = 2\int_0^{1/2} u(1, z)dz$, dimensionless flow rate parameter
$\bar{v}$ = tangential component of velocity (length/time)
$v = \bar{v}/\Omega r_0$, dimensionless tangential component of velocity
$V_0 = 2\int_0^{1/2} v(1, z)dz$, dimensionless tangential velocity parameter
$\bar{w}$ = axial component of velocity (length/time)
$w = \bar{w}/\Omega h$, dimensionless axial component of velocity
$\bar{z}$ = axial space coordinate (length)
$z = \bar{z}/h$, dimensionless axial space coordinate
β = dummy variable in integration
ξ = dummy variable in integration
μ = viscosity (force-time/length²)
Ω = angular velocity of disks (rad/time)
ρ = density (mass/length³)

Subscripts

i = inner radius of disks
o = outer radius of disks

Introduction

THE flow between corotating disks is of basic interest in fluid mechanics. In the case of steady laminar flow of a Newtonian incompressible fluid, with throughflow between the disks, the flow may be either radially inward or radially outward. In the case of radial outflow, the flow corresponds with that in a multiple-disk pump, which has been treated extensively in the literature [1, 2, 3, 4].[2] In the case of radial inflow, the flow corresponds with that in a multiple-disk turbine; the fluid is supplied at the outer periphery of the disks with a tangential component as well as a radial component of velocity, and with pressure greater than that at an inner exhaust radius. This flow has also received extensive investigation because of the potential applications of multiple-disk turbines and because of occurrence of the flow in certain fluidic elements. The feasibility of such turbines has been amply demonstrated experimentally [5, 6, 7].

Analysis of the turbine-type flow has also been extensive. Matsch and Rice obtained solutions for the limiting cases of potential flow [8] and creeping flow [9] between corotating disks with arbitrarily distributed (partial) admission at the outer periphery of the disks. The general laminar flow of a Newtonian fluid between corotating disks is described by the Navier-Stokes equations; the flow is strongly three-dimensional. It is subject to the no-slip boundary conditions at the disk surfaces, and the fluid may be supplied at the outer periphery of the disks in a wide variety of ways. Matsch and Rice [10] limited consideration to full (uniform) peripheral distribution of the fluid supplied. In reference [10], order of magnitude arguments are used to simplify the governing equations and an iterative solution is given for the asymptotic flow field; the concept of the asymptotic flow field is given in reference [10], and it is shown that the flow has two parameters, N_{RE} and U_0.

In a paper also concerned only with full peripheral admission, Boyd and Rice [11] presented a numerical method which is definitively accurate and complete for the turbine flow, including the entrance region as well as the asymptotic flow. The reduced equations, established in reference [10], were used; it was shown that the entrance region preceding the asymptotic region has the three parameters N_{RE}, U_0, and V_0 for any given disk-to-disk distribution of the velocity components. In reference [11], the nature of the entrance flow is clarified by examples. It is pointed out that the numerical method presented is sufficiently accurate to allow calculation of quantities of interest such as "turbine efficiency," torque, and power for the flow between corotating disks.

An experimental investigation reported by Adams and Rice [12] substantiated the adequacy of the problem statement and of the numerical solution of reference [11]; that is, very good agreement was obtained between the results of reference [11] and experiment over a wide range of combinations of N_{RE}, U_0, V_0.

At this point in the developmental history of multiple disk turbines, two further things are required to enable design of incompressible, laminar flow, full admission, multiple-disk turbines for given applications, and/or to calculate performance for comparison with conventional turbines: (*a*) experimental determination of the criterin for the occurrence of laminar flow rather than turbulent flow between corotating disks (all experimental flow encountered in reference [12] was laminar), and (*b*) calculation of numerous flow cases and extensive plotting of the results to yield performance maps for the rotors of multiple-disk turbines.

The second of the foregoing requirements could be fulfilled by using the numerical procedure of reference [11] since that procedure is definitively accurate. However, the method is not sufficiently efficient with regard to digital computer time, when a very large number of cases is contemplated.

The computational method presented herein can replace the procedure of reference [11] for solution of the same problem, with a reduction in required computer time to approximately 2 percent of that required by the numerical procedure of reference [11].

Furthermore, the method of solution presented herein is of interest in basic fluid mechanics since it applies to a flow which is strongly three-dimensional and not of a boundary-layer nature; that is, the pressure distribution is not prescribed in advance but is solved for simultaneously with the velocity field. It is an efficient and accurate method; it will be later shown that an accuracy equal to or greater than that of reference [11] is achieved. Because of this basic importance, the essential features of the method are reported in some detail here. The limitations of the method and its strengths are documented.

Finally, it is noted that a solution for the flow between corotating disks implies a solution for the flow between corotating conical surfaces [13]. Turbines with rotors composed of nested corotating cones have been operated experimentally [5].

Problem Statement

The geometry of the flow passage and a coordinate system are

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[2] Numbers in brackets designate References at end of paper.

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shown in Fig. 1. With the assumption that $u(1, z)$ and $v(1, z)$ are even functions of z, the problem can be stated in the half-space, $0 \leq z \leq 1$, as discussed in reference [11]. In dimensionless variables, the problem statement is

$$u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{dp}{dr} + \frac{1}{N_{RE}}\frac{\partial^2 u}{\partial z^2} \tag{1}$$

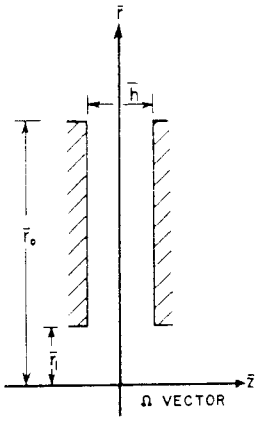

Fig. 1 Geometry and coordinate system

$$u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = \frac{1}{N_{RE}}\frac{\partial^2 v}{\partial z^2} \tag{2}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\left.\begin{aligned} u(1, z) &= u_0(z) \\ v(1, z) &= v_0(z) \end{aligned}\right\} \quad 0 \leq z \leq r_1 \tag{4}$$

$$\left.\begin{aligned} \frac{\partial u}{\partial z}(r_1, 0) &= 0 \\ \frac{\partial v}{\partial z}(r_1, 0) &= 0 \\ w(r_1, 0) &= 0 \\ u(r_1, r_1) &= 0 \\ v(r_1, r_1) &= v_1 \\ w(r_1, r_1) &= 0 \end{aligned}\right\} \quad r_1 \leq r \leq 1 \tag{5}$$

Here u_0 and v_0 are arbitrary, but prescribed, even functions of z; it follows that w is an odd function of z. To complete the problem statement, another equation is needed, and is obtained in convenient form by integrating equation (1) repeatedly with respect to z, using appropriate boundary conditions and using the fact that volume flow rate is conserved along the disk radius. The resulting equation is

$$\frac{dp}{dr} = -\frac{12U_0}{rN_{RE}} - 24\int_0^{r_1}\int_0^{z}\int_1^{\xi}\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right)d\xi\,dz\,dz \tag{6}$$

where U_0 is the mean value of u_0. Finally, as a matter of convenience, it is specified that $p(r_1, z) = 0$.

Development of the Solution Method

The solution is begun by integrating equations (1), (2), and (3) over the interval $0 \leq z \leq r_1$ to produce

$$\int_0^{r_1}\left[u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right]dz = -\frac{1}{2}\frac{dp}{dr}r_1 - \frac{1}{N_{RE}}\frac{\partial u}{\partial z}(r_1, 0) \tag{7}$$

$$\int_0^{r_1}\left[u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right]dz = \frac{1}{N_{RE}}\frac{\partial v}{\partial z}(r_1, 0) \tag{8}$$

$$\int_0^{r_1} u(r_1, z)\,dz = \frac{U_0}{2r} \tag{9}$$

Equations (6), (7), (8), and (9) constitute four integral equations in four unknowns.

Nth-order polynomial forms are chosen for the radial and tangential components of velocity, according to

$$u(r, z) = \sum_{i=1}^{N+1}(a_{i-1})z^{i-1} \tag{10}$$

$$v(r, z) = \sum_{j=1}^{N+1}(b_{j-1})z^{j-1} \tag{11}$$

where a_i and b_j are functions of r above. It follows from continuity that

$$w(r, z) = -\sum_{i=1}^{N+1}\left(\frac{a_{i-1}}{r} + a_{i-1}'\right)\frac{z^i}{i} \tag{12}$$

where the prime denotes differentiation with respect to r.

Equations (10)-(12) are inserted into equations (6)-(9) to produce

$$\frac{dp}{dr} = -\frac{12U_0}{rN_{RE}} - 24\sum_{i=1}^{N+1}\sum_{j=1}^{N+1}\frac{Q_{i,j}}{(i+j-1)(i+j+1)2^{(i+j+1)}} \tag{13}$$

where

$$Q_{i,j} = a_{i-1}a_{j-1}' - \frac{(j-1)a_{j-1}}{i}\left(\frac{a_{i-1}}{r} + a_{i-1}'\right) - \frac{b_{i-1}b_{j-1}}{r}$$

and

$$\sum_{i=1}^{N+1}\sum_{j=1}^{N+1}\frac{a_{i-1}a_{j-1}' + a_{i-1}'a_{j-1} + \frac{1}{r}(a_{i-1}a_{j-1} - b_{i-1}b_{j-1})}{(i+j-1)2^{(i+j-1)}} + \frac{1}{2}\frac{dp}{dr} - \frac{1}{N_{RE}}\sum_{i=1}^{N+1}\frac{(i-1)a_{i-1}}{2^{(i-2)}} = 0 \tag{14}$$

$$\sum_{i=1}^{N+1}\sum_{j=1}^{N+1}\frac{a_{i-1}b_{j-1}' + a_{i-1}'b_{j-1} + \frac{2}{r}(a_{i-1}b_{j-1})}{(i+j-1)2^{(i+j-1)}} - \frac{1}{N_{RE}}\sum_{i=1}^{N+1}\frac{(i-1)b_{i-1}}{2^{(i-2)}} = 0 \tag{15}$$

$$\sum_{i=1}^{N+1}\frac{a_{i-1}}{i2^i} = \frac{U_0}{2r} \tag{16}$$

Clearly equations (13)-(16) indicate the possibility of constructing a forward-stepping solution, starting with known velocity distributions u_0 and v_0 at $r = 1$. Each step necessitates a solution of the differential equations (13)-(16), for a_i and b_j. However, highly inflected velocity profiles occur in the solution; to represent these profiles, polynomials of at least fourth order are required. For fourth and greater-order polynomials, equations (13)-(16) are insufficient to solve for a_i and b_j. For this reason, additional relations between the a_i and b_j are generated by requiring the differential equations (1)-(3) to be satisfied in the flow to yield the correct number of equations to allow solution for a_i and b_j. The number of equations involved depends on the order of the polynomials chosen to represent the velocity components.

The method indicated above for generating the additional equations for a_i and b_j is the unique feature of this integral method, which allows it to be easily extended for use with polynomials of any order; this feature is also largely responsible for the excellent accuracy and very rapid computational ability of the method.

Specifically, to generate the needed additional relations, equation (1) is written as

$$\sum_{i=1}^{N+1}\sum_{j=1}^{N+1}\left[a_ia_j' - \left(\frac{a_i}{r} + a_i'\right)\frac{ja_j}{i+1} - \frac{b_ib_j}{r}\right]z^{i+j-2} + \frac{dp}{dr} - \frac{1}{N_{RE}}\sum_{i=1}^{N+1}(i-1)(i-2)a_iz^{i-3} = 0 \tag{17}$$

and equation (2) is written as

$$\sum_{i=1}^{N+1}\sum_{j=1}^{N+1}\left[a_ib_j' - \left(\frac{a_i}{r} + a_i'\right)\frac{jb_j}{i+1} - \frac{a_ib_j}{r}\right]z^{i+j-2} - \frac{1}{N_{RE}}\sum_{i=1}^{N+1}(i-1)(i-2)b_iz^{i-3} = 0 \tag{18}$$

The coefficient of z^j must be zero; hence, a needed relation is obtained from equations (17) and (18) for every value of j, according to

$$a_ja_1' - \frac{b_0^2}{r} - \frac{2a_2}{N_{RE}} + \frac{dp}{dr} = 0 \qquad \text{for } j = 0 \tag{19}$$

$$\sum_{i=0}^{j}\left[a_ia_{j-i}' - \left(\frac{a_i}{r} + a_i'\right)\frac{(j-i-1)}{(i+1)}a_{j-i} - \frac{b_ib_{j-i}}{r}\right] - \frac{(j+1)(j+2)a_{j+2}}{N_{RE}} = 0 \qquad \text{for } j = 1, 2, \ldots \tag{20}$$

$$\sum_{i=0}^{j}\left[a_ib_{j-i}' - \left(\frac{a_i}{r} + a_i'\right)\frac{(j-i-1)}{(i+1)}b_{j-i} - \frac{a_ib_{j-i}}{r}\right] - \frac{(j+1)(j+2)b_{j+2}}{N_{RE}} = 0 \qquad \text{for } j = 0, 1, 2, \ldots \tag{21}$$

In implementing the method, equation (19) is not used since satisfaction of equation (13), required in the method, automatically causes satisfaction of equation (19). In other words, equation (19) is redundant. (It is noted, however, that satisfaction (19) would not automatically cause satisfaction of equation (13).)

In implementing the method, equations (13)-(16) are used, together with equations (20)-(21), with j determined by N in the polynomial representations. For $N = 4$, $j = 1$ in equation (20) and $j = 0, 1$ in equation (21) should be used. For $N = 6$, $j = 1, 2, 3$ in (20) and $j = 0, 1, 2, 3$ in (21) should be used. For $N = 8$, $j = 1, 2, 3, 4, 5$ in (20) and $j = 0, 1, 2, 3, 4, 5$ in (21) should be used. These combinations, together with the boundary conditions, result in the proper number of independent equations to enable solution for a_i and b_j. It is easy to establish the proper combinations for $N > 8$, the ease of extension of the method to higher-ordered polynomial representations of the velocity profiles constitutes a main strength of the method, as earlier noted.

Application of the boundary conditions at $z = 0$ gives the result that $a_i = b_i = 0$ for all odd i; this greatly simplifies the equations for any value of N. The remaining unused boundary conditions, $u(r, r_1) = 0$ and $v(r, r_1) = v_1$, are used to complete the set of equations for a_i, b_j, for any N, and at every forward step.

At the entrance, the initial distributions u_0 and v_0 are applied by "fitting" the polynomials of the chosen order N, in equations (10) and (11), to obtain a_i and b_i at $r = 1$. It is noted that $w(1, z)$ is then also known.

Each step inward from a radius at which the field values are known involves solution of the set of ordinary differential equations earlier described for a_i and b_i at the inner-radial station. The equations earlier described constitute $2N + 2$ ordinary differential equations. For each radial step, solution can be made most efficiently by a predictor-corrector method. The primary advantages of a predictor-corrector method are the very rapid computational time and the readily estimated error which can be used as a basis for choosing step size. The primary disadvantage is that the solution must be known at r_{n-3}, r_{n-2}, r_{n-1}, and r_n in order to find the solution at r_{n+1}. Thus the method is not self-starting. For this reason, a Runge-Kutta-Gill [14] method was used to provide starting solutions and an Adams-Moulton [14] method was used following the starting solution. These methods are well documented elsewhere and can be implemented in many ways varying in detail, and are not further discussed here.

The solution was begun at $r = 1$ using an extremely small step size, $\Delta r = 10^{-6}$. In calculations for each forward step, the truncation error, E_K, was calculated, and compared with preset allowable error bounds, E_{max} and E_{min}. If $E_K > E_{max}$, the step size was halved and the calculation repeated, the R-K-G method being used to restart the solution. If $E_K < E_{min}$, the step size was doubled and the calculation repeated, the R-K-G method being used to restart the solution. For $E_{min} < E_K < E_{max}$, the step size remained constant and the A-M method was used to continue the calculations.

Results

The solution procedure described was evaluated by comparing results for $N = 4$, $N = 6$, and $N = 8$ with each other and with the results of references [11] and [12] for a wide range of combinations of the three parameters N_{RE}, U_0, and V_0. In all cases, at some radii the u-velocity component profile is highly inflected; for this reason the results for $N = 4$ were very poor. However, results for $N = 6$ were in good agreement with results for $N = 8$, thus showing that increasing N causes the method to converge to the correct solution, asymptotically. Furthermore, results for $N = 8$ were in excellent agreement with the results given in references [11] and [12]; that is, with a solution known to be valid and accurate, and with experiment. It is not feasible to delineate the very small differences between the various results; the differences are not meaningful for any combination of the parameters and in general are less than 0.01 percent, distributed randomly.

A comparison of the calculated radial pressure distribution using the present integral method, with calculated results obtained by Boyd and Rice [11] and with experimental results obtained by Adams and Rice [12], is given in Fig. 2. The comparison is typical of that obtained for all combinations of parameters; specifically $N_{RE} = 2.1737$, $U_0 = -0.1559$, and $V_0 = 1.1908$ in Fig. 2. It is noted that the calculated results agree so well that they appear as a single result in the figure. The difference between the calculated and the experimental results is well within the margin of experimental error over most of the radius; agreement is not expected near $r = 1$ for reasons discussed in detail in reference [12]. Velocity profiles have not been measured experimentally; those calculated for another typical combination of parameters, $N_{RE} = 5.0$, $U_0 = -0.05$, $V_0 = 1.1$, and using results from the present integral method and the results of Boyd and Rice [11], are presented in Figs. 3, 4 and 5. The results of the two calculation methods agree so well that the scale of the figures allows no differentiation between them.

The need for an accurate calculation method which can be executed extremely rapidly using a digital computer was earlier made clear. The method herein presented satisfies the need. Before the method was developed, conventional integral methods

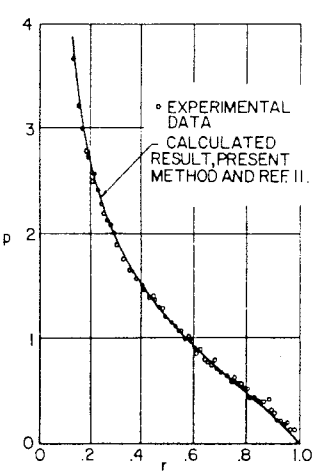

Fig. 2 Comparison of calculated results and experimental results for pressure distribution, for $N_{RE} = 2.1737$, $U_0 = -0.1559$, and $V_0 = 1.1908$. Calculated results from present and earlier methods are coincident.

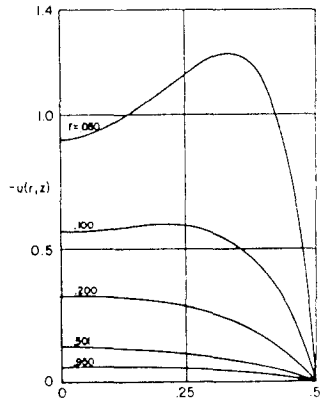

Fig. 3 Calculated results for radial velocity component profiles at selected radii, for $N_{Re} = 5.0$, $U_i = -0.05$, and $V_0 = 1.1$. Calculated results from present and earlier methods are coincident.

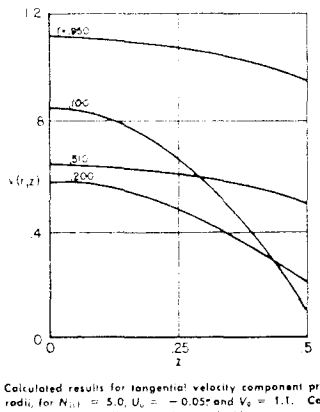

Fig. 4 Calculated results for tangential velocity component profiles at selected radii, for $N_{Re} = 5.0$, $U_i = -0.05$ and $V_0 = 1.1$. Calculated results from present and earlier methods are coincident.

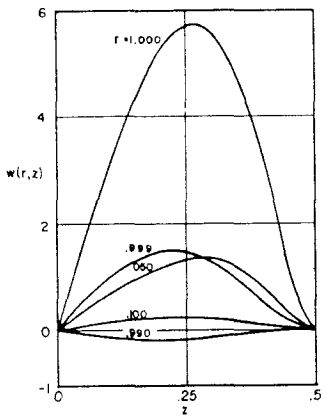

Fig. 5 Calculated results for axial velocity component profiles at selected radii, for $N_{Re} = 5.0$, $U_i = -0.05$, and $V_0 = 1.1$. Calculated results from present and earlier methods are coincident.

were investigated. Of existant integral methods applicable for three-dimensional, non-boundary-layer flow, the multiple-strip method of Pallone [15] proved most promising. The problem was programmed for digital computation using the multiple strip method with fourth-order polynomials corresponding to a single strip; *results were very poor because of inability of the fourth-order polynomials to represent the inflected u profiles.* Extension of the method to fifth-order polynomials (two strips) produced an inordinate increase in the computational time, with only a moderate increase in accuracy. This was because a much smaller radial step size was required with fifth-order polynomials, necessitating calculation at a much larger number of radial stations, and because a much larger matrix had to be inverted at each radial station. The calculation penalties do not seem to nearly the same extent as N is increased in the method described in the present paper. Thus conventional integral methods resulted in *prohibitive computational time while the method* herein described resulted in very nominal computational time.

Conclusions

An integral solution was developed for three-dimensional laminar flow of an incompressible Newtonian fluid in radially inward through-flow between parallel corotating disks. In such flow, the pressure gradient is not prescribed but must be solved for simultaneously with the velocity field. Integral solutions for such flows are rare, and the method presented can probably be adapted to other three-dimensional flows. The method presented is easily extended systematically to higher ordered polynomial representations of the velocity components. For this reason, the accuracy of the method can be easily determined by observing the convergence to an asymptotic limit as N is increased.

In the problem to which the method was applied, the method with $N = 8$ required approximately one-fifteenth of the computational time that was required by an accurate finite-difference procedure. For this reason, the computational method can be used to generate data useful in considering the design of multiple-disk turbines. The method can probably also be adapted for compressible flow calculations and for problems in heat transfer. It is also adaptable for boundary-layer flows. Further details concerning the method, with examples of use and displays of results, are contained in a dissertation by Boyack [16].

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Flow Regime Definition for Flow Between Corotating Disks

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Experimental data and results are presented for the flow of an incompressible Newtonian fluid, with full admission, between closely spaced corotating disks, and for both radially inward and radially outward throughflows. The data consist of the measured radial pressure distribution together with flow visualization by means of injected dye, over a very wide range of parameters descriptive of the flows. From the data, the combinations of the parameters corresponding with laminar and with turbulent flow are determined; that is, the data are sufficient to define the flow regimes. Furthermore, for laminar flow, both the pressure distribution data and the flow visualization data confirm the adequacy of earlier analytical results for use in the design of practical devices incorporating these flows, such as multiple disk pumps and turbines.

Nomenclature

h = distance between disks (length)

$\bar{n}$ = coordinate normal to radial coordinate $\bar{r}$

$N_{Re} = \Omega h^2/\nu$ = Reynolds number, dimensionless

$p_i = \bar{p}_i/\rho\Omega^2\bar{r}_0^2$ = dimensionless static pressure difference between local point and reference point, $\bar{r}_0$, in radially inward flow

$p_2 = \bar{p}_2/\rho\Omega^2\bar{r}_i^2$ = dimensionless static pressure difference between local point and reference point, $\bar{r}_i$ in radially outward flow

p_{i_1} = static pressure difference, in radially inward flow (force/(length)^2)

p_{2_1} = static pressure difference, in radially outward flow (force/(length)^2)

Q = volume flow rate ((length)^3/time)

$\bar{r}$ = radial distance (length)

$\bar{r}_i$ = inner radius (length)

$\bar{r}_0$ = outer radius (length)

$r_1 = \bar{r}/\bar{r}_0$ = dimensionless radial coordinate in radially inward flow

$r_2 = \bar{r}/\bar{r}_i$ = dimensionless radial coordinate in radially outward flow

$C_i = Q/2\pi\bar{r}_i^2 h\Omega$ = dimensionless flow rate parameter for radially outward flow

$C_0 = Q/2\pi\bar{r}_0^2 h\Omega$ = dimensionless flow rate parameter for radially inward flow

$V_i = V_i/h\Omega\bar{r}_i$ = dimensionless tangential velocity component at inner radius of rotor

$V_0 = V_i/h\Omega\bar{r}_0$ = dimensionless tangential velocity component at outer radius of rotor

V_i = tangential component of velocity, at inner radius of rotor (length/(time))

V_i = tangential component of velocity, at outer radius of rotor (length/(time))

ν = kinematic viscosity of fluid, ((length)^2/time)

Ω = angular velocity of disks (rad/time)

ρ = density of fluid (mass/(length)^3)

α = arctan (U_0/V_0) = nozzle angle for radially inward flow

θ = arctan (u/v) = angle between tangent to streamline and normal to radial line

Introduction

THE laminar throughflow of an incompressible Newtonian fluid between closely spaced corotating disks has been considered analytically by numerous investigators. This problem is distinguished from other flows in the presence of rotating disks by a throughflow of fluid between the disks. The flow can be supplied at the outer periphery of the disks and with a tangential component of velocity; in this case, the flow and the pair of disks constitute an element of a practical device known as a multiple disk turbine and torque is produced on the disk pair by the flow. If, instead, the flow is supplied at an inner radius of the disks and progresses radially outward, torque being supplied to the fluid by the disks, then the flow and the pair of disks are an element of a multiple disk pump.

A series of analytical contributions to understanding and describing laminar flows between corotating disks culminated in a computer-oriented analytical procedure by Boyack and Rice [1],[3] that paper recites the history of the problem and cites other contributions. The method due to Boyack was further examined, improved, and documented with respect to accuracy by Crawford [2]. The analysis produces the dimensionless velocity field and the radial dimensionless pressure distribution between the corotating disks, as a function of the dimensionless spatial location between the disks and of three dimensionless parameters (U_0, V_0, N_{Re} in the case of radially inward flow or U_i, V_i, N_{Re} in the case of radially outward flow) and of the shape of the velocity distributions at the flow inlet to the disks; all of these quantities are defined in the Nomenclature section. Early in the analytical history of such flows, the existence of an entrance region followed by an asymptotic region was recognized; the flow in the entrance region depends on all three parameters and on the inlet velocity distribution shape, while the flow in

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[3] Numbers in brackets designate References at end of paper.

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the asymptotic region depends only on U_0 and N_{Re} or U_i and N_{Re} (depending on the radial direction of the flow).

The essential correctness and accuracy of the analytical model and of the computational procedures employed by Boyack and

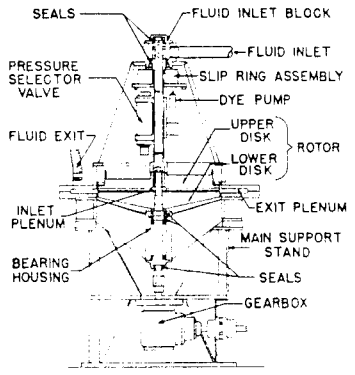

Fig. 1 Schematic diagram of experimental assembly in radially outward flow configuration

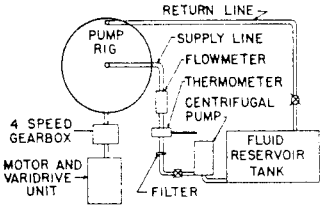

Fig. 2 Schematic diagram of fluid flow circuit for radially outward flow configuration

Rice [1] were verified by experimental work by Adams and Rice [3]. Excellent agreement between calculated and experimental radial pressure distributions was shown for radially inward flow over a range of flow parameters in which apparently only laminar flow as encountered. The experimental data and results given later in this paper were obtained with an improved version of the experimental apparatus originally constructed by Adams; modifications allowed a much wider range of flow parameters such that both laminar and turbulent flow occurred. Furthermore, means of dye injection was added to the experimental apparatus, and both radially inward and outward flows were investigated.

An improved version of the computer program made by Boyack was used by Crawford and Rice [4] to acquire computed results for the construction of performance maps for use in the design of multiple disk pumps. It was shown that such pumps have interesting characteristics and can have excellent performance. Lawn and Rice [5] used the same computer program to prepare extensive performance maps for multiple disk turbines. Some similar design information has also appeared due to independent work by Byal'yi [6]. The calculated performance data in these references [4, 5, 6] enable the rational design of multiple disk turbomachinery, provided that the flow between the disks is laminar; the analytical results on which the design information is based assume laminar flow.

The results presented herein define the occurrence of laminar and of turbulent flow and constitute the remaining needed information for utilization of the design information discussed

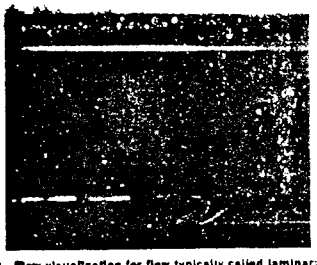

Fig. 3 Flow visualization for flow typically called laminar; flow radially inward with $N_{Re}=6.22$, $U_i=-0.098$

in the foregoing paragraphs. In addition, the flow visualization data enable additional substantiation of the earlier results by Adams and Rice [3] of the adequacy of the flow model and of the calculation procedures used by Boyack and Rice [1] when the flow is laminar.

No basic analysis has yet appeared for turbulent flow between corotating disks, or for either flow regime when the fluid is compressible. A wholly empirical turbulent flow model and computed results have been presented by Nendl [7]. Research results by Bakke [8] using a hot wire anemometer in a radially outward flow of air between large, rapidly rotating, widely spaced, corotating disks are of much interest in understanding the flow phenomena when the flow is turbulent. All flows encountered by Bakke were turbulent according to his deduction from the measurement of the relative turbulent intensity at various radial stations. Thus he did not deduce a criterion for the occurrence of laminar flow; furthermore, only radially outward flow was considered (flow with adverse pressure gradient).

Design of Experiment and Apparatus

The experimental apparatus designed and originally used by Adams and Rice [3] was modified and used in the investigations reported herein. An elaborate and reliable analysis of sources of experimental error and uncertainty was made by Adams. This was further used by the present investigators to estimate uncertainty in experimental values of N_{Re}, U_0, V_0, and p.

The apparatus due to Adams as modified for use by the present investigators, was constructed using precision machining techniques, massive structural parts, and accurate bearing alignments and clearances. It consists basically of a pair of extremely flat disks with excellent dimensional tolerances with respect to flatness and smoothness arranged to corotate on a shaft while supplied with fluid flow between the disks (Fig. 1); quantitative details are given in references [3, 11, 12]. Large bearings were utilized producing rotation in which the disks were parallel and concentric as confirmed by direct measurement. The disks were nominally 16.000 in. O.D., 2.000 in. I.D., and spaced 0.060 in. apart. The lower disk was coated with white epoxy before machining; this formed an excellent background against which to observe injected dye in the space between the disks. The upper disk was made of $\frac{1}{2}$ in. thick transparent Lexan polycarbonate plastic backed by a massive steel spider. The upper disk was provided with static pressure taps of the size and design used by Adams. Ten dye injection stations, providing the possibility of injecting dye in both the entrance and the asymptotic flow regions, were also provided, consisting of through-holes into which removable brass "dummy" plugs were fitted. In order to inject dye at a chosen radial station, one of the "dummy" plugs was replaced with a plug equipped with a dye injection tube made of 0.012 in. O.D., 0.006 in. I.D. hypodermic tubing; the injection tube entered the space between disks normal to the disks and

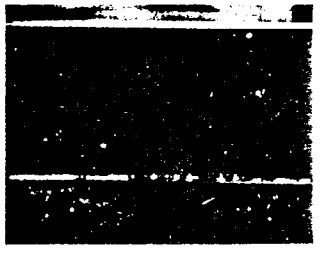

Fig. 4 Flow visualization for flow typically called turbulent; flow radially inward with $N_{Re}=4.98$, $U_i=-0.399$

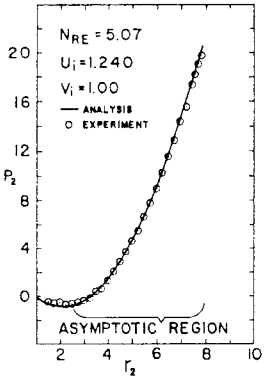

Fig. 5 Comparison of calculated and measured dimensionless pressure distribution for flow typically called laminar; flow radially outward with $N_{Re}=5.07$, $U_i=1.24$, $V_i=1.00$

then was bent at a right angle. The dye leaving the tubing was thus injected in the direction of fluid moving along the midplane streamline between the two disks. Each plug supporting a dye injection tube could be rotated to cause the tube direction to coincide with the local streamline direction. Each transparent window of the upper disk, between arms of the supporting spider, was equipped with an accurately scribed grid for dimensional reference when observing dye traces. Injection of dye was accomplished when desired by triggering the motor drive of a hypodermic pump mounted on the rotating disk assembly.

Experiments with radially inward flow were first carried out; for these, flow was supplied by a nozzle ring which received the flow from a plenum chamber and directed the flow through 30 discrete nozzles at a fixed small angle between the flow and a tangent to the disk circumference. The flow exhausted from the disks at an inner radius through a hollow bearing shaft, into an exhaust plenum which was maintained full of fluid.

Small clearances were provided between the rotating disk assembly and the housing; carbon seals effectively prevented leakage between the stationary housing and the rotating assembly. The choice of rotor dimensions, fluid properties and angular velocity was made to minimize experimental uncertainty over the parameter ranges of interest; consequently, the experimental fluid used was a water-ethylene glycol mixture having the desired value of ν in any given experiment; ν was varied in the totality of experimental cases to allow determination of the effect of viscosity on the occurrence of transition from laminar to turbulent flow. This fluid also has a reasonably small coefficient of viscosity change with temperature which is desirable to minimize errors due to viscosity change with the small tempera-

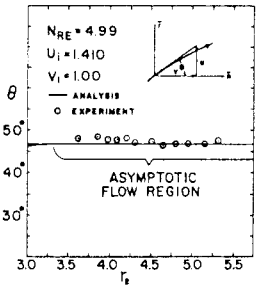

Fig. 6 Comparison of calculated and measured local streamline slope for flow typically called laminar; flow radially outward with $N_{Re}=4.99$, $U_i=1.41$, $V_i=1.00$

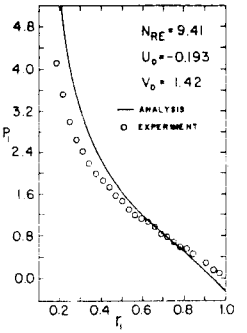

Fig. 7 Comparison of calculated and measured dimensionless pressure distribution for flow typically called turbulent; flow radially inward with $N_{Re}=9.41$, $U_0=-0.193$, $V_0=1.42$

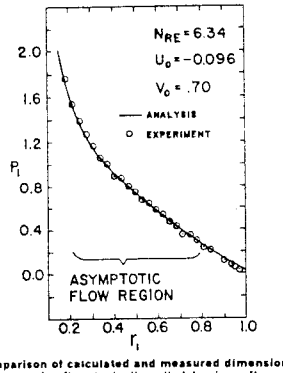

Fig. 8 Comparison of calculated and measured dimensionless pressure distribution for flow typically called laminar; flow radially inward with $N_{Re}=6.34$, $U_0=-0.096$, $V_0=0.702$

ture rise through the flow caused by viscous dissipation. An uncertainty analysis also resulted in the choice of an electronic turbine-meter for measurement of the volume flow rate and an electronic means of measuring the angular velocity.

Experiments with radially outward flow were also carried out; for these, the nozzle ring was removed and other minor modifications were made to allow the fluid to enter at the inner radius of the rotor and exhaust into a plenum at the outer periphery. In either flow direction, the torque exerted by the flow on the disks was small compared with the torque due to seals, bearings, etc. Consequently, the disk assembly was rotated at any desired speed by a variable-speed gearmotor, indicated in Fig. 1.

Immediately before operation to obtain data, the density and viscosity of the water-ethylene glycol mixture were measured. Density was determined by use of a hydrometer; viscosity was determined using a Ubbelohde viscosimeter in a constant temperature bath. The dye used for flow visualization was adjusted for neutral buoyancy in the primary fluid mixture.

The system for pressure measurement described in detail by Adams and Rice [3] was also used in the present investigation. The overall pressure measurement system, consisting of pressure taps, remote tap selector, unbonded strain gage type pressure transducer, FM telemetry system, and stationary recorder was calibrated immediately before each collection of data. All data reduction was conducted by digital computer using programs written for that specific purpose.

A storage reservoir, a centrifugal pump and appropriate valves were provided to circulate and control the fluid at desired rates. Fig. 2 presents a schematic diagram of the fluid flow loop and certain instrumentation elements for the radial outflow experimental configuration.

Procedures

In each experimental case investigated, pressure distribution data were recorded and photographs were made of filaments of injected dye. The analytical solution for laminar flow was determined for each case, corresponding with nominal intended values of Reynolds number, flow rate parameter, and the dimensionless tangential velocity. The analytical solution yielded the calculated dimensionless static pressure and local slope of a streamline as functions of dimensionless radial location between the disks.

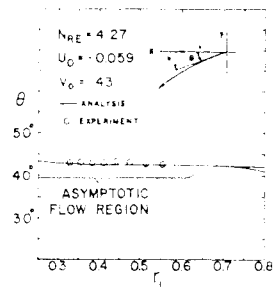

Fig. 9 Comparison of calculated and measured local streamline slope for flow typically called laminar; flow radially inward with N_{Re} = 4.27, U_0 = −0.059, V_0 = 0.43

For cases in which the flow was laminar (as deduced from the characteristics of the dye filaments, as discussed later herein), the injected dye filaments were streamlines in the midplane of the flow. Photographs of the streamlines were mechanically measured to gain the spatial coordinates of points on the streamlines. These coordinates were then entered into a digital computer program which calculated the local slope of the streamline using a variety of elementary methods of calculation (for comparison). This procedure allowed comparison of the calculated with the experimentally obtained local slope of streamlines in the laminar flow regime.

Visual inspection of photographs of the dye filaments obtained for each experimental case investigated was used to deduce the flow regime for each case, i.e., whether the flow was nominally laminar or nominally turbulent. To assist in such interpretation a special apparatus was built and the experiments of Osborne Reynolds were repeated locally, using the techniques to be later used in rotating disk flow investigation. The behavior of the dye was noted in relation to the classically accepted flow regimes occurring in the experiment. Following this experience, it was noted (as expected) that similar dye behavior was observed in various different flow cases in the rotating disk flows. Fig. 3 presents a typical photograph for which the flow was called laminar; from such photographs, the local slope of the streamline was also determined, as earlier discussed. Fig. 4 presents a typical photograph for which the flow was called turbulent; no streamline is evident since the dye disperses very rapidly downstream from the injection point. Direct observation during the experiments, using a strobe light, indicated that the dye dispersed randomly with time in eddying swirls and showed all of the accepted visual characteristics of turbulent flow.

It was further observed that systematic variation of any parameter which differed greatly for laminar and for turbulent flow, (such as Ω for fixed ν, or ν for fixed Ω) produced a mixed flow condition for a range of values of the parameter between the extremes. In this mixed flow, or transition region, the flow appeared laminar and intermittently turbulent, randomly with time. This region is delineated in results presented later herein.

In the region of laminar flow, excellent agreement between analytical results and experimental results was expected, following the conclusions of Adams and Rice [3]. Throughout the region called laminar according to flow visualization results, extremely good agreement was obtained with regard to both local pressure gradient and local slope of a streamline. Figs. 5 and 6 present such comparisons for a typically laminar flow case, for radially outward flow with V_i = 1.0. For reasons exhaustively discussed in references [1, 2, 3], comparison of results should only be made in the radial region referred to as asymptotic flow (away from the flow entrance). The flow in the entrance region depends to a great extent upon the entrance velocity distributions which were not precisely known in the experiments. However, analytical results calculated for the entrance conditions thought to be present experimentally showed good agreement with experimental results over most of the entrance region as well as the asymptotic region, as shown by Fig. 5 and Fig. 8. Similar agreement between the analysis and experiment were obtained throughout the parameter region called laminar, for radially inward flow.

In the mixed flow, or transition region, the results of the laminar flow analysis departed systematically from the experimental results, with regard to both local pressure gradient and local streamline slope. They were distinctly different in the flow region called turbulent. Fig. 7 shows typical results, for radially inward flow, in the turbulent flow region. Similar results were demonstrated for radially outward flow and corresponded with the flows in which flow visualization indicated the flow to be turbulent.

Results

The excellent agreement between experimental results and analytical results using the methods and computer program of reference [2] were earlier pointed up in Figs. 5 and 6, for radially outward flow when the flow is laminar. Figs. 8 and 9 show the similar agreement that was obtained for radially inward flow when the flow is laminar. The data displayed in these figures are typical of a large number of combinations of the flow parameters for which the flow was laminar and indicate agreement of both the radial pressure gradient and the local streamline slope as functions of the radial coordinate. The agreement reinforces the conclusions of Adams and Rice [3] concerning validity of the referenced analyses and points up the confidence with which the design information of Crawford and Rice [4] and Lawn and Rice [5] may be used to predict the performance of multiple disk turbomachines using laminar flow.

A summary of the results delineating the regimes of flow for radially outward flows with V_i = 1.0 is shown in Fig. 10. The parameter range over which the flow is laminar is extensive and includes the N_{Re}, U_0, α combinations of most interest as indicated in references [4, 6]; pumps to be operated with high efficiency use small values of U_0, generally below the threshold of mixed (transition) flow, and U_0 values of approximately unity.

A summary of the results delineating the regimes of flow for radially inward flows is shown in Fig. 11; in this case the nozzle angle α is also a parameter, where α = arctan (U_0, V_0), but the dependence of transition on α is so very weak that it is meaningless to account for it systematically. Instead, the data of Fig. 11 should be used for all nozzle angles occurring in practice (approximately $0 < \alpha \leq 15$) deg as a conservative procedure. The

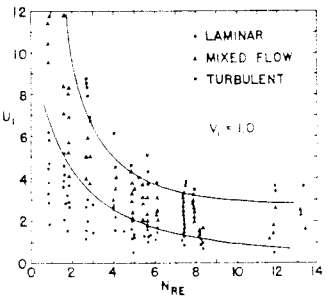

Fig. 10 Experimental data and the flow regimes delineated, for radially outward flow

parameter range over which the flow is laminar is extensive also for radially inward flow and includes the N_{Re}, U_0, α combinations of most interest as indicated in reference [5]; turbines for high efficiency will use small values of U_0 generally below the threshold of mixed (transition) flow. However, multiple disk turbines for some applications may infringe on the transition flow range for some off-design-point operation unless care is taken in the design choices to minimize U_0 as N_{Re} is increased. Furthermore, certain applications may emphasize minimization of size and weight rather than efficiency; the data shown in Fig. 11 then become crucial as guidance for design choices, to assure laminar flow operation of the turbine.

In the procedures used to determine the results in Figs. 10 and 11, the individual values of Q, Ω, and ν were widely varied in obtaining the parameter combinations. Only $\bar{r}_i$, $\bar{r}_0$, and h remained constant. This is sufficient to establish correlation of transition between disks having the physical geometry described.

Closure

While the empirical data presented herein is adequate for design guidance for multiple disk turbomachinery, it is not satisfying or definitive in a basic way regarding the occurrence of the flow regimes. Some effort has already been made to predict the stability of flows between corotating disks [9]; further analyses, together with basic experimentation, are needed to improve understanding of the wide variety of phenomena occurring in the general flow situation. In radially inward flow, the turbulent flow issuing from the turbine nozzles is stabilized because of the favorable pressure gradient and becomes laminar in the rotor soon after entrance, if the N_{Re}, U_0 combination is in the laminar flow zone indicated in Fig. 11. At smaller values of r however, the flow tends to become turbulent again since the local Reynolds number of the flow increases as the local velocity increases. For radially outward flow, with a N_{Re}, U_i combination in the laminar flow zone indicated in Fig. 10, flow supplied at the entrance is stabilized in the rotor soon after the entrance and tends to become more stable as the flow progresses to larger values of r_i. The data due to Bakke [8], for radially outward flows correspond generally with $N_{Re} > 100$ and $U_i > 25$ in the Nomenclature of the present paper; Bakke found only turbulent flow in his experiments and the data of Fig. 10 are consistent with that finding.

Kreith [10] reported relaminarization in radially outward flows between stationary disks; Bakke [8] produced data to qualify Kreith's observations concerning turbulent-to-laminar transition in flows between stationary disks. However, these investigators did not establish transition criteria or the presence or absence of relaminarization for flows between corotating disks. Additional experimental work is needed regarding these questions for rotating flows.

Extensive further details and documentation of the experiments and conclusions summarized herein are given in a dissertation by Pater [11] and a thesis by Crowther [12].

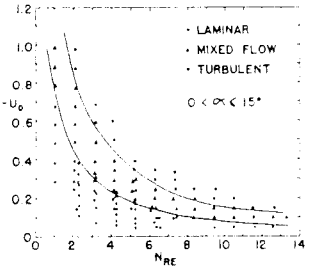

Fig. 11 Experimental data and the flow regimes delineated, for radially inward flow

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Calculated Design Data for the Multiple-Disk Turbine Using Incompressible Fluid

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Earlier analyses of the laminar radially inward throughflow of Newtonian incompressible fluid between parallel corotating disks have been used to calculate the performance of multiple-disk turbines using such flow passages as the rotor. Such turbines are characterized by certain dimensionless parameters and a large number of computerized calculations has enabled preparation of turbine performance maps for turbines idealized as having no losses external to the rotor (except for assumed zero pressure recovery in the turbine exhaust). These maps show the quantitative dependence of turbine efficiency, total pressure and delivered power on the turbine geometry and speed, the turbine nozzle direction and pressure drop, and on the fluid properties; full admission around the periphery of the rotor is assumed. Conventional loss information for the nozzles, and conventional bearing, seal and "disk friction" loss information, must be applied in the design process to provide prediction of actual turbine performance and comparison with conventional turbines.

[1]Numbers in brackets designate References at end of paper.

Contributed by the Fluids Engineering Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS and presented at the Joint Fluids Engineering & CSME Conference, Montreal, Quebec, Canada, May 13–15, 1974. Manuscript received at ASME Headquarters February 7, 1974. Paper No. 74-FE-9.

Nomenclature

Dimensional Quantities

H = operating head for turbine, length
h = spacing between adjacent disks, length
p_n = pressure change in nozzles, force/(length)^2
p_r = pressure change through rotor, force/(length)^2
p_t = *stagnation pressure change* through turbine, force/(length)^2
P = power delivered to turbine rotor, force-length/time
Q = volume flow through entire turbine, (length)^3/time
Q_i = volume flow between pair of adjacent disks, (length)^3/time
$\bar{r}$ = radial coordinate, length
$\bar{T}_i$ = torque applied by fluid between one pair of disks, force-length
$\bar{u}$ = $\bar{u}(\bar{r}, \bar{z})$ = *radial component of* velocity distribution, length/time
$\bar{U}_o = \frac{2}{h_i}\int_0^{h/2} \bar{u}_o(\bar{z})d\bar{z} = \frac{Q_i}{2\pi\bar{r}_o h}$, average value of radial component of velocity at rotor entrance (nozzle exit), length/time
$\bar{v}$ = $\bar{v}(\bar{r}, \bar{z})$ = tangential component of velocity distribution, *length/time*
$\bar{V}_o = \frac{2}{h_i}\int_0^{h/2} \bar{v}_o(\bar{z})d\bar{z}$ = average value of tangential component of velocity at rotor entrance (nozzle exit), length/time
$\bar{w}(\bar{r}, \bar{z})$ = axial component of velocity distribution, length/time
$\bar{z}$ = axial space coordinate, length
ρ = *density of fluid, mass/(length)*^{3}
μ = viscosity of fluid, force-time/(length)^2
Ω = angular velocity of rotor, rad/(time)

Dimensionless Quantities

N = number of spaces between disks, in set of disks composing a turbine rotor
$N_{RE} = \frac{\rho\Omega\bar{r}_o^2}{\mu}$, Reynolds number
$p_n = p_n/\rho\Omega^2\bar{r}_o^2$
$p_r = p_r/\rho\Omega^2\bar{r}_o^2$
$p_t = \bar{p}_t/\rho\Omega^2\bar{r}_o^2$
$r = \bar{r}/\bar{r}_o$
$T_i = \bar{T}_i/\rho\bar{r}_o^4 h\Omega^2$
$u = u(r, z) = \bar{u}(\bar{r}, \bar{z})/\Omega\bar{r}_o$
$U_o = \bar{U}_o/\Omega\bar{r}_o = \frac{Q_i}{2\pi\bar{r}_o^3\Omega h_i}$ = volume flow rate parameter
$v = v(r, z) = \bar{v}(\bar{r}, \bar{z})/\Omega\bar{r}_o$
$V_o = \bar{V}_o/\Omega\bar{r}_o$ = tangential velocity parameter
$z = \bar{z}/h$
$\eta = \frac{T_i}{2\pi U_o p_t}$ = efficiency

Subscripts

i = inner (exhaust) radial station
o = outer (nozzle exit, rotor entrance) radial station

Introduction

The conception of multiple-disk turbomachinery is credited to N. Tesla early in this century [1–3], and the devices were widely discussed in the semitechnical press of that time [4–7]. Subsequently, such turbomachinery received scant attention for many years but the list of references included in this paper indicates the extent of a revival of interest. A number of experimental and analytical feasibility studies have been reported for the turbine configuration of multiple-disk turbomachinery [8–15]. The papers reporting on these studies make the operating principle clear and indicate that laminar flow of fluid in the rotor, rather than turbulent flow, is of the most practical interest.

A recent sequence of analytical contributions has made possible calculation of the performance of multiple-disk turbines, according to an idealized model, provided that the fluid is Newtonian and incompressible and that the flow is laminar. The essential part of the flow in a multiple-disk turbine is that between a pair of corotating disks; the turbine consists of a number of such flows in parallel, together with fluid supply

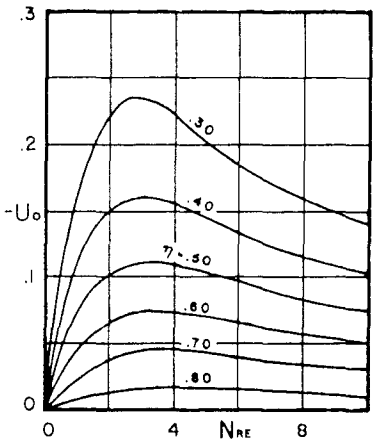

Fig. 1 Constant efficiency lines on N_{RE}, U_o coordinates for V_o=1.1, r_i=0.3 and parabolic inlet velocity distributions

A numerical (finite-difference) scheme for calculating the entrance flow between corotating disks, for radially outward flow corresponding with a multiple-disk pump was presented by *Breiter and Pohlhausen [18]. A similar method of calculation* was applied for radially inward flow, corresponding with the multiple-disk turbine, by Boyd and Rice [19]; the fluid was considered admitted uniformly around the outer periphery of the disks through nozzles providing tangential, radial, and axial components of velocity. In [19], the disk-to-disk spatial distributions of the velocity components at the outer radius $\bar{r}_o$ (provided by the nozzles to the rotor) are considered arbitrary but specified, and these are additional parameters for the specification of a *case of flow in the entrance region between the disks.*

Furthermore, [19] introduces a simplification in the parameterization; with the general shape of the disk-to-disk velocity component profiles specified (uniform, parabolic, etc.), then the three parameters N_{RE}, U_o and V_o are sufficient to characterize a case of entrance region flow. Here V_o is the average value of the dimensionless tangential component of the velocity of the flow supplied by the nozzles to the rotor, corresponding with the "fluid-to-blade velocity ratio" of conventional turbine practice. *The calculation procedure detailed in [19] also is applicable for* the asymptotic flow region and overcomes the limitation in accuracy in the earlier methods of calculating that region. The disadvantage of the method is the excessively lengthy computation time required by digital computer.

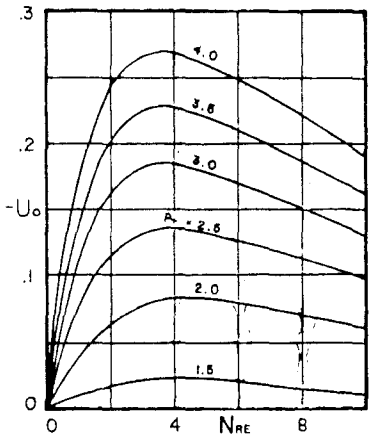

Fig. 2 Constant dimensionless total pressure lines on N_{RE}, U_o coordinates for V_o=1.1, r_i=0.3 and parabolic inlet distributions

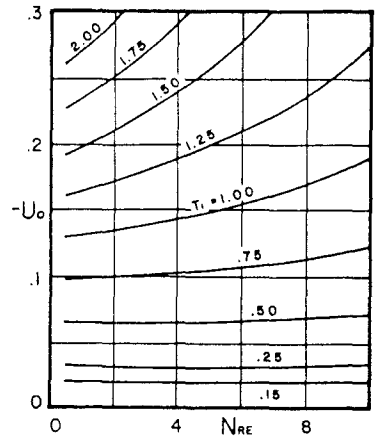

Fig. 3 Constant dimensionless torque lines on N_{RE}, U_o coordinates for V_o=1.1, r_i=0.3 and parabolic inlet velocity distributions

The adequacy of the calculation model and of the numerical computations was confirmed experimentally by Adams and Rice [20].

Boyack and Rice [21] developed an integral method for the flow model used by previous investigators and a computer program which executes extremely rapidly. The method yields results essentially identical with those of the lengthy computations of references [18, 19] and therefore also confirmed experimentally in reference [20]. Use of this computer program enabled calculation of the very large number of flow cases necessary to obtain the design data presented herein. Crawford [22] improved the ability of the method of Boyack and Rice to handle uniform inlet profiles, exhaustively examined the correctness of results in the entrance region, and also further decreased the execution time required for digital computation by improved programming.

Model of Turbine for Calculations

The mathematical model and calculation procedure for the flow between a pair of corotating disks is that used by Boyack and Rice [21] discussed above. *The multiple-disk turbine consists* of many such corotating disks arranged on a common shaft with the flow passages in parallel. The choice of the number of disks is determined by individual design requirements; the design information presented herein is based on the flow between a single pair of corotating disks. The flow is considered to be a laminar flow of an incompressible Newtonian fluid. The calculations provide realistically for the thermodynamically irreversible flow effects in the space between the disks. The pressure change in the flow between the disks, p_r, is composed of that due to radially-incurred area change (with consequent velocity change) and that due to irreversible (viscous) effects.

The dimensionless parameters N_{RE}, U_o and V_o for the flow between a pair of corotating disks carry over to become also the independent parameters for the turbine model. The turbine is assumed provided with nozzles around the entire periphery of the rotor (full admission). The nozzles provide the specified values of U_o and V_o to the space between a pair of disks from a plenum chamber having the dimensionless stagnation pressure p_t. The stagnation pressure is assumed conserved in the flow in the nozzles in the turbine model, that is, the flow in the nozzles is considered to be thermodynamically reversible. Part of p_t is the

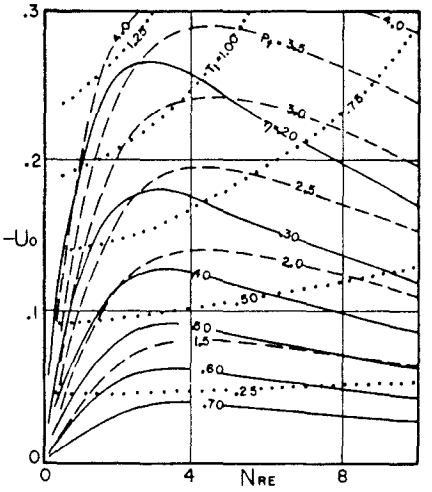

Fig. 4 Performance data for V_o=0.8, r_i=0.3 and parabolic inlet velocity distributions

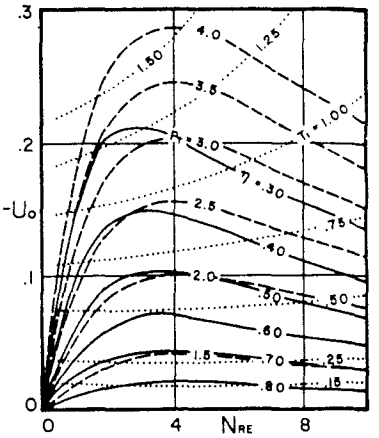

Fig. 5 Performance data for V_o=1.0, r_i=0.3 and parabolic inlet velocity distributions

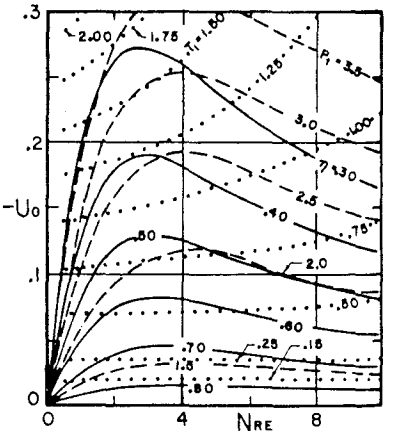

Fig. 6 Performance data for V_o=1.1, r_i=0.4 and parabolic inlet velocity distributions

dimensionless pressure drop in the reversible nozzles, p_n, given by

$$p_n = \frac{1}{U_e}\int_e^{1/2} [u_e^2(z) + v_e^2(z)]u_e(z)\,dz \qquad (1)$$

where the contribution due to $u_n(z)$ is considered to be negligibly small. It is seen that p_n depends on the disk-to-disk distribution of the velocity components.

The fluid can be supplied to the rotor of a multiple-disk turbine

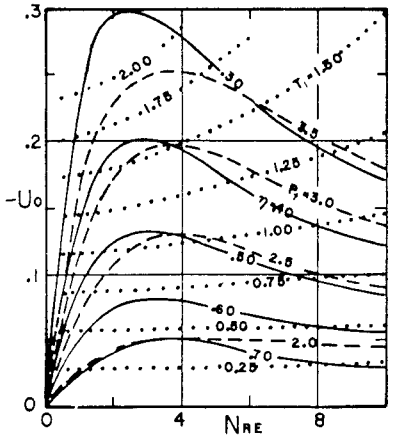

Fig. 7 Performance data for V_e=1.3, r_i=0.4 and parabolic inlet velocity distributions

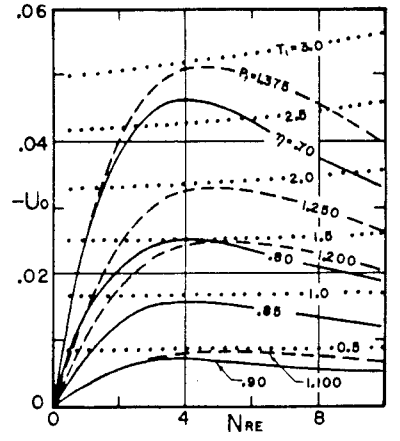

Fig. 8 Performance data for V_e=1.1, r_i=0.3 and uniform inlet velocity profiles

through nozzles arranged in various ways. On the one hand, a separate nozzle ring may be provided for each disk space; in this case the disk-to-disk distribution of the velocity components $u_e(z)$ and $v_e(z)$ will be approximately parabolic. On the other hand, the nozzle ring may consist of slots extending across the rotor or at least across several disk spaces; in this case, disk-to-disk distribution of the velocity components $u_e(z)$ and $v_e(z)$ will be approximately uniform.

As an idealization considered of interest in practice, the distribution of $u_e(z)$ and $v_e(z)$ assumed herein is parabolic generally; only where specifically noted later, the distribution is assumed uniform in order to demonstrate the effect of the distribution on the performance of a multiple-disk turbine. However, in applications the disk spacing frequently is so small that separate nozzles are impractical and a single admission slot must be used. The uniform distribution is then a better idealization of the supply flow.

In the turbine model, the fluid is considered to exhaust from the rotor into a stagnant exit plenum chamber, without the benefit of an exhaust diffuser. Thus, with the nozzles considered to be supplied with fluid from a stagnant condition,

$$p_i = p_n + p_r \qquad (2)$$

and p_r is the dimensionless stagnation pressure difference required across the turbine. The dimensional quantity p_r/ρ is, then, the available energy of the fluid per unit mass and is the proper denominator for the expression of turbine efficiency, η.

The torque delivered to the rotor by the fluid can be calculated by two fundamentally different means; both involve use of the calculated velocity field between the disks. As one possibility, the local shear stress over the surface of the disks can be derived from the velocity field and fluid properties and hence the local torque due to the shear force can be integrated over the surface of the disks to obtain the rotor torque for the flow between a pair of disks. As an alternative, a finite control volume can be used between the entrance and exit radii and the torque can be calculated from the change of the moment of momentum of the fluid between those radial stations. The latter procedure is used here since it is more direct and accurate mainly because it avoids differentiation of the fluid velocity profiles. The dimensionless torque T_i (for one space between a single pair of disks) is also the dimensionless work delivered to the rotor by the fluid and is given by

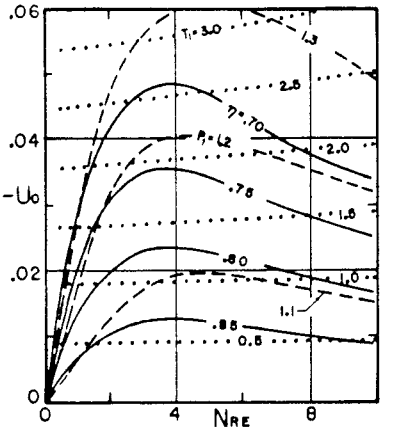

Fig. 9 Performance data for V_e=1.1, r_i=0.4 and uniform inlet velocity profiles

$$T_i = 4\pi\left[r_e^2\int_e^{1/2} u(r_e, z)v(r_e, z)dz - \int_e^{1/2} u_i(z)v_i(z)dz\right]. \qquad (3)$$

Hence the turbine efficiency is given by

$$\eta = \frac{T_i}{2\pi U_e p_i} \qquad (4)$$

The computer program used to implement the analysis of Boyack and Rice [21] was arranged to accept N_{RE}, U_e and V_e as input parameters with the shape of $u_e(z)$ and $v_e(z)$ specified, and to calculate p_i, T_i and η as output results for selected dimensionless exit radii r_i such that $1 \geq r_i \geq 0.05$ with $\Delta r_i \approx 0.05$. Thus each combination of input parameters corresponds with an infinity of dimensionless turbines having different exit radii; furthermore, each combination of N_{RE}, U_e, V_e and r_i correspond with an infinity of dimensional (actual) turbines.

The turbine model does not allow for resisting torques acting on the turbine rotor due to rotor-to-housing "disk friction," bearings, seals, etc. Furthermore, the model ignores pressure losses that will occur in an actual turbine due to interference of the disk edges with the nozzle exit flow and due to irreversibility in the nozzle flow, and due to exit passage area changes and frictional losses. (The latter "loss" may actually be a "gain" if the exit passage can be designed to produce a pressure recovery from the kinetic energy exhausting from the rotor, since the turbine model assumes no such pressure recovery.)

The design information presented thus far, therefore, accounts for losses only in the flow between the disks and leaves to the designer the taks of accounting for the loss of torque and efficiency due to nozzle efficiency, nozzle-rotor interference, rotor-to-housing "disk friction," bearings, and exhaust passage pressure change. The pressure, efficiency, and torque values given herein are therefore upper limits of the performance of actual multiple-disk turbines and must be used with conventional loss information for the turbine parts external to the rotor to calculate

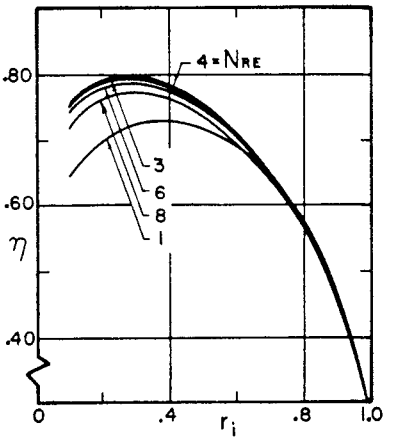

Fig. 10 Turbine efficiency as a function of r_i with N_{RE} as a parameter, for V_e=1.1, U_e=−0.02, and parabolic inlet velocity distributions

the performance of actual turbines. A design example given later herein includes references for such loss information.

Results—Design Information Summary

The computer program was executed for a very large number of combinations of N_{RE}, U_e and V_e, each yielding values of p_i, T_i and η for values of r_i such that $1 \geq r_i \geq 0.05$. The results were cross-plotted by hand in order to produce data for plotting lines of constant p_i, T_i and η for a fixed value of r_i and of one input parameter, and using the remaining two input parameters as coordinates. A large amount of judgment was exercised in making additional computer program executions to "fill in"

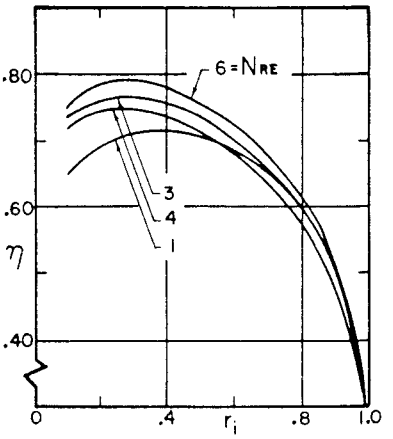

Fig. 11 Turbine efficiency as a function of r_i with N_{RE} as a parameter, for V_e=1.3, U_e=−0.02 and parabolic inlet velocity distributions

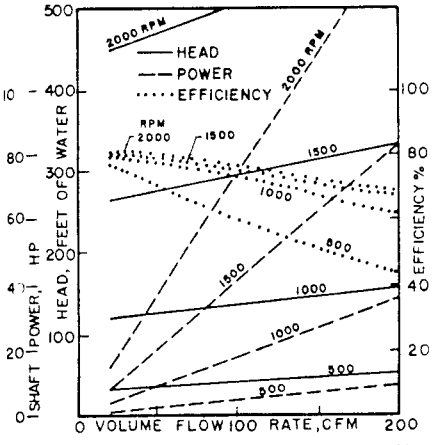

Fig. 12 Calculated performance for an actual water turbine with a rotor dia of 12 in.

the crossplots with definite additional data points wherever rates of change were found to be large during the crossplotting process.

Typical results are shown in Figs. 1, 2 and 3 which give lines of constant p_i, T_i and η using U_e, N_{RE} coordinates and for V_e = 1.1 and r_i = 0.3 and for parabolic velocity profiles at the rotor entrance (nozzle exit). From a very large number of plots such as those of Figs. 1, 2 and 3, dimensionless performance maps were produced.

The dimensionless performance maps are voluminous beyond the possibility of journal publication. They are available in the form of a University Report [23]. The results given therein are in four groups, arranged according to four values of the parameter V_e: 0.8, 1.0, 1.1 and 1.3. This is thought to be the useful and interesting range for V_e with the values of most interest being near and just exceeding unity. The data for each value of V_e have r_i as a parameter and are presented in the form of sets of lines for constant p_i, T_i and η on U_e, N_{RE} coordinates.

Figs. 4–7 present typical samples of the results referenced above, for the case in which the turbines have parabolically shaped disk-to-disk velocity components at the rotor entrance (nozzle exit). The Report [23] also gives voluminous results in the same form for turbines having uniform disk-to-disk velocity components at the rotor entrance. Typical samples of these results are presented in Figs. 8 and 9. Comparison of the various results yields information concerning the effect of the shape of the entrance profiles on turbine performance. The obtaining of the results in the Report [23] and of similar results for other parameter values is expensive in terms of digital computer usage but once done, eliminates the need for further analysis of the flow within the rotor of a multiple-disk turbine; only very modest algebraic calculations are needed to apply the results to the design of multiple-disk turbines.

Discussion

Some insights into multiple-disk turbine design characteristics are afforded through crossplotting of the performance results. Fig. 10 presents the dependency of turbine efficiency on the dimensionless inner (exhaust) radius r_i, with N_{RE} as a parameter, for V_e = 1.1 and U_e = − 0.02. (These are reasonable and interesting values for these parameters.) The optimum inner radius is approximately 0.3 for $3 \leq N_{RE}$ and is approximately 0.4 for N_{RE} = 1; the figures also shows that the greatest efficiency occurs for N_{RE} = 4 for this parameter combination. An

increased value of the volume flow rate parameter reduces the efficiency at all values of N_{RE} and increases the value of r_i for maximum efficiency.

The effect on efficiency of an increased value of V_o is detailed in Fig. 11, for $U_o = -0.02$. The maximum efficiency is somewhat decreased by the higher value of V_o but the turbine characteristics are generally unchanged.

Further details concerning these matters are contained in the Report [23]. For $U_o = -0.02$ and $N_{RE} = 4$, maximum efficiency occurs near $V_o = 1.0$ and the effect of increasing r_i above 0.3 is to reduce efficiency and to cause the local maximum efficiency to occur at higher values of V_o. Furthermore, both T_1 and p_1 are almost-linear functions of V_o, increasing as r_i is decreased. Throughout the parameter range, changing the N_{RE} values changes the turbine performance quantitatively but all trends remain similar to those for $N_{RE} = 4$.

It is necessary to realize that the results pertain to the flow between a single pair of disks; for a turbine with N spaces between disks, $((N + 1)$ disks), the total flow rate, Q, will be

$$Q = NQ_1 = NU_o\, 2\pi r_o^2 \Omega h \tag{5}$$

and the power delivered to the rotor by the fluid will be

$$P = NT_1 \rho r_o^4 h \Omega^2 \tag{6}$$

which can be found alternatively from the equation

$$P = \eta Q p_1. \tag{7}$$

Both equations (6) and (7) refer to the power delivered to the rotor by the fluid; the shaft power will be less because of rotor-to-housing "disk friction," bearing losses, seal losses, etc. Also, for an actual turbine, the stagnation pressure required will be more than p_1 because of irreversible pressure changes in the fluid supply nozzles and between the nozzles and the spaces between the disks. Consequently, the actual turbine efficiency will be somewhat less than that indicated in the figures. The designer will allow for these facts in the use of the design data to predict turbine performance.

There are numerous sources of information of value in estimating the various losses. General design features and the performance of bearings, seals, and nozzles are reviewed in references [24, 25]. Daly and Nece [26] give sufficient detail concerning the rotor-to-housing "disk friction" to allow optimization of the clearance and estimation of the power loss from that source.

An Example of Turbine Design and Performance

Using the reference material earlier cited and the data in the figures, a very simple digital computer program can be used to determine the physical dimensions of turbines and the performance characteristics. As an example, it is assumed that a water turbine is desired which will produce 12 shaft horsepower when operated at 1000 rpm with a total head of 130 ft of water. A suitable design point is chosen as a compromise between efficiency, physical size, and ease of executing design details; the design point is somewhat arbitrary and several should be investigated by the designer. In this case, the point chosen is $U_o = -0.02$, $N_{RE} = 4$ and $r_i = 0.30$. It is further assumed that the turbine will use well-designed nozzles to supply fluid to the rotor with an approximately parabolic disk-to-disk distribution of velocity, carbon ring seals, and conventional bearings, and will have the optimum casing-to-rotor clearance for least "disk friction."

The foregoing assumptions and choices result in a turbine having a rotor dia of 12.0 in., a rotor exhaust dia of 3.6 in., and 479 disks spaced 0.0083 in. apart. If a disk thickness of 0.004 in. is arbitrarily chosen, this results in a rotor with an axial length of 5.89 in.; the total volume of the turbine is approximately 1200 cu in. including the turbine casing and other structure.

The performance characteristics are shown in Fig. 12.

The very small disk thickness used in this design example is realistic; each disk carries very small torsional and radial loading and no axial loading. The disks can be spaced correctly and the rotor made more rigid by dimpling the disks to achieve the required average disk spacing.

In this example, the maximum turbine efficiency is approximately 81 percent and occurs at lowest flow rate and highest speed. This value of efficiency is by no means the maximum achievable in multiple-disk turbines. A different choice of the design point or of speed, power or head at the design point, will result in a different set of performance curves and characteristics.

As an extension of the foregoing example it is considered that the turbine is to be designed to use a light oil ($\rho = 54$ lb$_m$/ft^3, $\nu = 0.0001$ ft^2/sec) rather than water ($\rho = 62.3$ lb$_m$/ft^3, $\nu = 0.0000106$ ft^2/sec). All other specifications being the same, the oil turbine requires 197 disks and has a spacing between the disks of 0.02345 in. which results in a rotor length of 5.41 in., and an efficiency of 0.76 and volume flow rate of 75 cfm at 1000 rpm and 130 ft head. The computerized evaluation program would easily generate the off-design point characteristics of this turbine also.

In contrast with conventional turbines, the performance of the rotor and hence good estimates of the turbine characteristics can be realistically calculated for multiple-disk turbines for both design point and off-design point operation, with ease using only algebraic calculations.

Conclusion

While the design data presented herein are sufficient to allow calculation of the performance of multiple-disk turbines and comparison with conventional turbines, the designer will finally want more detail of the flow field between the disks. At such point of interest, the designer should use references [21, 22] to obtain a computer-oriented analysis which easily and economically yields the desired detailed flow information.

Further details concerning the generation of the cross-plots and of design procedures are given in a thesis by Lawn [27] as well as the Report [23].

The multiple-disk turbine can be designed with high efficiency for fluid with any viscosity and density; it remains to be established by further investigation and by practice whether or not the turbine has substantial advantages when compared with conventional turbines, in the various extremes of application. For all of the data presented herein, it is assumed that the flow in the rotor is laminar. It is known from investigation by Adams and Rice [20] that laminar flow occurs in the flow between rotating disks over a wide range of the flow parameters. The criteria for prediction of laminar and of turbulent flow has been established for multiple-disk turbines; it is known at this time that laminar flow persists to high values of N_{RE} for low values of U_o, and to high values of U_o for low values of N_{RE}; high values of both parameters promote turbulent flow. Details of the delineation of the laminar region and of the experiments leading to it are available in a University Report [28] and will appear subsequently in a journal publication.

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AN INTEGRAL SOLUTION FOR COMPRESSIBLE FLOW THROUGH DISC TURBINES

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Abstract

The three-dimensional, compressible, laminar, inward flow between corotating discs has been modeled. The method of solution involves the fitting of eighth-order polynomials to the radial and tangential velocity profiles at each incremental step in the radial space variable. A fit is accomplished through satisfaction of the boundary conditions and the appropriate differential equations in both integrated and non-integrated form.

Results, which are applicable to disc turbine design, have been obtained for various values of Mach number (M), Reynolds number (Re), and average dimensionless inlet velocities in the radial (U_o) and tangential (V_o) direction. They show that optimum turbine efficiency is obtained for M<0.6, V_o=1.10, and minimum values of U_o. Since torque decreases with U_o, efficiency can be increased only at the expense of increased machine size. Reynolds number had only a moderate effect on efficiency and no practical effect on torque or pressure ratio. For the range of parameter values of greatest interest, the machine operates typically at low pressure ratios.

Introduction

The disc turbine, bladeless turbine, or Tesla turbine has been given renewed attention the past few years. The advantages over conventional bladed turbines include reduced noise and simplicity of construction (reducing costs), hopefully at little or no loss of turbine efficiency or increase in machine size. Some applications exist even at the expense of lost turbine efficiency and/or increased size. Another possible advantage is an increase in thermal efficiency through use of ceramic discs which allow high temperature operation. Problems associated with the fabrication of these discs are reduced considerably if the need for blades is eliminated.

The machine consists of a row of closely spaced, thin, circular disks mounted on a single shaft. The discs have openings at an interior radial location through which the flow exits axially. The working fluid is expanded through nozzles directed tangent to the outer periphery of the discs. As the working fluid flows tangentially and slightly inward on its path between the discs it transfers angular momentum via the shear forces at the surface. Due to the proximity of the discs, the flow can be thought of as a confined type rather than a boundary layer type.

The concept was patented by Tesla in 1913. Several attempts have been made in the last 20 years to build a turbine of this type in order to obtain performance characteristics. Many of these attempts were disappointing, but were based upon limited knowledge of the flow field. Some were just based on faulty reasoning; i.e., roughening disc surfaces "to increase friction." Turbines were built by Armstrong[2], Beans[3], Gruber[6], Leaman[9], Muhlstein and Read[14], and Rice[16]. Most of the analysis pertient to the problem has been accomplished by Rice and coworkers[4,5,8,10-13,17] for incompressible flow. Results of these studies show that it is desirable to operate the turbine under conditions of laminar flow with low flow rate between disc pairs for maximum efficiency. Data obtained by Pater, Crowther, and Rice[15] on a solution of ethlylene glycol and water show pertinent transition criteria for both cases of radial inward and radial outward flow between a pair of rotating discs. The remaining problem of interest is that of steady, laminar, compressible flow through this turbine.

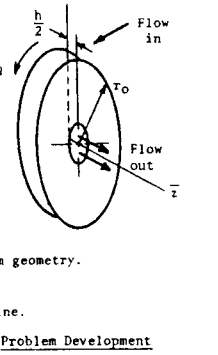

Figure 1. Problem geometry.

Problem Development

The geometry of the problem is shown in Fig. 1. The equations of motion for a constant viscosity fluid flow are:

$$\bar{\rho}\frac{D\bar{\vec{v}}}{Dt} = \bar{\vec{F}} - \nabla(\bar{p} - \tfrac{1}{3}\mu\nabla\cdot\bar{\vec{v}}) + \mu\nabla^2\bar{\vec{v}}. \tag{1}$$

The continuity equation is:

$$\frac{D\bar{\rho}}{Dt} + \bar{\rho}\nabla\cdot\bar{\vec{v}} = 0 \tag{2}$$

Finally, the equation for polytropic flow provides an equation of state.

$$\frac{\bar{p}}{\bar{\rho}^n} = \text{constant}. \tag{3}$$

If the assumption is made that disc thickness, $h \ll r_o$, the disc radius; and nondimensional terms:

$$u = \bar{u}/r_o\Omega \qquad v = \bar{v}/r_o\Omega \qquad w = \bar{w}/h\Omega$$
$$r = \bar{r}/r_o \qquad z = \bar{z}/h$$
$$p = \bar{p}/p_o \qquad \rho = \bar{\rho}/\rho_o$$

are used, equations (1) reduce to the following:

$$u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{dp/dr}{kM^2\rho} + \frac{\partial^2 u/\partial z^2}{Re\ \rho} \tag{4}$$

$$u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = \frac{\partial^2 v/\partial z^2}{Re\ \rho} \tag{5}$$

The continuity equation becomes,

$$\frac{\partial}{\partial r}(\rho r u) + \frac{\partial}{\partial z}(\rho r w) = 0. \tag{6}$$

All θ-dependent terms have been dropped from equations (4-6) by reason of symmetry. If the isentropic flow equation of state is chosen, n=k, and equation (3) becomes:

$$p = \rho^k \tag{7}$$

This choice does not imply that we are assuming the turbine is operating isentropically. It is merely a convenient and reasonable choice of state equation for most gases. Boundary conditions for this flow are:

$$u(r,1/2) = w(r,1/2) = 0 \tag{8a}$$

$$v(r,1/2) = r \tag{8b}$$

$$\frac{\partial u}{\partial z}(r,0) = \frac{\partial v}{\partial z}(r,0) = 0 \tag{8c}$$

$$u(1,z) = u_o(z) \tag{8d}$$

$$v(1,z) = v_o(z) \tag{8e}$$

$$p(1,z) = 1 \tag{8f}$$

Equations (4-7) and boundary conditions, equations (8), constitute a statement of the problem. The equations are similar to those for incompressible flow used by Matsch and Rice[13], Boyd and Rice[5], and Boyack and Rice[4]. Differences include the addition of Mach number as a parameter and the relevance of the local value of pressure on the solution.

Method of Solution

Boyd and Rice[5] solved the incompressible flow equations using finite difference techniques. However, the method was expensive in terms of computation time. Boyack and Rice[4] used polynomial approximations for the velocity components and a unique method of solution to solve the same equations reducing computation time by a factor of 50. The latter method was chosen for solution of the present equations.

Since the paper by Boyack and Rice contains some errors, the method is repeated here. Eighth-order polynomials were chosen to represent the tangential and radial velocities:

$$u(r,z) = \sum_{i=0}^{8} A_i(r)z^i \tag{9}$$

$$v(r,z) = \sum_{i=0}^{8} B_i(r)z^i \tag{10}$$

Flow symmetry in the z-direction commands that all odd i coefficients are zero. Thus, if the inlet profiles, Mach number, and Reynolds number are specified, solution can be obtained at each r (starting at the inlet) provided 5 independent equations for the A_i coefficients, 5 independent equations for the B_i coefficients, and an additional equation containing pressure are chosen.

First, w and ρ are eliminated from equations (4) and (5) using equations (6) and (7). Integrating equation (6) with respect to z and using equations (9-10):

$$w = -\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)\sum_{i=0}^{8} A_i\frac{z^{i+1}}{i+1} - \sum_{i=0}^{8} A'_i\frac{z^{i+1}}{i+1} \tag{11}$$

where primed terms refer to differentiation with respect to r. Substitution of equations (9-11) directly into equation (4) gives:

$$\sum_{i=0}^{8}\sum_{j=0}^{8}\left\{A_iA'_j - \frac{B_iB_j}{r} - \frac{i}{j+1}\left[\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)A_iA_j + A_iA'_j\right]\right\} z^{i+j}$$
$$+ \frac{dp/dr}{kM^2p^{1/k}} - \frac{1}{Re\ p^{1/k}}\sum_{i=0}^{8} i(i-1)A_i z^{i-2} = 0 \tag{12}$$

In order that this equation be satisfied, the coefficient of each power of z must vanish.

$$\sum_{i=0}^{n}\left\{A_iA'_{n-i} - \frac{B_iB_{n-i}}{r} - \frac{i}{n-i+1}\left[\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)A_iA_{n-i} + A_iA'_{n-i}\right]\right.$$
$$\left. - \frac{(n+2)(n+1)A_{n+2}}{Re\ p^{1/k}}\right\} + \frac{dp/dr}{kM^2p^{1/k}} = 0;\ n=0,2,4,\ldots,16 \tag{13}$$

Up to 9 equations can be obtained from equation (13); however, not all will be needed.

Substitution of equations (9-11) into equation (5) yields:

$$\sum_{i=0}^{8}\sum_{j=0}^{8}\left\{A_iB'_j + \frac{A_iB_j}{r} - \frac{i}{j+1}\left[\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)B_iA_j + B_iA'_j\right]\right\} z^{i+j}$$
$$- \frac{1}{Re\ p^{1/k}}\sum_{i=0}^{8} i(i-1)B_i z^{i-2} = 0 \tag{14}$$

From this is obtained:

$$\sum_{i=0}^{n}\left\{A_iB'_{n-i} + \frac{A_iB_{n-i}}{r} - \frac{1}{n-i+1}\left[\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)B_iA_{n-i} + B_iA'_{n-i}\right]\right.$$
$$\left. - \frac{(n+2)(n+1)B_{n+2}}{Re\ p^{1/k}}\right\} = 0;\ n=0,2,4\ldots,16. \tag{15}$$

Another 9 equations are available here.

Rearranging equation (4), integrating to the midpoint between discs, and using the boundary conditions equations (8), we obtain:

$$\int_0^{1/2}\left[2u\frac{\partial u}{\partial r} - \frac{v^2}{r} + u^2\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)\right]dz = -\frac{dp/dr}{2kM^2p^{1/k}}$$
$$+ \frac{\partial u/\partial z}{Re\ p^{1/k}}\Bigg|_0^{1/2} \tag{16}$$

which is:

$$\sum_{i=0}^{8}\sum_{j=0}^{8}\frac{2A_iA'_j - \frac{B_iB_j}{r} + \left(\frac{1}{r} + \frac{dp/dr}{pk}\right)A_iA_j}{(i+j+1)\ 2^{i+j+1}}$$
$$+ \frac{dp/dr}{2kM^2p^{1/k}} - \frac{1}{Re\ p^{1/k}}\sum_{i=0}^{8}\frac{iA_i}{2^{i-1}} = 0 \tag{17}$$

Similar treatment of equation (5) yeilds:

$$\int_0^{1/2}\left[u\frac{\partial v}{\partial r} + \left(\frac{2}{r} + \frac{dp/dr}{pk}\right)uv + v\frac{\partial u}{\partial r}\right]dz = \frac{\partial v/\partial z}{Re\ p^{1/k}}\Bigg|_0^{1/2} \tag{18}$$

which becomes:

$$\sum_{i=0}^{8}\sum_{j=0}^{8}\frac{[A_iB'_j + A_iB_j\left(\frac{2}{r} + \frac{dp/dr}{pk}\right) + B_iA'_j]}{(i+j+1)\ 2^{i+j+1}}$$
$$- \frac{1}{Re\ p^{1/k}}\sum_{i=0}^{8}\frac{iB_i}{2^{i-1}} = 0 \tag{19}$$

Integrating equation (6) to the midpoint between discs and using the boundary conditions,

$$\int_0^{1/2}\frac{\partial}{\partial r}(\rho r u)dz = 0 \tag{20}$$

or

$$\frac{\partial}{\partial r}\left[r\rho\int_0^{1/2} u\,dz\right] = 0 \tag{21}$$

It is convenient to introduce a mean radial flow parameter at the inlet:

$$U_o = 2\int_0^{1/2} u(1,z)dz = 2\int_0^{1/2} u_o(z)dz \tag{22}$$

Using equations (21-22) and the boundary conditions, the result is:

$$\sum_{i=0}^{8}\frac{A_i}{(i+1)2^i} = \frac{U_o}{rp^{1/k}} \tag{23}$$

Boundary conditions, equations (8a, b) yield:

$$\sum_{i=0}^{8}\frac{A_i}{2^i} = 0 \tag{24}$$

$$\sum_{i=0}^{8}\frac{B_i}{2^i} = r \tag{25}$$

Finally, equation (4) is integrated 3 times with respect to z in the following manner:

$$\int_0^{1/2}\int_{1/2}^{z}\int_0^{z}\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right)dz\,dz\,dz = \frac{-dp/dr}{24kM^2p^{1/k}}$$
$$- \frac{U_o}{2r\ Re\ p^{2/k}} \tag{26}$$

which becomes:

$$24\sum_{i=0}^{8}\sum_{j=0}^{8}\left\{\frac{A_iA'_j - \frac{B_iB_j}{r} - \frac{i}{j+1}\left[\left(\frac{1}{r} + \frac{dp/dr}{pk}\right)A_iA_j + A_iA'_j\right]}{(i+j+1)(i+j+3)2^{i+j+3}}\right\}$$
$$+ \frac{dp/dr}{kM^2p^{1/k}} + \frac{12\ U_o}{r\ Re\ p^{2/k}} = 0 \tag{27}$$

The equations chosen were equations (13) with n=2 and 4, equations (15) with n=0, 2, and 4, and equations (17, 19, 23, 24, 25, 27). The zeroth order equation from equations (13) was not used as it is redundant when equation (27) is included. Since more equations are available than needed (overspecified), only fourth order satisifaction of equations (12) and (14) is achieved. Still, the unique quality which sets this type of solution apart from conventional integral techniques is that the parent differential equations are satisified within the flow (at least up to the fourth degree).

Since the chosen equations contain first order derivatives in r, it was necessary to solve them numerically. The Adams-Bashforth predictor-corrector method was chosen for its superior stability behavior. Truncation error was improved by using a modifier step before each operation. The solution was started using the Runga-Kutta method. Those equations which did not have derivatives in them were differentiated once with respect to r.

Initial values of the velocity profile coefficients were generated using equations (8a, 8b, and 22) and:

$$V_o = 2\int_0^{1/2} v(1,z)dz = 2\int_0^{1/2} v_o(z)dz \tag{28}$$

Since the most probable type of entry profile for a turbine is a uniform flow across the discs, all of the runs were made by approximating that condition as closely as possible (within the constraint of the 8th order assumption) using:

$$u_o(z) = A_o + A_8z^8 \tag{29}$$

$$v_o(z) = B_o + B_8z^8 \tag{30}$$

For the case of parabolic entry profiles to match the experimental data,

$$u_o(z) = A_o + A_2z^2 \tag{31}$$

$$v_o(z) = B_o + B_2z^2 \tag{32}$$

were used. For the "uniform" entry case equations (29, 30) become:

$$u_o(z) = (\tfrac{9}{8} - 288z^8)U_o \tag{33}$$

$$v_o(z) = 1 + (\tfrac{9}{8} - 288z^8)(V_o - 1) \tag{34}$$

The parabolic entry pair are:

$$u_o(z) = (\tfrac{3}{2} - 6z^2)U_o \tag{35}$$

$$v_o(z) = 1 + (\tfrac{3}{2} - 6z^2)(V_o - 1) \tag{36}$$

Thus, solution was effected using the initial profiles and initial pressure, stepping inward with the Runga-Kutta method, obtaining the needed 11 derivatives by solving the selected equations (13, 15, 17, 19, 23, 24, 25, and 27) simultaneously with the Crout method. When the sufficient number of starting values had been obtained, the Adams-Bashforth P-C method was employed to continue the solution. Although 11 equations were needed to obtain the derivatives at each step, not all 11 derivatives were needed. To save machine time, only the A_0, A_2, A_4, B_0, B_2, B_4, B_6, and p values were obtained using the R-K or P-C methods at each step. The A_6, A_8, and B_8 values were obtained using equations (23, 24, 25) after solving for the other coefficients.

Dimensionless torque was obtained by employing conservation of angular momentum:

$$\bar{T}(r) = 2\int_0^{h/2}\bar{r}\bar{v}\bar{\rho}\bar{u}(2\pi\bar{r}d\bar{z}) - 2\int_0^{h/2} r_o\bar{v}_o\rho_o\bar{u}_o(2\pi r_o d\bar{z}) \tag{37}$$

$$T(r) = 4\pi\left[r^2\rho\int_0^{1/2} uvdz - \int_0^{1/2} u_{o}v_{o}dz\right] \tag{38}$$

The conventional turbine efficiency was computed from

$$\eta_t(r) = \text{Power out}/\Delta H_i \tag{39}$$

except that the static exit pressure was used in lieu of the stagnation exit pressure (a conservative choice). This was done to make the term compatible with previous work by Lawn and Rice[8]. The resulting relationship is:

$$\eta_t(r) = \frac{(k-1)TM^2}{2rU_o(p_{to}^m - p^m)}\ ;\ m = \frac{k-1}{k} \tag{40}$$

where:

$$p_{to} = \frac{\bar{p}_{to}}{p_o} = \left[1 + \frac{(k-1)}{2}(V_o^2 + U_o^2)M^2\right]^{1/m} \tag{41}$$

Stability, Roundoff, and Truncation Error

An indication of the relative stability of the solution to each unknown was monitored using the estimate

$$S_n = \frac{\Delta f_n}{\Delta r_n} \geq -1 \tag{42}$$

If more than 2 of the solutions violated this condition, the step size was halved. Reasonable control over the stability of the solutions was maintained using this criteria.

Since typically 10K to 20K steps were taken in the r-direction; it was necessary to protect against round-off error by using double-precision in all calculations of variables which showed r-variation.

Truncation error was controled using the following criteria:

$$e_L \leq \frac{\epsilon_n}{f_n} \leq e_H \tag{43}$$

where e_L and e_H were chosen prior to starting the solution. Typically, 10^{-4} and 10^{-6} were used; however, in some cases 10^{-5} and 10^{-7} were imposed. When the relative error ϵ_n/f_n was less than e_L, the step size was halved; and, of course when the upper limit was violated, it was doubled. An estimate of ϵ_n was obtained using:

$$\epsilon'_n = (f^p_n - f^c_n) - (f^p_{n-1} - f^c_{n-1}) \tag{44}$$

Recall that the predictor and the corrector were "modified" at each step by adding the truncation error for that step to them (see Hamming[7], p. 206). Thus, equation (44) provided the best estimate available of the truncation error for the composite method. In order that successive halvings and doublings be avoided, the minimum number of steps between them was controled.

Results

No experimental data for compressible flow are available to measure the present solution. However, some verification of the solution was obtained by using it (substantially unaltered) to generate a solution to the incompressible flow problem. Specifically, $M^2 = 10^{-16}$ and $k = 10^{16}$ were imposed. This effectively

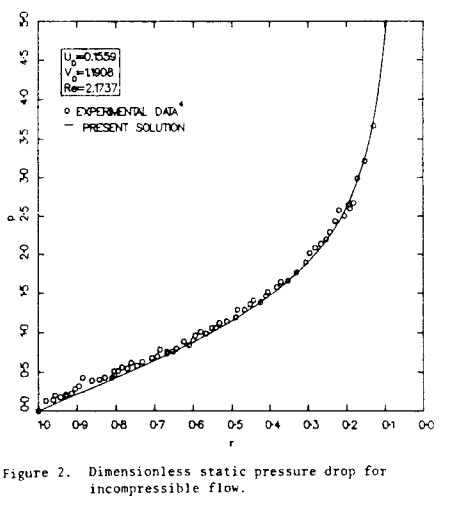

Figure 2. Dimensionless static pressure drop for incompressible flow.

transforms equations (4-7), and thus the working equations, to the incompressible form. The lone consequence of this "transformation" is an implied change in the dimensionless pressure term, p. More conveniently it becomes

$$p = (p_o - \bar{p})/\rho_o r_o^2 \Omega^2 \qquad (45)$$

if the boundary condition

$$p = (1,z) = 0 \qquad (46)$$

is imposed in lieu of boundary condition (8f).

Adams and Rice[1] have obtained data on the incompressible flow of an ethylene glycol-water mixture between a pair of rotating discs. Figure 2 shows data, apparently from the Adams and Rice investigation, but reported elsewhere[4]. Dimensionless pressure drop (equation 45) is plotted against radius ratio, r, for the case of U_o= -0.1559, V_o= 1.1908, and Re= 2.1737. The line shows results from the present analysis using parabolic entry profiles. The generally higher location of the data can be explained by the difference in entrance conditions for the experiment and the analysis. The analysis assumes full periphery admission, whereas admission in the experiment was accomplished using 30 nozzles equally spaced adjacent to the edge of the 16-inch diameter discs. In view of this difference, it is more pertinent to compare slope (pressure gradient) in this plot.

Figures 3, 4, and 5 show the performance characteristics of turbine efficiency, dimensionless torque, and pressure ratio (total inlet to static outlet) as a function of radius ratio for the condition of U_o=0.015, V_o=1.10, Re=6.00, and selected values of Mach number. In this case and all subsequent cases, a value of k=1.4 was used. Note that peak efficiency drops off rapidly for M>0.6. Another trend is that the peak efficiency shifts to a higher value of exit radius as M is increased. The approximate location of the peak efficiency, r=0.3, holds for the range of parameters of greatest interest. The effect of M on torque is not discernable in figure 4; however, the pressure ratio increases rapidly for M>0.6. Note that this is typically a low pressure ratio machine.

Figures 6, 7, and 8 show the performance characteristics for the condition of M=0.500, V_o=1.10, Re= 3.00, and selected values of U_o, the dimensionless inlet radial velocity. Since U_o is a direct measure of flow rate, that effect is demonstrated. This parameter shows the most spectacular effect of any on the performance of the machine. The trend is to sharply reduce peak efficiency as U_o increases. Also as U_o increases, peak efficiency occurs at larger values of exit radius. Torque increases with increasing U_o, thus setting up a tradeoff between machine size and efficiency for a given application. The effect of U_o on

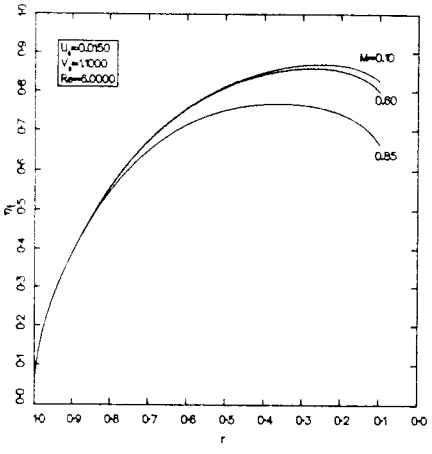

Figure 3. Effect of Mach number on turbine efficiency.

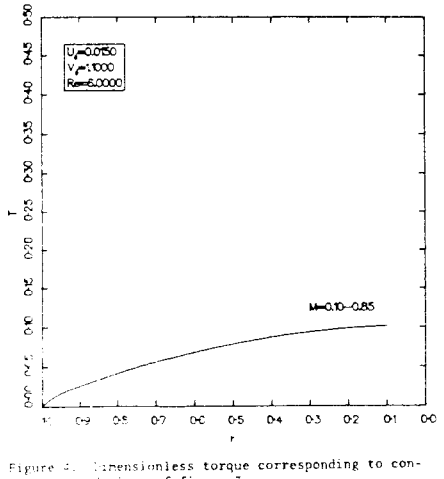

Figure 4. Dimensionless torque corresponding to conditions of figure 3.

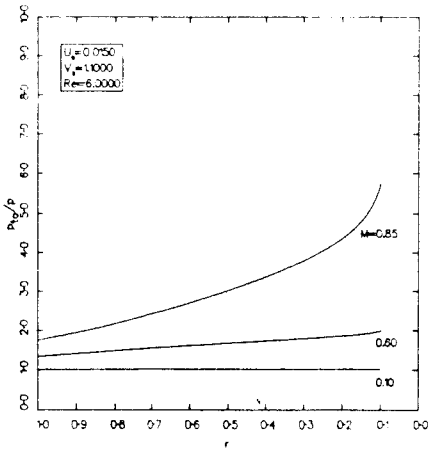

Figure 5. Effect of Mach number on pressure ratio.

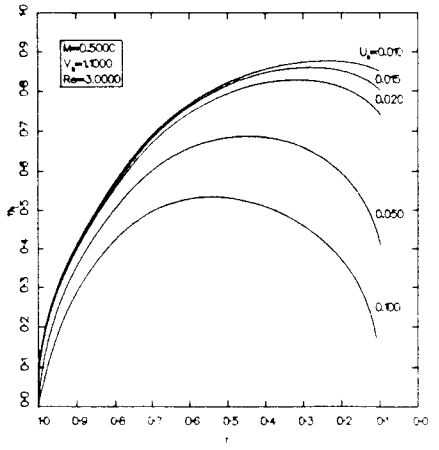

Figure 6. Effect of dimensionless inlet radial velocity on turbine efficiency.

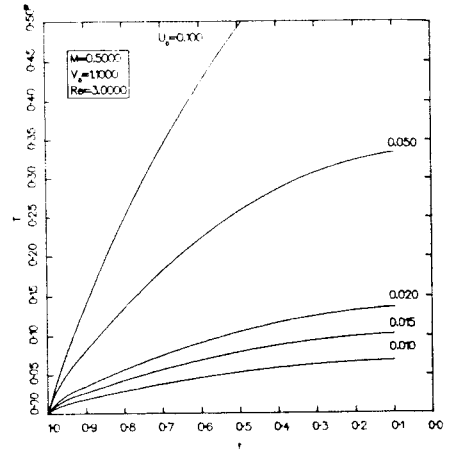

Figure 7. Effect of dimensionless inlet radial velocity on dimensionless torque.

pressure ratio is moderate near the radius of interest (r=0.3).

The effect of dimensionless inlet tangential velocity, V_o, on performance is shown in figures 9, 10, and 11 for the condition U_o=0.015, M=0.600, and Re= 6.00. Note that for V_o=0.9, a region exists near the inlet where the fluid bulk flow is slower than the local disc velocity. Hence, momentum is transfered to the fluid in this region resulting in a slight negative torque (not distinguishable) and efficiency. However, of greater interest is the effect of V_o on peak efficiency. An optimum value of V_o in the vicinity of 1.10 appears to be typical. Note that torque (following some adjustment near entry) and pressure ratio are near linear functions of V_o.

The effect of Reynolds number on performance is shown in figures 12, 13, and 14 for M=0.500, U_o=0.015, and V_o=1.10. The lowest case run, Re=3.0, showed the highest efficiency. Other cases of Re=4.0 and 6.0 (not shown) fell in between the plotted curves. Attempts to use values of Re<3.0 resulted in excessive

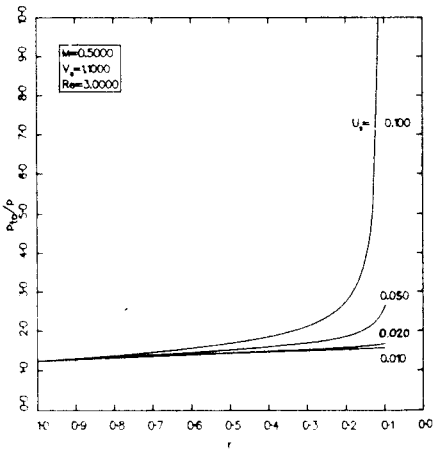

Figure 8. Effect of dimensionless inlet radial velocity on pressure ratio.

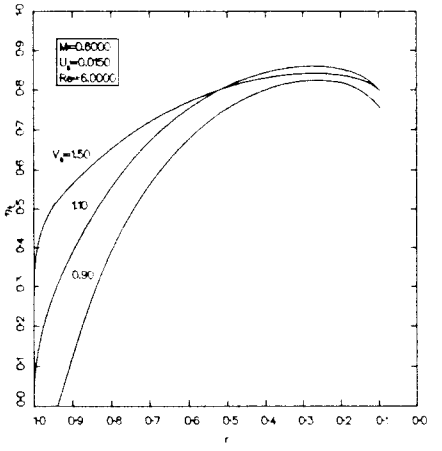

Figure 9. Effect of dimensionless inlet tangential velocity on turbine efficiency.

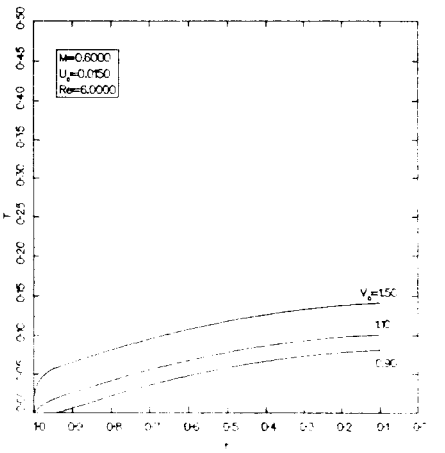

Figure 10. Effect of dimensionless inlet tangential velocity on dimensionless torque.

run time and were terminated prior to completion. Variation of Re had very little effect on torque and pressure ratio, certainly not enough to allow discrimination on a plot. The trend was toward lower torque and higher pressure ratio as Re was increased.

Velocity profiles have not been displayed; however, they are similar to those shown by Boyack and Rice[4] and Boyd and Rice[5] for incompressible flow. Although for the incompressible flow cases, the present solution successfully executed inflected profiles, no inflected profiles were obtained for the compressible flow conditions run. In cases where inflected profiles might be expected, the solution terminated early (r>0.1) due to a sign change in the pressure term. After exhaustive review of the solution and the program for correctness, it was concluded that a solution did not exist beyond these points for that set of inlet parameters. Early termination occured in runs which were of little interest in the actual design of a turbine

Spot comparisons with the performance maps of Lawn and Rice[6] using the incompressible flow solution provided fair agreement, typically within 5%. With the present solution, efficiency and torque were typi-

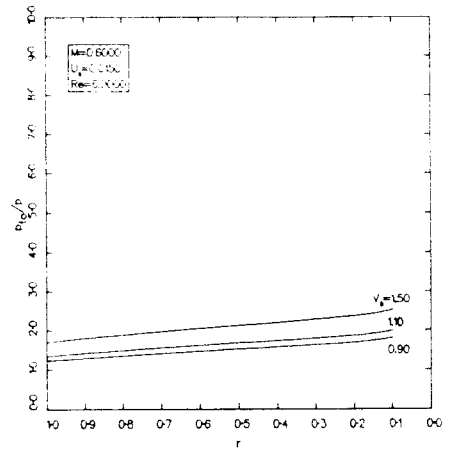

Figure 11. Effect of dimensionless inlet tangential velocity on pressure ratio.

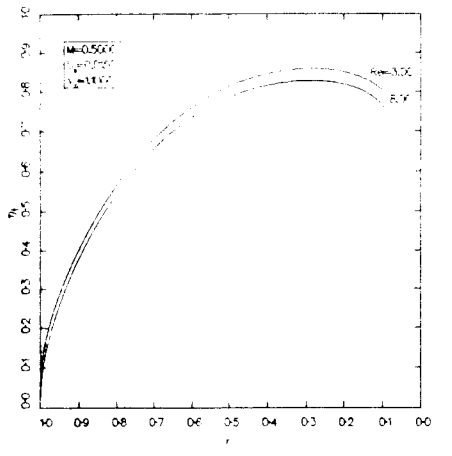

Figure 12. Effect of Reynolds number on turbine efficiency.

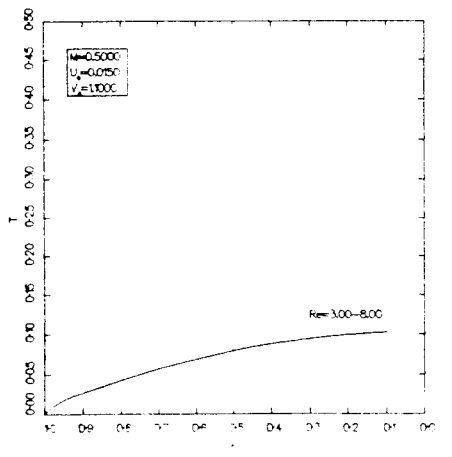

Figure 13. Dimensionless torque corresponding to conditions of figure 12.

calls higher and total pressure drop lower, given the constraint of interpolating the maps. The difference might well lie in the way "uniform" inlet profiles were defined at the periphery. Indeed, after the solution for the case shown in figure 2 was run, a uniform entrance solution was made. Differences on the same order as cited above were noted, the parabolic profile case providing higher efficiencies and torques.

Discussion

Losses associated with inlet nozzle configuration, inlet boundary scrubbing, and windage have not been addressed by this investigation. It is felt that sufficient criteria to predict these losses and optimize a design exist in the literature. These problems could represent a formidable design obstacle. At first load, scrubbing losses might be as large as 15% for high inlet velocities.

The problem of predicting laminar flow operation is not at all settled. Although the data of Bakr, Crowther, and Rice[15] are very helpful, no data are available on flow regime definition for the inward flow of gases between co-rotating discs. This problem (transition) will probably define the upper limits on Re, other factors being a weak function of this parameter.

Supersonic operation of the turbine was not considered, since a shock must occur somewhere prior to exit. Not only are large losses involved; but consideration must be given to possible structural problems induced by the shock. It is interesting to conjecture

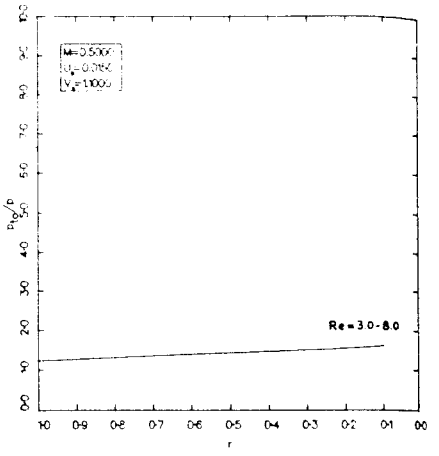

Figure 14. Pressure ratio corresponding to conditions of figure 12.

what shape the shock might take. If supersonic flow could be maintained into the space between the discs, the shock must evidently assume a circular shape at a critical radial position.

From a cursory inspection of the results of this investigation it appears that typically a compressible flow disc turbine would be a high speed, multistage machine with hundreds of closely spaced (2 or 3 mils) thin discs. An alternative to staging for this low pressure ratio machine is regeneration. A closer view of the details needs to be made for each particular application trading off machine size for efficiency.

Conclusions

A solution to the laminar, compressible flow between rotating discs has been accomplished. Results are applicable to disc turbine design. Results show that:

1. Maximum efficiency is obtained as the flow rate (U_o) between discs is minimized. However, since torque is adversely affected by minimizing U_o, a compromise between efficiency and turbine size will be necessary.

2. Optimum efficiency is obtained for $M_o \leq .6$; efficiency rising slightly as M is reduced further. There is no effect on torque and a moderate effect on pressure ratio for $M_o \leq .6$.

3. Depending somewhat on the other parameters, optimum efficiency is obtained for $V_o \approx 1.1$. Both torque and pressure ratio are virtually linear functions of V_o.

4. Reynolds number has only a moderate effect on efficiency, at least in the range 3.0-8.0. Although it is maximum at Re=3.0, the criteria for selection of operating Re might be better based on transition data. Reynolds number has no practical effect on torque or pressure ratio.

5. The shape of entrance profiles can alter performance parameters as much as 5%.

6. For the range of parameters of most interest, optimal radius ratios for optimum performance are $.25 \leq r \leq 0.30$.

Nomenclature

A_i Coefficients in the radial velocity polynomial, $u(r,z)$

B_i Coefficients in the tangential velocity polynomial, $v(r,z)$

e_H Upper limit imposed on the relative truncation error, ϵ_n/f_n

e_L Lower limit imposed on the relative truncation error, ϵ_n/f_n

f_n Function, f(r), evaluated at $r=r_n$.

f_n^c Corrector value of f_n

f_n^p Predictor value of f_n

$\bar{f}$ Body force vector

h Disc spacing

ΔH_i Difference in time rate of change of isentropic stagnation enthalpy across turbine

k Ratio of specific heats, c_p/c_v

m Ratio, $(k-1)/k$

M Mach number, $r_o\Omega/\sqrt{kp_o/\rho_o}$

p Dimensionless pressure, $\bar{p}/p_o$

p_o Inlet pressure

p_{to} Dimensionless total pressure at inlet, $\bar{p}_{to}/P_o$

r Dimensionless radial coordinate, $\bar{r}/r_o$

r_n Radial coordinate evaluated at n^{th} step

r_o Disc outside radius

Re Reynolds number, $\Omega h^2/\upsilon$

S_n Partial derivative, $\partial f/\partial r\ (r_n)$

t Time

T Dimensionless torque for disc pair, $\bar{T}/\rho_o r_o^5 \Omega^2 h^3$

u Dimensionless radial velocity, $\bar{u}/\Omega r_o$

u_o Radial velocity profile at inlet, $u_o=u_o(z)=u(1,z)$

U_o Inlet radial velocity parameter, $2\int_0^{1/2} u_o(z)dz$

v Dimensionless tangential velocity, $\bar{v}/r_o$

v_o Tangential velocity profile at inlet, $v_o=v_o(z)=v(1,z)$

$\bar{V}$ Velocity vector

V_o Inlet tangential velocity parameter, $2\int_0^{1/2} v_o(z)dz$

w Dimensionless axial velocity, $\bar{w}/\Omega h$

z Dimensionless axial coordinate, $\bar{z}/h$

ϵ_n Truncation error at n^{th} step

η_t Turbine efficiency

μ Dynamic viscosity at inlet

υ Kinematic viscosity, μ/ρ_o

ρ Local dimensionless fluid density, $\bar{\rho}/\rho_o$

ρ_o Density at inlet

Ω Disc angular velocity

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Power, December 1975, p. 51-3

New efforts aim at practical application of multiple-disk-pump concept in industry

A Michigan company, experimenting with the 60-year old Tesla idea, now offers several pumps. Trials with compressors, blowers, and even turbines also show promise for the bladeless impeller

By William O'Keefe, Associate Editor

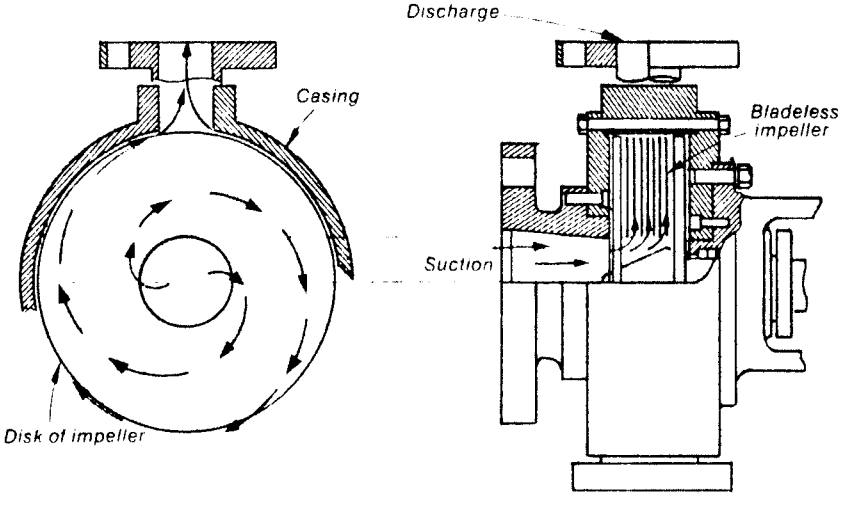
The centrifugal pump as we know it today moves fluid through its impeller by the rotating action of curved blades. The effect of the blades on the fluid is easy to understand. However, the possibility of a smooth, flat impeller drawing fluid in near its center and expelling the fluid at high speed from the periphery is not so easy to visualize. Nevertheless, the idea is not new; it was patented by Nikola Tesla many years ago.

At that time, the multiple-disk or bladeless-wheel machine seemed to offer little, compared with the conventional centrifugal pump or other available types, which were already at a satisfactory level of performance for the period's applications. But about 20 years ago, because the multiple-disk concept had certain potential advantages in aerospace work, analysis and development began in earnest.

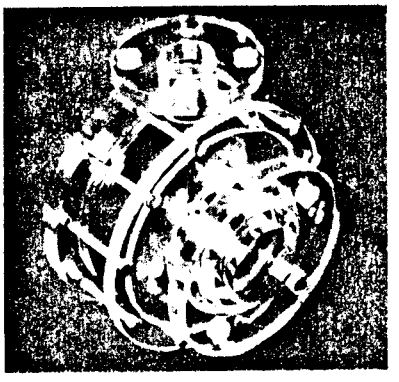
Several working models showed the feasibility of the idea, even though pump efficiency was low. By then, the work had a firm enough R&D basis so that even the demise of the aerospace application for the pump several years ago did not destroy practical interest in the idea.

Today, a small firm in Vernon, Mich, Molecular Kinetics Inc, is carrying the effort forward, and has reached the point at which pumps are being offered commercially. The efforts of the firm center on practical research and also on conducting experiments on blowers and turbines that utilize the bladeless-wheel concept.

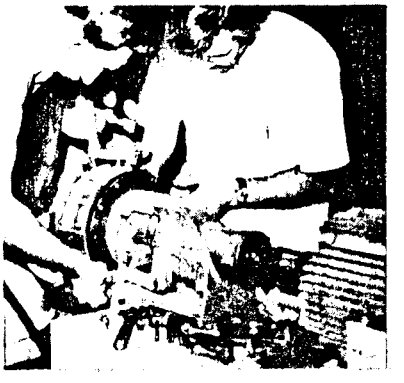
Impeller disks drag fluid through pump in spiral outward path



Bladeless wheel is heart of Molecular Kinetics pump, but fluid entry and exit have received close attention, too. Casing outlet is on radius, as in AVS pumps



Symmetry and simplicity of pump components are apparent in this model

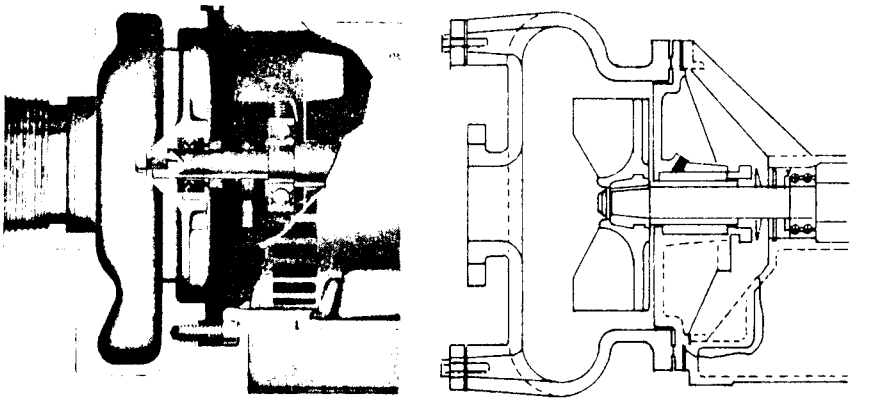


Conventional frame and drive adapt easily to pump's casing and rotor

In conventional turbomachinery, such as pumps, compressors, and turbines, either blades or shaped passages produce momentum exchange between fluid and rotor. The resulting force comes from pressure distribution over the rotor. In the multiple-disk rotor as it is used in a pump, however, all force transmission between fluid and rotor results from shear stress exerted on the fluid by the rotor.

The rotor consists of several thin, closely spaced disks, bolted together. Each disk has a center hole, through which liquid or gas passes on its way to the narrow spaces between disks, as figure at left shows. The boundary layer of fluid on each disk face revolves with the disk, and accelerates incoming elements of liquid in a spiral path until the tip velocity of the impeller is reached. The liquid leaves the open periphery of the rotor, travels through a narrow annular space between casing and rotor, and finally exits at a discharge connection mounted radially on the casing.

As in conventional bladed-impeller pumps, the peripheral speed of the disk pack determines the head developed. Capacity in a multiple-disk pump depends on the number of disks and the total surface area. Stacking up of disks is the logical way to build capacity. Not only liquids but also air and other gases can be "pumped" by the machine, although the effects of compression complicate design. Finally, the bladeless wheel can be rotated by steam from peripheral nozzles to deliver power, thus becoming a turbine.



Vanes in some modern centrifugals are deliberately moved back from volute passage to handle solids and slurries better. Impeller effect transmits through the fluid

Centrifugal blower is another possible application for multiple-disk principle

What is the reason for renewed interest in this idea? On the face of it, the best that could result would be a younger brother of the conventional centrifugal pump, blower, compressor, or bladed turbine. This is not the situation in practice, however. Multiple-disk machines have several advantages that can win a place for them. For pumps, let's look at where these machines stand in comparison with bladed-impeller centrifugals.

First, the multiple-disk pump can move liquids with a wide range of viscosities. In fact, wherever fluids have viscosity or density greatly different from water's values, the multiple-disk pump will be worth scrutiny. Slurries are also a good possibility. And abrasives could be handled with less chance of damage to the rotor, because the abrasive particles do not impinge directly on blades, as in regular centrifugals. Pumping of fragile material particles in a liquid could be another end use for the pump.

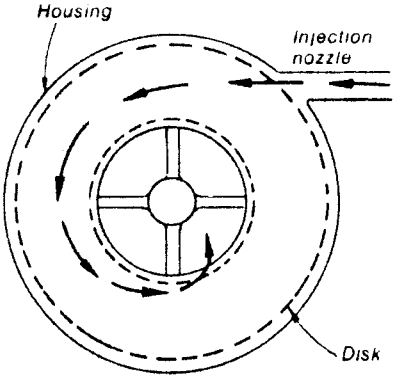
Lack of blades in the direct path of the fluid is not unique to the multiple-disk pump, of course. Some centrifugal pumps, like the two shown above, rely on recessed but still bladed impellers to develop a vortex in the pump casing. Such recessed-impeller pumps can handle slurries, abrasives, and fragile materials, although efficiency is low. (One extra advantage of the recessed-

Bladeless impellers function as a turbine for wet steam

If it is provided with steam-admission nozzles around the casing, a properly designed multiple-disk impeller can act as a turbine. Steam (or liquid or gas) spiraling inward to the hole at disk center exerts a shear force on the disks and rotates them.

One big advantage of such a turbine is its ability to take very wet steam without harm. (Wet steam can quickly injure a conventional bladed turbine.) This characteristic has made the machine interesting to process plants that have had drive-turbine troubles involving poor-quality steam.

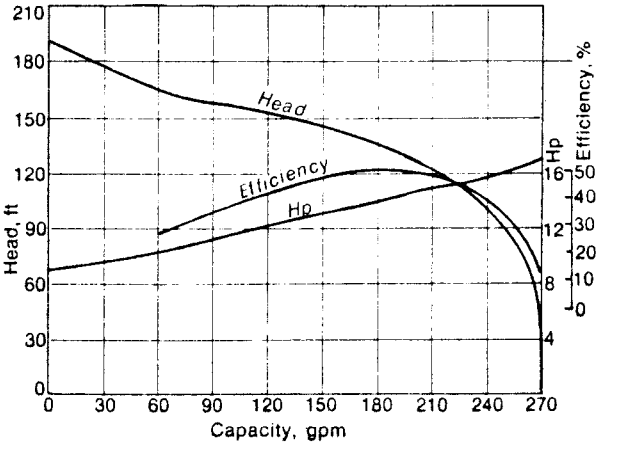
Disk spacing in multiple-disk turbines is often close, on the order of a few thousandths of an inch. Stainless steel is a good corrosion-resistant material for disks. Theoretical efficiencies are as high as 80%, but actual overall efficiencies of experimental machines are lower than for conventional turbines, because of losses of available energy in nozzles and exhaust, admission blockage, and shear forces on rotor outer surfaces.



Competitive performance is emerging for new production versions

Although the multiple-disk pump might find minor uses merely because of its unique characteristics, wide acceptance will demand good performance. This means a reasonable range of heads and capacities, and reasonable efficiencies. This is part of the development under way now. Graph at right is a head-capacity chart for one of the Molecular Kinetics pumps on test. With a 3600-rpm motor drive, the pump delivers around 200 gpm at about 64-ft head. Efficiencies are not so high as those of the best centrifugals, but are near those of many specialty pumps.

The pump operates smoothly and quietly down to shutoff, with no vibration. Noise is hard to estimate, because the cooling fan on the drive motor makes much more noise than the pump. At the present time, top capacity is estimated to be around 640 gpm, assuming a 3600-rpm drive, which is quite common in conventional centrifugal-pump work. This size range should allow the new pump to get consideration in most process work.



impeller pump is its ability to handle large solids, of course.)

High suction specific speed is a plus for the multiple-disk pump. This property means low cavitation sensitivity, which can be important with liquids of high vapor pressure, or where suction heads are low.

Because the closely spaced disks in a multiple-disk rotor can exert a drag on any liquid or gas between them, the pump can draw in air from its suction connection, and this is useful in self-priming. Molecular Kinetics says it can achieve vacuum up to 25 in. Hg, and the effect is enough to self-prime the pump even if the casing is less than half full.

Small centrifugal pumps have always been at an efficiency disadvantage compared with larger units, and this gives the multiple-disk pump another opportunity. Low-volume applications are now being worked on.

Production problems should be fewer with multiple-disk pumps, if present development is a good clue to the future. A simple cylindrical casing and a series of flat disks and spacers on a shaft allow the manufacturer to eliminate patterns and castings. This is an attractive saving, especially if designers have to match a pump very closely to its service.

In the multiple-disk pump, addition of disks raises capacity, and increases in disk OD or rotor speed build head.

Disadvantages of the multiple-disk pump still center on its low efficiency. So improvement in efficiency is the big task that is taken on by practical developers. Progress to date is apparent when you compare the graph at center of the previous page with the 12-21% efficiencies of early experimental pumps. Nevertheless, much practical work remains. Remember that theoretical efficiencies of a single disk pair do not take into account such influences as interference of disk edge with flow at entrance and exit of passageway, losses in the entrance plenum and in the casing, and disk/casing-side friction.

It is quite possible that improvements in efficiency will complicate design and manufacture and increase costs. For example, devices introduced around the pump eye could help smooth the sharp right-angle turn into the disk spaces, but pump cost would go up. For the casing, a different outlet nozzle or a special shape could look attractive in theory, and even in experiments, but added manufacturing costs or the need for castings could hurt, along with loss in design flexibility. Locating the rotor off-center in an enlarged circular casing is another possibility, but this would have to be closely examined to determine its effect on efficiency.

A compressor or blower operating on the bladeless-impeller concept is even more foreign to ordinary engineering experience than is the pump. Yet a stack of closely spaced spinning disks can produce a flow of air by shear action. So far, the blower looks easier to develop than a compressor. As with pumps, efficiency is retained in very small sizes, giving a potential advantage where air volumes are small and good efficiency is sought.

Turbines of multiple-disk design are feasible, too, and some authorities think this may be the most promising use of the principle.

The geothermal area is one illustration of how the multiple-disk principle can serve a demanding application. Wet steam and cavitation are expected to be problems in geothermal work, but the multiple-disk machines should be equal to this. The turbine can handle wet steam, and the pump resists cavitation. Other applications are actively being sought now, and the future looks bright at last for an idea that originated around the turn of the century. **WOK**

The theory of the multiple-disk pump has developed in past 20 years

When Nikola Tesla obtained the original US patents for the multiple-disk pump in 1913, the engineering and scientific community took little interest in the idea. It was not until the 1950s that such investigators as Warren Rice, at Arizona State University, began to work on the theory and development of the machine. Since then, several pumps, turbines, and blowers or compressors have been built to test analytically derived ideas.

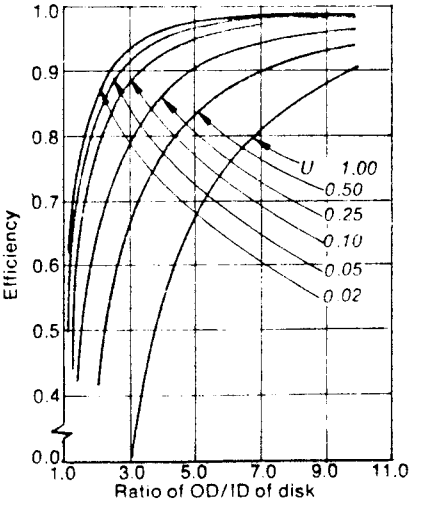
Most of the theory centers on what happens in a single passage between two disks. Assumption is that flow in all disk spaces is similar, a condition believed to be reasonably attained in real machines. The fluid is assumed to be a two-dimensional flow, axially symmetric, in the plane of the smooth disks, which rotate at constant angular velocity, Ω. Theoretical analyses often assume that a reversible diffuser collects fluid from the rotor and decelerates it to a stagnation condition. Viscous forces can be taken into account by expressing the shear stress at surfaces of contact between fluid and disks in terms of a constant factor, f.

Solution of the differential equations of motion for fluid flow between the disks requires a computer. For effective conversion of theory to practice, parameters are advisable. Reynolds number, N_{Re}, is one of these. For the multiple-disk pump, it is $\rho\Omega h^2/\mu$, where ρ is the fluid density, Ω the disk angular velocity, h the spacing between disks, and μ the fluid viscosity. Another parameter, which Rice calls a volume flow-rate parameter, is U_n. This is $Q/(2\pi r_i^2\Omega\ h)$, where Q is the volume flow rate, and r_i the disk's center-hole radius.

High efficiency will be important in making the multiple-disk pump a success, and analysts must sort out the various parameters and variables to show what engineers and designers need to take into account. Graphic plots of efficiency against Reynolds number, or ratio of disk outer radius to inner radius, are a convenient aid.

Graph at right is an example. Here the factors are arranged to show how efficiency varies with the design-important factor of ratio of disk-hole diameter to disk diameter. Several curves show the variation for a range of U_n values. In this graph, N_{Re} is assumed to be a constant 5.0. Maximum efficiency of multiple-disk pumps occurs at low Reynolds numbers. One researcher concluded that an N_{Re} of π^2 gives the highest efficiency, but others believe that an even lower value yields the maximum. The lower value is based on experimental results, and any added progress now will have to come from enlightened development work.

Except for the rotor, the principal design problem will center on getting effective diffusion of flow leaving the rotor. Roughly half of the rotor-imparted energy is kinetic energy as the fluid enters the diffuser. In addition, flow angle of fluid entering the diffuser is low, because the radial component of velocity is low compared with the tangential component, so pressure recovery and diffuser efficiency tend to be low.



THE TESLA TURBINE

Kathleen Joyce — Walter P. Baumgartner

This article will acquaint the reader with one of Nikola Tesla's very important contributions in the field of applied mechanics, the rotary machine which he invented around the turn of the century.

Nikola Tesla's most ingenious invention of polyphase transmission of electrical power and his great accomplishment in the transmission of electrical energy without wires using high frequency electric oscillations were fundamental in the development of radio, T.V. and radar.

It is most unfortunate that Tesla's ingenious turbine design, which exploited the well-known phenomenas of viscosity and adhesion, was never really recognized in engineering circles. Our jet engines would be much more powerful with a lot less fuel consumtion.

In 1912 Tesla developed a small turbine which he referred to as: "a power house in a hat."

"What I have done is to discard entirely the idea that there must be a solid wall in front of the steam and to apply in a practical way, for the first time, two properties which every physicist knows to be common to all fluids but which have not been utilized. These are adhesion and viscosity.

"To say nothing of it being a new application of mechanical principles, it has many decided advantages. First of these is its simplicity. It is comparatively inexpensive to construct, because nothing but the bearings need be accurately fitted, and exact clearances are not essential. Then there is nothing to get out of order and the disks can easily be replaced by any competent mechanic. It can be reversed with-

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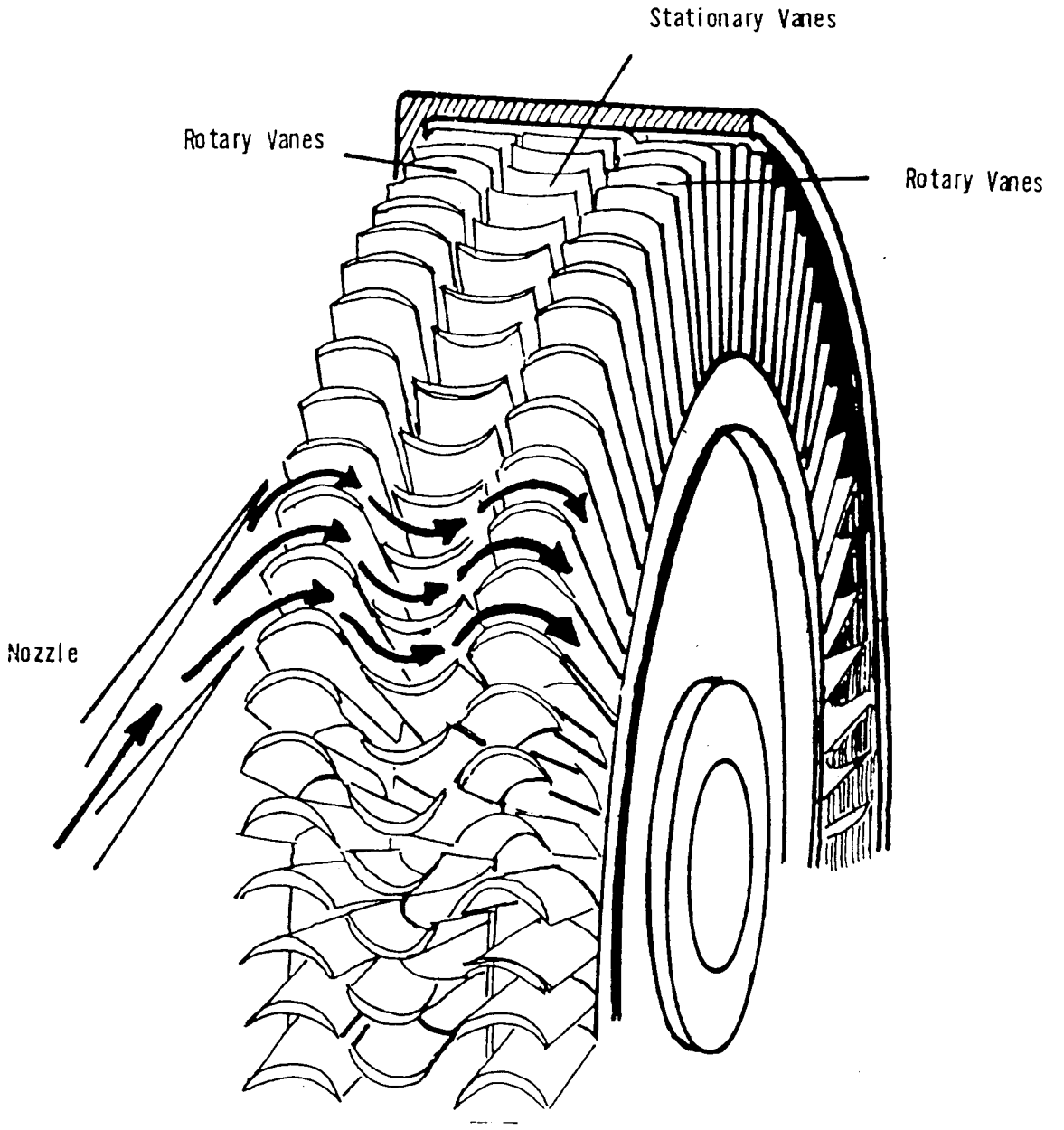


Principle of the conventional turbine with its velocity staging.

out complex or cumbersome apparatus-all that is needed is a two-way valve to let the steam in at one side or the other, as desired. Reversing an ordinary turbine is next to impossible.

Even though Tesla's name was hardly ever mentioned in our educational institutions, Tesla occupies one of the outstanding places in the history of modern science and techniques. For his scientific achievements, Nikola Tesla received many outstanding recognitions from distinguished scientific institutions throughout the world, and was presented with many doctor's degrees.

INTRODUCTION

A working one-stage model of a revolutionary new turbine (based on Nikola Tesla's design) can be built which will provide at least 30% efficiency in energy conversion as compared to the 20-25% efficiency of present-day turbines. (See Figure 1 - Principle of Conventional Turbine Operation and Figure 2 - Principle of New Turbine Design). A two-stage model would provide an efficiency of at least 50% in energy conversion.

DESCRIPTION OF NEW TURBINE DESIGN

The turbine consists of a set of flat metal disks mounted on a shaft and rotating within a casing, with a fluid or gas entering with high velocity at the periphery of the disks, flowing between them in free spiral paths, and finally escaping through exhaust ports at their center.

The new turbine has three major parts (see Figure 4 - Sketch of Single-Stage New Rotary Turbine):

1) the rotor, a set of closely spaced metal disks on a common shaft,

2) the casing containing the rotor

3) a mechanism to introduce a moving fluid or gas under pressure into the rotor. This can be either a compression chamber (see Figure 3 - Design of Combustion Chamber of New Turbine) or an input valve.

Kinetic energy (and optionally thermal energy) from the moving fluid or gas (energy medium) is converted into mechanical energy for use by devices attached to the drive shaft. The fluid or gas is injected at the periphery of the rotor and spirals inward between the disks toward the center where it exits. The vortex of energy set up by the energy medium drags the rotor along with it, thus turning the drive shaft and producing mechanical energy. (See Figure 5 - Spiral Vortexual Movement of Energy Medium between Disks of New Turbine).

For more efficient energy conversion, the exhaust from the first stage can be utilized to power a second stage of the turbine. The second stage would be similar to the first stage but the rotor disks would be larger in diameter and there would be more of them.

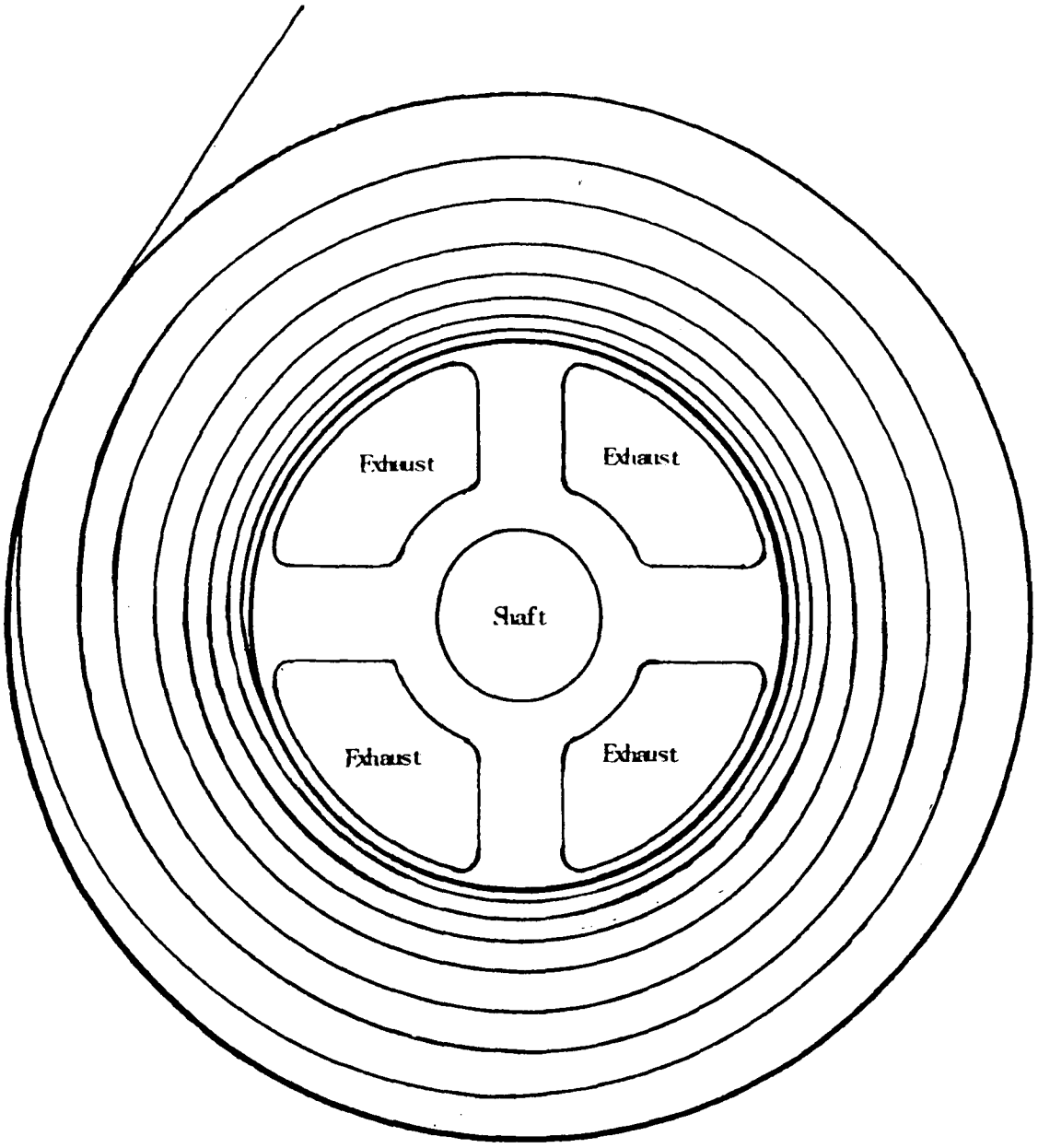
OPERATING PRINCIPLES OF NEW TURBINE DESIGN

The new turbine design is based on the vortex principle and the use of the adhesion and viscosity properties of fluids and gases.

THE VORTEX PRINCIPLE

Fluids and gases in motion are used as an energy carrier in the conversion and practical application of mechanical power. A body placed in a moving fluid is forced in the direction of fluid movement. Normally a fluid in mo-

NO TURBINE CAN BE EFFICIENT UNLESS DESIGN IS BASED ON THE SPIRAL MOVEMENT OF NATURE



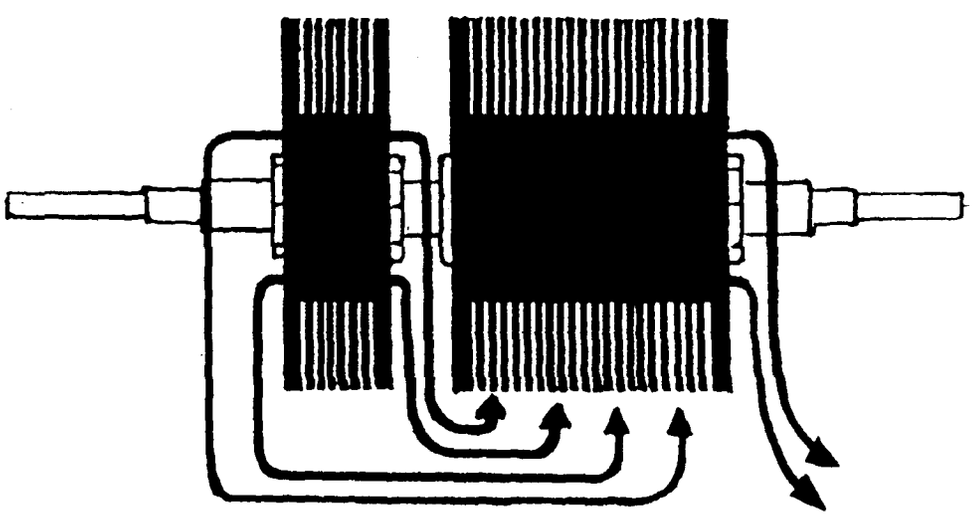
This side view of a single disk shows the energy medium travelling along its spiral vortexual path moving towards the exhaust ports at the center. As illustrated the medium approaches the periphery at a tangent which allows it to create its own spiral path. Energy conversion on to the shaft is therefore smooth and simple compared with the conventional blade design in turbines.

tion, introduced under pressure, will gradually lose its velocity. This is partially compensated for by utilizing the vortex principle and introducing the fluid at the large open end of a vortex. As the fluid moves in a tighter and tighter spiral, its speed increases. This is illustrated in the common example of water exiting out the drain of a sink.

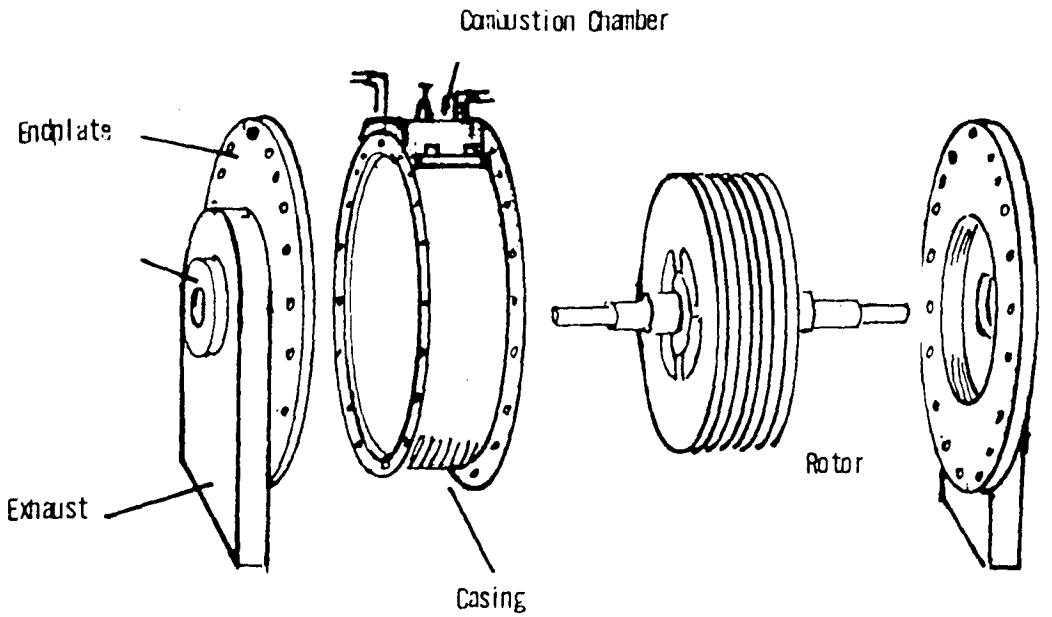
In the turbine design, the motion of the spirally exiting fluid causes the disks submerged in the fluid to revolve on their common axis, converting the kinetic energy of fluid movement to mechanical movement of a rotating shaft. If steam or gas is used, the resistance to the passage of the energy medium between adjoining plates is approximately proportional to the square of the relative speed, which is at a maximum toward the center of the disks and is equal to the tangential velocity of the energy medium. Thus the resistance to radial escape is very great, being furthermore enhanced by the centrifugal force acting outwardly.

Change in the velocity and direction in the fluid or gas should be as gradual as possible. In apparatus currently utilized, more or less sudden changes, shocks and vibrations are unavoidable. Devices used to extract energy from fluid motion (pistons, paddles, vanes and blades) introduce numerous defects and limitations and add to the complications, cost of production and maintenance of energy generating machines.

With existing rotational machines, the working fluid or gas is forced to follow both moving and stationary channels. This movement is influenced by the hydraulic resistances in the different channel cross sections which produce eddy currents through variation of direction and speed



Rotor showing two stage disk assembly for compound effect producing up to 65% efficiency in the conversion of heat into mechanical energy.



within the channels. These resistances diminish considerably the efficiency of present-day machines.

By allowing the propelling fluid to move in natural paths or stream lines of least resistance, unimpeded by vanes or kindred devices, we can eliminate those resistances and find a much better energy conversion based on the natural vortexual flow which occurs between the disks in a spiral movement. The fluid thus changes its velocity and direction of movement by imperceptible degrees and avoids losses due to sudden variations.

This invention has a much more efficient energy conversion ratio since it utilizes the vortex principle.

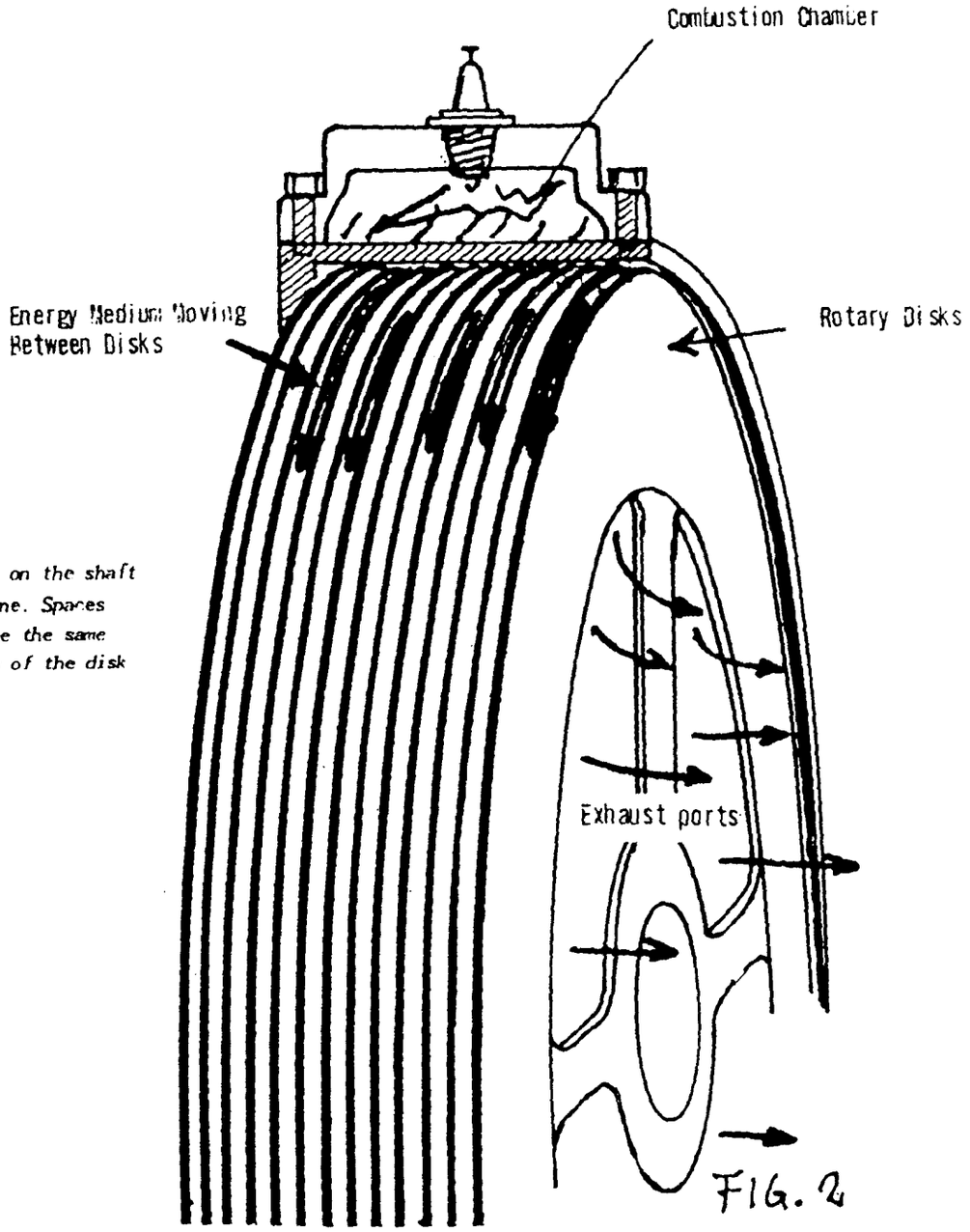
ADHESION AND VICOSITY PROPERTIES OF FLUIDS AND GASES

A new principle, which has not been exploited so far in turbine design, is the active use of the cohesion and viscosity properties of fluids and gases. As the inner and outer friction of the fluid or gas is dependent on cohesion and viscosity, so also the resistance of the moving body or rotor is dependent on this same trait. Machines of this design could be called frictional machines.

Metals and gases have a peculiar affinity for each other which causes the molecules of gases to cling to metals. Consider a set of parallel metal disks mounted on a common axis and immersed in a spirally revolving gas. The disks will turn on their axis due to the movement of the gas. The disk movement is enhanced by the adhesion of the moving molecules of gas to the metal surface. This adhesion principle is utilized by placing the metal disks very close together. The dimensions of the device as a whole, and the spacing of the disks in any given machine will be determined by the conditions and requirements of special cases. The intervening distance between disks should be greater, the larger the diameter of the disks, the longer the spiral path of the fluid and the greater its viscosity. The spacing should be such that the entire mass of the fluid, before exiting, is accelerated to a nearly uniform velocity, not much below that of the peri-

phery of the disks under normal working conditions, and almost equal to it when the outlet is closed and the particles move in concentric circles.

When the fluid under pressure is allowed to pass through the input valve, the disks will be set in rotation in a clockwise direction, the fluid traveling in a spiral path and with continuously diminishing velocity until it reaches the orifices through which it is discharged. If the disks are allowed to turn freely, on nearly frictionless bearings, the disk rims will attain a speed closely approximating the maximum of that of the adjacent fluid and the spiral path of the particles will be comparatively long, consisting of many almost circular



Disk arrangement on the shaft of a Tesla turbine. Spaces between disks are the same as the thickness of the disk material.

FIG. 2

turns. If load is put on and the disk thus slowed down, the motion of the fluid is retarded, the turns are reduced and the path is shortened.

In general, the torque will be directly proportional to the square of the velocity of the fluid relative to the disks and to their effective area, and, inversely, to the distance separating them. The machine will, generally, perform its maximum work when the effective speed of the disks is one-half of that of the medium; but to attain the highest economy, the relative speed or slip, for any given performance, should be as small as possible. This condition may be to any desired degree approximated by increasing the active area of and reducing the space between the disks.

One of the interesting features of this turbine is that it is self-regulating. The periphery velocity of the disks cannot exceed the velocity of the entering medium.

PREVIOUS WORKING EXPERIMENTAL MODELS OF THIS TURBINE.

Nikola Tesla built 7 different working models of this turbine design. The vital statistics on these models are given in TABLE 1, VITAL STATISTICS OF WORKING MODELS OF TESLA TURBINE.

In 1972, Mr. Walter Baumgartner built an experimental model driven by compressed air. This model turned 18,000 RPM. The turbine tested was coupled by gear drive to an AC generator. The rotor consisted of a stack of very thin disks six inches in diameter made of high grade steel. The disks were one thirty-second of an inch thick and were separated by spacers of the same metal and thickness but of much smaller diameter which were cut in a form of a cross with a circular center section. The extended arms served as ribs to brace the disks. There were eight disks and the edgewise face of the stack was approximately one-half inch across. They were mounted on the center of a shaft about six inches long. The shaft was nearly an inch in diameter in the mid-section and was tapered in steps to less than half an inch to the ends. The rotor was set in a casing made in four parts bolted together.

The generator was loaded to test horsepower on the turbine. On test bench, the 6" model gave 15 horsepower with cold air (non-expanding gas) and with just one input nozzle. The compressed air entered at 180 lb./sq. in. pressure. A six-inch model, one-stage, one nozzle, steam injection, would give 30 horsepower.

In the case of expanding gases, several designs of combustion chambers are possible depending upon the turbine application. To attain a long operating life for the disks, the combustion chamber is designed to have the actual combustion occurring in an auxiliary chamber with water injection. This will decrease the high temperature but increase the expansion factor since water will split partially into hydrogen and oxygen. An increase in the number of combustion chambers would also contribute to a greater gain of power. Axial aircompressers could be introduced which again would bring up horsepower and increase efficiency.

In another model, a jet of steam rushing at high velocity between very closely spaced disks was slowed down by the friction, but the disks, being capable of rotation, moved with increasing velocity until their speed was almost equal to that of the steam. Utilization of the attraction between gasses and metal surfaces enabled the moving steam to grip the metal of the discs more ef-

TABLE 1 VITAL STATISTICS OF WORKING MODELS OF TESLA TURBINES

MODEL ID.	1906	1906	1910	1911	WATER-SIDE	ALLIS	ALLIS CHALMER
H.P. OUTPUT	30		100	110	200	200	675
EFFICIENCY							38%
R.P.M.	20,000	35,000	10,000	9,000	9,000	10,000-12,000	3,600
NUMBER OF DISKS	12	8				20	15
DISK DIAMETER	5"	6"	12"	9.75"	18"	18"	60"
DISK THICKNESS		1/32"					1/8"
DISK STACK THICKNESS		1/2"		2"			
DISK MATERIAL		GERMAN SILVER					
MEDIUM USED	COMPRESSED AIR				SATURATED STEAM	STEAM	
SUPPLY LINE PRESSURE				125 LB./SQ.IN.	125 LB./SQ. IN.	80 LB.	80 LB.
CONSUMPTION					38 LB/HP/HR		
INPUT TEMPERATURE							100°F SUPER-HEATED
HEAT DROP					130 B.T.U		
TURBINE WEIGHT	10 LB.				400 Lb.		
ROTOR WEIGHT	1½ LB.						
ROTOR HP/WEIGHT RATIO	20 $\frac{HP}{LB}$						

fectively and to drag them around at high velocities. This almost frictionless machine transmitted almost the entire expansive energy to the shaft.

A model with disks enlarged to diameter to 10 inches and stacked to 2 inches with only one port or combustion chamber can be expected to produce a minimum of 200 horsepower. Tesla claimed, "Under proper conditions and arrangements, the performance might increase to 1000 horsepower. There is almost no limit to the mechanical performance (transformation or conversion) of such a machine design. This engine will work with gas, as in the usual type of explosion engine used in automobiles and aircraft, even better than it did with steam."

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WATCH FOR THE ARTICLE ON: "WIRELESS TRANSMISSION OF ELECTRICAL ENERGY" - HOW IT CAN BE ACCOMPLISHED AND THE IMPACT IT WOULD HAVE ON OUR SOCIETY - STARTING WITH NEXT ISSUES.

by **WALTER P. BAUMGARTNER**

POSSELL BLADELESS PUMPS

by Kathleen Joyce

TESLA'S BOUNDARY LAYER CONCEPT

The boundary layer concept for the transfer of energy in pumps and turbines was first suggested by Nikola Tesla, the father of alternating current technology, in the early 1900's. Tesla utilized closely spaced parallel rotating disks in his turbine. His boundary layer technology was based on the following principles:

1) The adhesive and viscous properties of any fluid cause it to be attracted to the surface of the rotating disks thus forming a boundary layer.

2) The resistance of the fluid particles to molecular separation causes the fluid to be "dragged along" with the turning disks.

Since Tesla was primarily an electrical genius whose efforts were largely concentrated in this field and since metallurgy was far from todays state of development, his efforts in the boundary layer transfer field languished and became public domain.

ANALYTICAL STUDIES OF BOUNDARY LAYER

From time to time since then, Tesla's efforts have been reviewed by many academic institutions and additional analytical work has been done, highlighting the advantages and potential of this unique method of energy transfer. **These analytical works were of excellent quality and indicated that this technology was capable of producing results of exceptional magnitude.**

However, when academic institution efforts were made to reduce this work to practical hardware, the results were far less than the analytical projections. This was frequently due to lack of practical experience in constructing turbines. This resulted in the mistaken opinion that the boundary layer (bladeless) pump and turbine were generally capable of efficiencies far below that of comparable "bladed" equipment. This is definitely not true.

PRACTICAL PERFORMANCE OF NEW PUMP

Irrefutable data (see Figure 1) shows this equipment is performing in specific applications with efficiencies equal to or greater than conventional equipment made to handle the same specific application.

There are many applications in the "real world" where "bladeless pumps" perform the most difficult or IMPOSSIBLE pumping requirements while conventional "lifting surface" pumps cannot function at all.

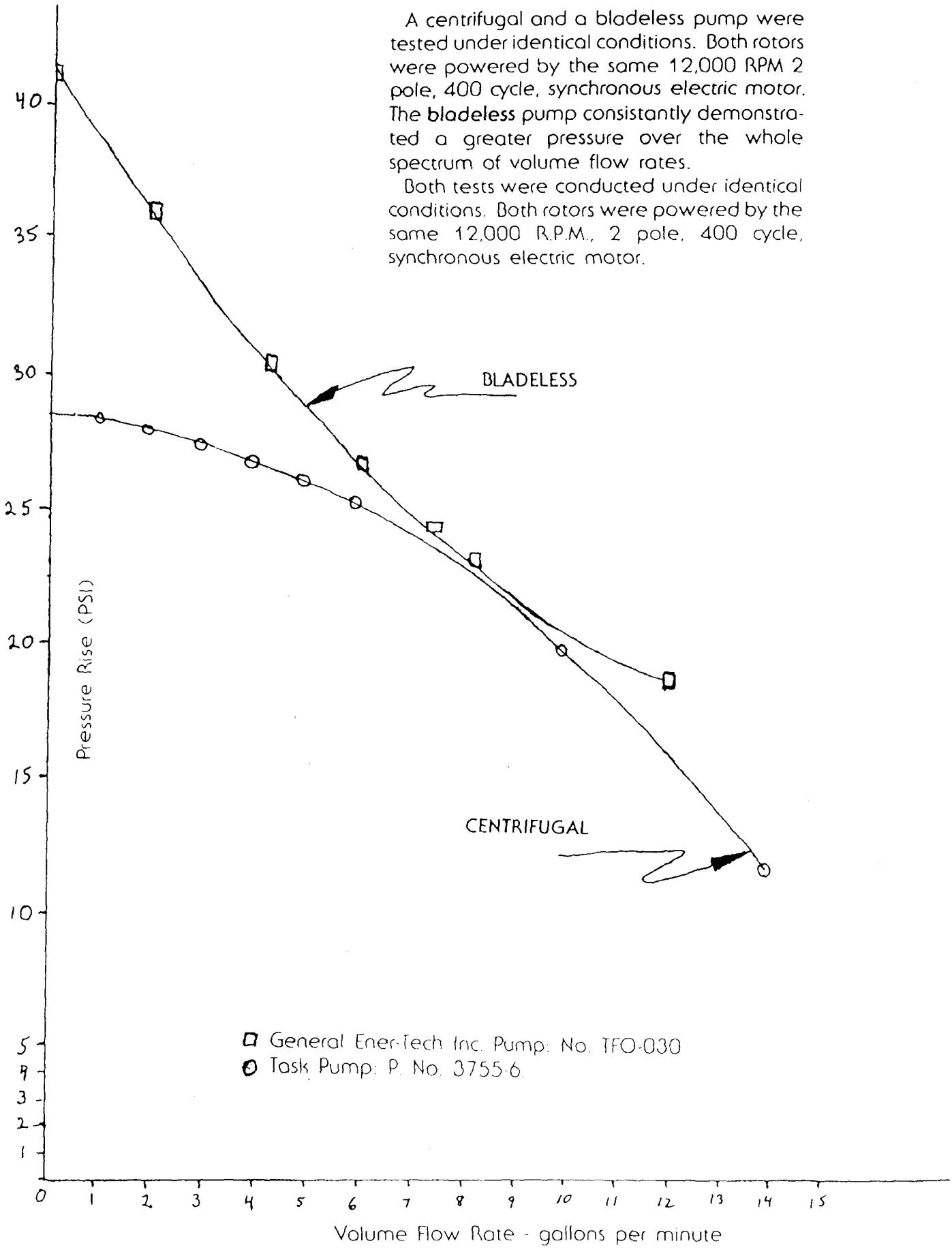
THE POSSELL BLADELESS PUMP PUMPS MATERIALS NORMALLY NOT EVEN ASSOCIATED WITH PUMPING. Examples of this are the successful pumping of live goldfish in water and many vegetables such as potatoes, tomatoes, squash, avocados and apples. All of this was recently demonstrated with absolutely no damage to the very delicate materials being pumped.

Current "in service" applications of the Possell Bladeless Pump include: coal slurry, geothermal '212°' salt brine, down hole pump for gas wells, powered glass, concrete grout, produce, sewage, fire pump (3,000 gpm, 170 psi) and blood pump for medical use.

Page 26 ENERGY UNLIMITED NO. 5, ENERGY UNLIMITED, Rt. 4, Box 288, Los Lunas NM 87031 USA
Jan-Mar 1979, p. 26-28

A centrifugal and a bladeless pump were tested under identical conditions. Both rotors were powered by the same 12,000 RPM 2 pole, 400 cycle, synchronous electric motor. The bladeless pump consistantly demonstrated a greater pressure over the whole spectrum of volume flow rates.

Both tests were conducted under identical conditions. Both rotors were powered by the same 12,000 R.P.M., 2 pole, 400 cycle, synchronous electric motor.



BLADELESS VERSUS BLADED PUMPS

The Possell Bladeless Pump is a relatively new type of pump which has been developed to overcome most of the disadvantages of conventional bladed or impeller pumps. Conventional pumps use the lifting surface of blades on their impellers to move fluid. Such pumps are subject to blade impingement of solids when abrasive materials are conveyed in the pumped fluids. The resulting erosion of the impeller blades causes the bladed pump to be subject to significant down time for repairs and resultant high maintenance costs and incipient pump failure. **The Possell Bladeless Pump overcomes these limitations of conventional lifting surfaces of bladed pumps by completely eliminating all lifting surfaces. Instead, energy is imparted to the pumped fluids by the boundary layer drag principle.**

CONVENTIONAL PUMP CAVITATION PROBLEM

One of the most difficult problems facing conventional bladed pumps is cavitation. This phenomena occurs when the suction pressure on the pump inlet drops below the "vapor pressure" at the inlet temperature of the fluid being pumped. The resultant "vapor bubbles" in the pumped fluid are impacted by the lifting surface of the impeller. These bubbles collapse and the liquid rushes into the voids left by the collapsed bubbles and the resultant "implosion" tears away at the surface of the impeller blade, causing rapid erosion of the impeller and a rapid reduction in pump performance. It also causes severe pressure fluctuations on the discharge side of the pump which can damage sensitive equipment downstream of the pump. If cavitation increases further, it will ultimately cause the pump to lose its prime and stop pumping completely.

HOW THE POSSELL BLADELESS PUMP WORKS

The basic design of the Possell Bladeless Pump incorporates one or more flat or concave discs, attached to a shaft which is rotated by any type of prime mover. The material which is being pumped is introduced into the center of the rotating discs where the boundary layer drag, on either side of each disc, imparts energy to the pumped material and the fluid moves outward in a helical path to a suitably designed discharge opening into a diffuser where the kinetic energy of the fluid is traded for static pressure.

The size, number, spacing and speed of the discs will vary, depending on the characteristics of the fluid being pumped and the desired performance. Since there is little relative motion between the fluid boundary layer and the surface of the discs, there is no impeller erosion even when pumping the most abrasive fluids. With no cavitation, **THE POSSELL BLADELESS PUMP CAN EFFECTIVELY PUMP GASES, LIQUIDS, SOLIDS OR ALMOST ANY COMBINATION OF 2 OR 3 PHASE FLOWS.**

POSSELL BLADELESS PUMP MANUFACTURE

The Possell Bladeless Pump offers significant production cost savings in terms of manufacturing costs. The impeller discs can be stamped from sheet material while the circular pump housing can be fabricated or cast with equal facility. The inherent simplicity of the bladeless pump makes it easy to assemble during construction and later enables maintenance to be accomplished simply and efficiently due to the ease of disassembly.

POSSELL BLADELESS PUMP PATENTS

The Possell Bladeless Pump has been invented and developed by C.R. Possell who holds various patents on numerous applications of the Boundary Layer Technology. Additional patents are pending on turbines, pumps, compressors and fans, all utilizing the boundary layer technology.

Readers desiring to purchase these Possell Bladeless Pumps and/or Turbines may write to this magazine for more information and prices. ###

news release

ENERGY UNLIMITED

Issue #20, p. 48-52

NEWS RELEASE Summer '85

NON-DESTRUCTIVE TECHNOLOGY DEMONSTRATED

ENERGY UNLIMITED would like to inform the public of the present development of a piece of hardware called an IMPLOSION MOTOR which will be a demonstration model to show the natural principle of VORTEXIAN ENERGY - Key to Motion, Propulsion and Power. Designing, developing and testing are in progress at the present by "Energy Unlimited's Research Team.

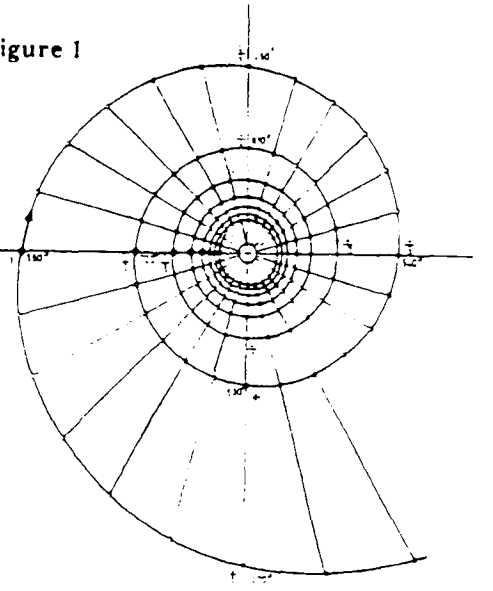
DISCOVERY RATHER THAN INVENTION

Their attitude being discovery rather than invention has led them to develop an observation of natural phenomena. If you want to have awesome power and lightning speed look at the masters of such in our universe. Nature shouts its secrets to us daily, all we have to do is imitate those mechanics.

VORTEX IS THE SHAPE OF MOTION Figure 1

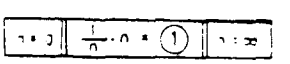
Why the vortex? A vortex is a manifestation of two opposing forces which exist constantly in our universe. It is the primary form or shape of energy movement. It is a geometrical spiral. It is the gathering tool and holding power of Third Dimension form. Motion is the bridge from space fabric to a physical existence. All life is an expression through motion and nothing else. Reflection and Curvature are used by nature to create this motion or action. All action is expressed through vortex motion. It occurs the same in all mediums. "Energy Unlimited's" team is learning to imitate this motion and translate it into machinery. Today man practices a straight line motion. We wrestle with nature, trying to force it to act along straight channels in our machinery, whether water or electricity. This is not its normal tendency when moving! Mass moved along straight lines increases its resistance with increasing speed. This method of moving mass or medium reverses the process of evolution. Through increasing resistance, energy, which is supposed to perform work, can only express itself partially; its freedom being controlled by its opposition and resistance.

Figure 1



A VORTEX IS AN OPEN GEOMETRICAL SPIRAL

HAVING THE FORMULA =



THE NATURAL GENERATIVE MOVEMENT IS AN INWARD MOVING, CENTRIPETAL, OPEN SPIRAL. THE OTHER HALF OF THE CYCLE IS THE CENTRIFUGAL, EXPANDING, RADIATING MOVEMENT WHICH IS PRESENTLY EMPLOYED IN OUR TECHNOLOG

IMPLOSION: SECRET TO POWER MULTIPLICATION

Within a spiral motion resistance is at a minimum in a "generative system" and therefore energy or motion finds expression to a high degree. To sustain and encourage motion in a system, we have to adapt apparatus or fit our machinery into the type of motion which expresses IMPLOSION. Implosion is the secret to power multiplication.

LOOK AT NATURES VORTEX ACTION

Let us look at observed vortexian influence in nature. All the intervening stages and manifold variations of the vortex formation can be discovered everywhere in nature, from the minute, microscopic and electric shapes and their movement, to the fully developed organ in all its detail, to the largest observable systems of galaxies. Our genetic makeup of life is stored and propagated with this spiral form. Our smallest biological units and simplest sea creatures up to snails, jellyfish and dolphins use their ppendages to create vortices for their movement. All creatures nerves, bone structures, auditory, digestive and circulatory systems are formed in spirals and operate in spirals. In the air, the mysterious flight of the bumblebee, hummingbird, and all birds is possible because they generate vortices and channel and control this turbulence. Even polarized fluids such as saps in plants and mountain water sources rise spirally to the tips of their systems.

What has remained the most enchanting shape in art of the ancients and the rock artists of today but the psychedelic, pulsating, whirling spiral?

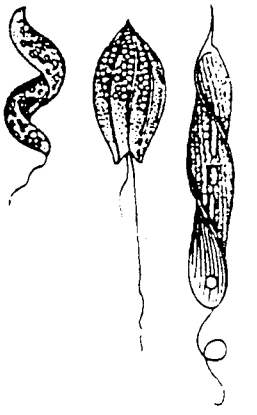
OBSERVATION PAYS OFF Figure 3

After many years of practical observation and investigation, hitherto unrelated facts began to fuse in one of those bits of intellectual fission that is called a stroke of genius.

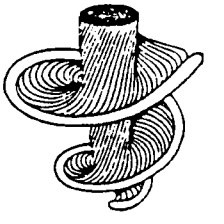
Walter P. Baumgartner, head of "Energy Unlimited"s Research Team, known for "getting the facts", has built working devices for the theories of Reich, Schauberger, Tesla, Russel, built his own, and duplicated the work of many others to "find out the truth for himself".

LOOK AT NATURE'S VORTEX ACTION

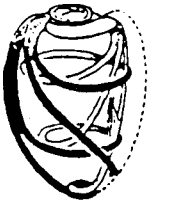
SHELLS GROW IN SPIRAL PATTERNS



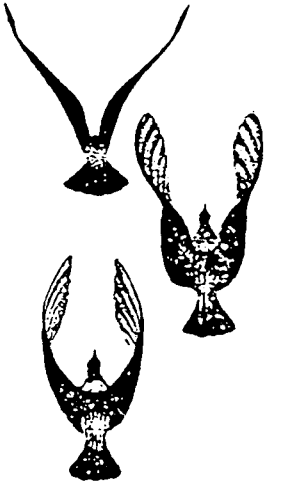
MARINE LIFE FORMS USE VORTEX PROPULSION



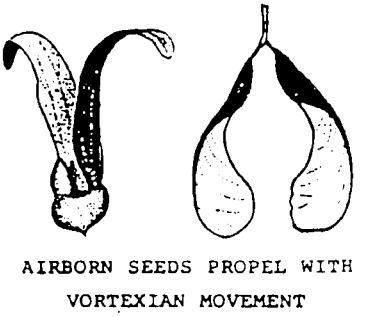
FIBRES IN THE AUDITORY NERVE ARE ARRANGED SPIRALLY JUST LIKE A LIQUID VORTEX



FIBRES AND FLOW OF BLOOD IN THE HUMAN HEART USE THE VORTEX PATTERN.



THE PHASES OF WING MOVEMENTS CAUSE VORTICES AROUND BIRDS TO MAKE FLIGHT POSSIBLE



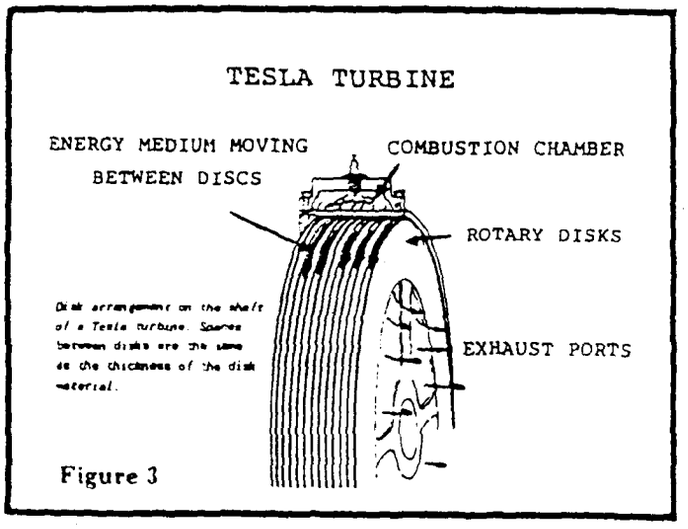
AIRBORN SEEDS PROPEL WITH VORTEXIAN MOVEMENT

AIR AND WATER CURRENTS MOVE SPIRALLY AS THESE OVER WOODLAND AND AN OPEN FIELD AND IN THE WATER FUNNEL.

As publisher of "Energy Unlimited" (a quarterly magazine which has served as a clearing house for researchers to exchange ideas and information) Mr. Baumgartner has been sent many of the world's finest attempts at "free energy" and spent more and more time personally finding the merit or false hopes in these attempts.

As a lecturer and educator, he has provoked brain storming sessions on natural technology, sharing with others understanding as he gains it. He is a perpetual student. He knows there is reciprocating, perpetual motion and power - he sees it evidenced everyday. "Can we deny the mysteries of water movements, birds and insects power, tornadoes and electricity; or can we help explain the common CAUSE these share? Why can't we apply these causes to our movements?", Mr. Baumgartner asks.

Resulting from a study of Nikola Tesla's works in 1967, Baumgartner developed Tesla Turbines which are based upon the vortexian suction, IMPELLER Principle and are one of the major components of the current research.


TESLA TURBINE
ENERGY MEDIUM MOVING BETWEEN DISCS
COMBUSTION CHAMBER
ROTARY DISKS
EXHAUST PORTS
Disk arrangement on the shaft of a Tesla turbine. Spaces between disks are the same as the thickness of the disk material.
Figure 3

TECHNOLOGY ADOPTS VORTEX

In proving the power of this principle, Mr. Baumgartner came across others whose works have used and proved the valuable vortexian power. Tesla, the electrical genius of the 19th Century, used the spiral shape to produce the most efficient spiral coil transformer to multiply voltage astronomically. Vicktor Schauberger, the Austrian naturalist, imitated the vortexian principle, improve waterways and agriculture Europe and later to propel saucer crafts during World War II. Henri Coanda, a Rumanian who developed the first jet propulsion systems and furthered our aircraft industry along, constantly advocated and demonstrated vortexian, implosion, suction power, instead of brut force. Walter Russell, the great artist and genius of our century, showed the universal mechanics and called the vortex the "basic tool of creation". Dr. Hans Jenny has demonstrated that sound travels through its mediums in a spiral path. Other applications of vortex power are used in air purifiers, water purifiers and desalinators, muffler systems, atomizers, solar collector silos, waste disposal, mining extractors and even an orthodonist's method to straighten teeth successfully. As more and more see the value of this principle, we will find the applications are endless.

IMPLO---WHAT WAS THAT WORD AGAIN?

Implosion and Impansion are two terms which are little known today. They are the opposite of explosion and expansion. While an upward temperature and dilation indicate expansion and explosion, a lowering of the temperature and contraction accompanies impansion and implosion.

Until now technology has recognized only one type of motion, the type which raises the temperature through friction and pressure. Chinese firecrackers, modern rockets, internal combustion motors, jet engines, the way we generate electricity, nuclear explosions are all examples of this pressure and friction explosion technology.

LET'S COMPARE: EFFICIENCY, COSTS, ENVIRONMENTAL IMPACT

An IMPLOSION MOTOR is in every respect the opposite of an explosion motor. When pressure and explosion are applied the particles they set in motion are often heated until they are white-hot. A so called heat barrier is produced by the resulting enormous friction resistance

orces. One of the greatest problems in the space program, for example is how to overcome this barrier.

In the case of the IMPLOSION MOTOR there is no heat barrier, no sound barrier, because with friction almost lacking, no heat is generated. On the contrary, due to the spiral contraction of the flowing medium, there is cooling and this creates a vacuum which INCREASES the suction pull and actually allows energy increase.

An example: when nozzles 1mm in diameter are used, the flowing medium detaches itself, when its rotation reaches 1200 rpm, from, the walls of the spiral channel and its flow rate rises to approximately 1290 m/sec. This corresponds to nearly 4 times the speed of sound.

In an implosion engine having a diameter of 1 meter, the lower surface of a suction rotor measuring slightly over 3 meters with 600 outlets installed, and with the rotor turning at the rate of 1200 rpm, the lower rotor shaft could be supplied with some 10,740 HP in the form of a cyclone-like projecting or lifting force. Such an engine output exceeds even the present possibilities of a nuclear engine.

A comparable nuclear engine with a reactor .9m long, .6m wide, and.9m high, while smaller than the above mentioned IMPLOSION MOTOR, will have a capacity 4000 HP lower. Furthermore, the nuclear reactor will have to be surrounded by tons of screen in hopes to eliminate at least part of its deadly radioactive emmissions. The cost to run the reactor will be tremendous. The IMPLOSION MOTOR runs on a small quantity of water or air uninterruptedly. Another advantage is that the implosion motor runs noiselessly without smoke, exhaust, fumes or danger. The implosion, suction, hydro turbine once started draws in water without further assistance and therefore requires only a fairly shallow basin for inflow and outflow.

Water consumption being equal, the power output of suction turbines is 700-800 percent higher than the power output of the compression turbines.

HOW IS IT PUT TOGETHER? Figure 4

See the diagram for the basic features of "Energy Unlimited's" IMPLOSION MOTOR. It is a combination of the Tesla Suction Impeller, Coanda's nozzles for vortexian current production, and Baumgartner's design to hold the vortexian envelope around the body of the motor.

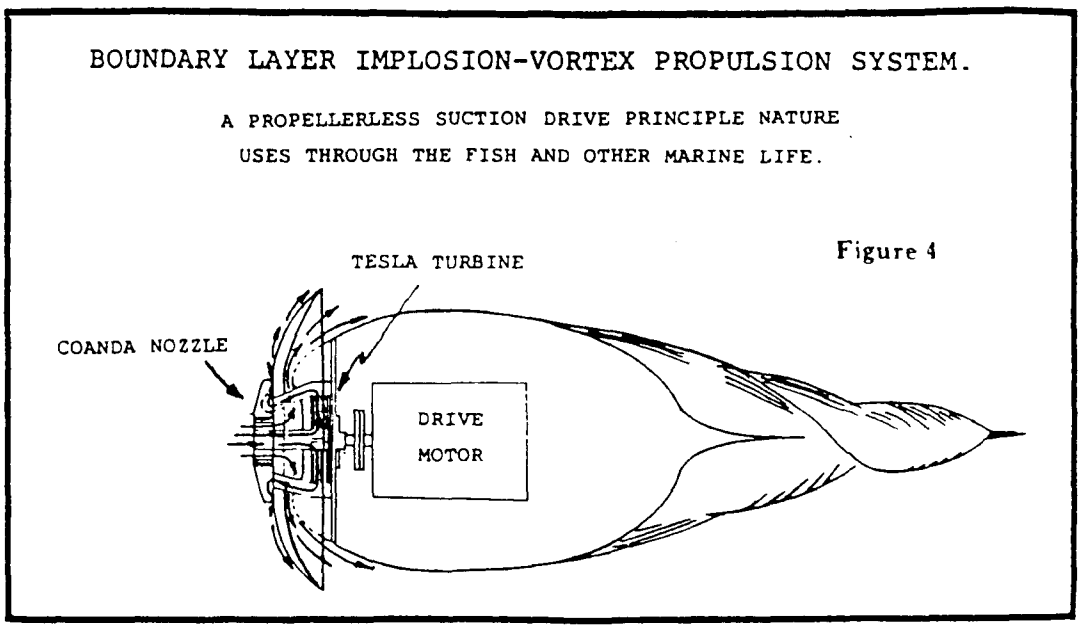


Figure 4

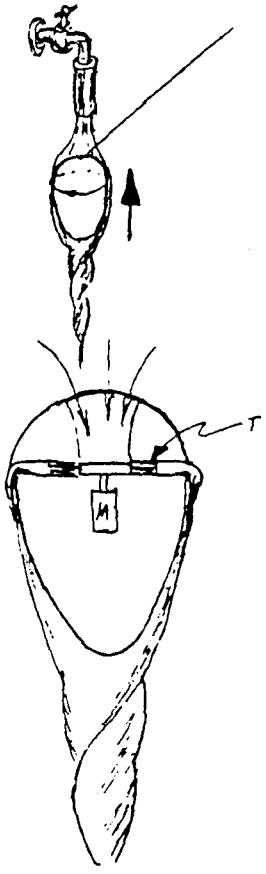
This motor can be modified to accomodate water or air propulsion or permanently mounted for power generation and multiplication.

THE OSCILLATORY CHAMBER

A further exciting application of this vortexian power principle is planned for the electro-magnetic medium. "Energy Unlimited's" Research Team has worked in the field of Magneto-Hydro-Dynamics (the science of the generation and controlled flow of a field of charged particles). With the recent discovery of "The Oscillatory Chamber" by Dr. Jan Pajak, the team and Dr. Pajak are researching a device which will begin this mechanical spin in an electric vortex.

The name "Oscillatory Chamber" is given to a principle of magnetic field production which employs the effects of the oscillation of electric sparks. These sparks circulate around the internal perimeter of a cubical chamber made of an electric insulator and filled with a dielectric gas. The four segmented electrodes joined to the inner surfaces of opposite walls of this chamber perform alone the function of two oscillatory circuits with a spark gap. The appropriate formation of mechanisms of oscillatory discharges occurring in both these circuits achieves: (1) the formation of circulating electric sparks which produce a bipolar magnetic field; (2) the elimination of electro magnetic forces acting on the structure of the chamber; (3) the absorbing and bonding of the energy initially supplied to this device; (4) the elimination of conversion of energy into heat; and (5) the chanelling of the destructive consequences of the accumulation of huge electric charges into the desired direction. As the final result of such a formation of the oscillatory chamber, this device will be able to raise the value of a produced magnetic flux to a level unlimited by theoretical premises. Read more about the projection of Dr. Pajak's powerful system applied to man's greatest desire "free propulsion through our immediate mediums and the rarified mediums of space" in MAGNOCRAFT - "Energy Unlimited's" latest publication.

ENERGY UNLIMITED IS HERE AND WE ARE HELPING BRING UNLIMITED ENERGY TO ALL!



TRY IT YOURSELF :

THE EGG SHAPE GIVES RISE TO A FORWARD OR UPWARD MOTION AGAINST THE MAIN STREAM. AS SHOWN IN THIS EXPERIMENT THE EMPTY EGGSHELL WILL MAINTAIN ITS POSITION AGAINST GRAVITY IF KEPT CENTERED BY A THREAD.

Integrated Gasification/Combined-Cycle Plants

History and Description of the Industry

The integrated gasification/combined-cycle plant (IGCC) is a relatively new combination of components—gasifiers, gas turbines, and steam turbines—which themselves have been around in some form for a long time. Steam turbines have been used to generate electrical power since 1930 in the United States, and now are used to generate more electrical power worldwide than any other technology.

A Reprint From THE SPOTLIGHT THE PAPER YOU CAN TRUST

Tesla's Turbine Technology Now a Reality

The National Spotlight, March 1984, p.1 & 11

By Tom Valentine

C. R. Possell is a doer. Where others have talked and mused, he has built hardware that works. And Possell's story finally looks toward a happy ending despite Big Brother-style suppression and unimaginable corporate and bureaucratic ignorance.

More than 20 years ago, Possell investigated the "myth" of the "Tesla turbine,"* and his innovative genius, like that of the great Nikola Tesla, ran into a brick wall in the form of the big business/big government partnership.

However, perseverance seems to be paying off.

"The Navy has granted my company a contract," Possell told The SPOTLIGHT in an exclusive interview recently. "And now maybe we can show the world what Tesla had in mind back in 1909."

The Navy contract is for Possell to build a geothermal electrical generating station on the Navy base at Fallon, Nevada. "Geothermal" refers to hot water and steam within the earth, as in geysers. "Old Faithful" at Yellowstone National Park is the most famous example of unharnessed geothermal energy.

"This project will put all my Tesla turbine and pump technology on the map, where it belongs," Possell mused.

But Big Brother's partners do not surrender their suppressions easily. The granting of the contract brought a protest from big business interests. It was argued that Possell's company was "too small" to finance and handle the contract properly. The protest placed more hurdles in Possell's path, but perseverance is Possell's long suit, along with his superior technology.

OPENS A FLOODGATE

Geothermal energy is tremendously abundant, and Possell's particular technology opens a floodgate that could very well render nuclear, coal and oil fueled electrical generation facilities much less attractive. While this is a negative for today's power structure, it could mean energy salvation for the world.

"The big oil companies know all about geothermal," Possell pointed out, "and they have moved to buy up as much of the obvious potential as they can."

Chevron, for example, has purchased thousands of the best potential geothermal locations. However, insiders among the Big Oil firms have leaked information many times that the oil moguls have long-range plans drawn up that indicate they intend to exhaust oil and coal reserves first, before allowing geothermal to expand to its true potential.

An official of Phillips Petroleum in Bartlesville, Oklahoma, who requested anonymity, told The SPOTLIGHT: "Our company has a 250-year plan, and geothermal isn't part of the program for at least 35 more years."

Possell considers geothermal nature's best source of energy. "There's more practically free energy bubbling up from within the earth than the Earth needs," he said. "It's readily available around the world; and, if we had to, we could dig down and tap it just about anywhere."

The technology for generating electrical power from geothermal has been around for 90 years. The first facility (in Laradello, Italy) has been producing since before the turn of the century.

EFFICIENT AND CHEAP

"My turbine and pump technology, which was inspired by Tesla's original turbine, is patented around the world and makes geothermal electrical generation more efficient and economical than any other source," Possell added matter-of-factly.

It has been a long, arduous journey for the "bladeless turbines" first dreamed up in the ingenious mind of Tesla back in 1909.

Tesla, the "father of alternating current," was such an inventive genius that

*A turbine is simply the ancient principle of the water wheel, brought up to modern-day speeds and power. Possell is an expert on turbines, having formerly worked for several conventional turbine companies.

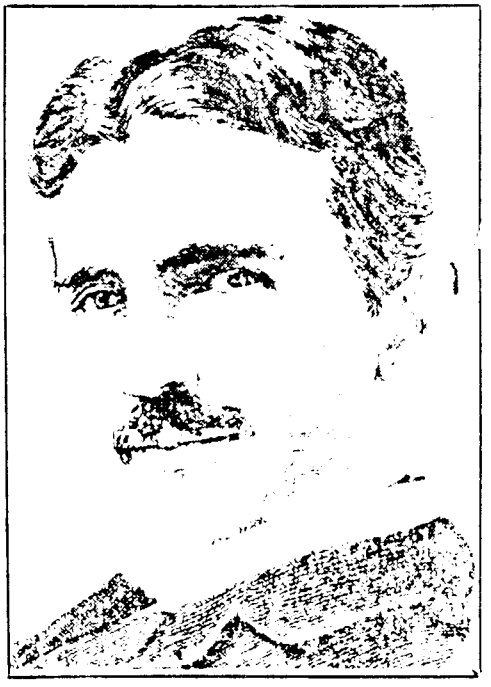
ENERGY IDEAS

today a virtual cult has grown up around the mystery of his life and work.

There is no doubt that history books deliberately obscured his achievements, and many people today believe much of his technology has been spirited away by monopolistic powers that desire to control technology to suit their long range goals.

LOOK MA, NO BLADES

The physics of electricity was Tesla's particular realm. But, in 1908-09, he took time to apply some thought to mechanical engineering. He designed and patented the first bladeless turbine, which evolved into a device of mythic proportions. The Tesla turbine is probably the most innovative engine scheme ever to be ignored, yet virtually canonized as legendary at the same time.



NiKOLA TESLA
. . . His theories become reality.

"Although Tesla's original ideas were clearly delineated in his patent application," Possell explained, "his invention had become nothing more than a term paper project used by a few college professors. Students would do very good analytical work, but their hardware was poor in quality, and the models invariably failed to perform as predicted by the analytical papers."

Possell's lifelong specialty has been reducing paper ideas to practice—making drawings into viable hardware. He was a young engineer more than 20 years ago, when he decided to make a model Tesla turbine.

All conventional turbines have "buckets," which is what the fan blades are called (in reference to the water wheel). But the Tesla/Possell designs do not have buckets--they are bladeless.

USING THE DRAG

"Tesla considered the problem known as boundary layer drag," Possell explained. "When a plane goes through the atmosphere, a thin layer of air sticks to the surface of the wings and body. Planes could fly 40 percent faster if this boundary layer of dragging air could be removed. Tesla, genius that he was, thought that such a phenomenon could be used by turning the unwanted concept around 180 degrees."

Tesla designed and patented a turbine that used the boundary layer drag instead of buckets. Possell brought Tesla's legendary design to life. His hardware proved that Tesla knew what he was talking about. The bladeless turbines are far less costly to make and maintain—and, in many cases, are more efficient than conventional (bladed) varieties.

"The first thing anyone needs to know about modern turbine technology," Possell added, "is that the energy stems from heat, not pressure. Working with the high temperatures necessary for effi-

Standing by one of his smaller industrial pumps, Jake Possell proudly announces that his hardware can pump boiling water (making it safer for nuclear power plants) and numerous materials without the wear and tear of cavitation and "water hammer."

ciency poses material and design problems that drive the cost up. It has taken millions of man hours and probably a trillion dollars to arrive at today's efficient jet aircraft turbines."

To explain what he meant about heat being the key rather than pressure, Possell illustrated: "If we had a hypothetical steel sphere and somehow bottled 20,000 pounds of pressure per square inch inside of it, and we had a pressure gauge attached, the gauge would drop immediately once a pinhole released the cold pressure.

"But if that pressure was superheated, it would take a long time before the pressure gauge would start going down. The equation is: 'BTUs** equal horsepower,' not 'Pressure equals horsepower'."

Possell added that a dramatic example of this fact of energy was brought home a few years ago when he served as a consultant to a firm that wanted to consider the electrical generating capacity of geopressure zones. Geopressure zones are not geothermal—there is tremendous cold pressure buildup, which may be dangerous to oil drilling equipment, but it isn't heated pressure.

"They didn't want to accept my analysis that the megawatt output of all that pressure was surprisingly low. But I was right, and the project was dropped."

So, the problems of high temperatures and turbines make such engines extremely capital intensive. No wonder, then, that the Big Brother partnerships of government and business so easily monopolized the turbine-essential industries.

"However, with bladeless turbines," Possell stressed, "the engineering problems are much more easily solved."

MAJOR COST BREAKTHROUGHS

For example, without the intricacies of bucket design, materials could be used that withstand far greater temperatures, and therefore put out far greater efficiencies.

Possell accomplished this major cost breakthrough in turbines nearly three decades ago. His first models were amazingly efficient, and he obtained a military contract to develop an expendable bladeless tur[illegible].

However, working with the military-industrial complex power brokers had certain drawbacks. Possell had observed the suppressive might of the power brokers when he witnessed the destruction of John Northrop's revolutionary flying "wing" aircraft. Suppressive, monopolistic politics, not failed technology, resulted in the destruction of flying wing technology and hastened the death of Northrop.

"I put my gas turbine technology on a back burner and turned my attention to pumps," Possell said.

Today Possell manufactures bladeless pumps that do not suffer the engineering design maladies of all other pumps. The boundary layer principle lowers costs, raises efficiency and obviates wear and tear.

"Bauxite (aluminum ore) is the toughest stuff imaginable," Possell said, "and standard pumps are worn out in a few weeks' time. One of our pumps has been moving bauxite for 10 months so far, and the wear is minimal."

Enter geothermal and today's electrical generating technology—a technology for which Big Brother wants the consumer to pay more and more (SPOTLIGHT, Nov. 28).

"Since bladeless pumps avoid cavitation (wearing away of metal by the action of water), we can pump boiling water at atmospheric pressure. No other pumps can do that. It's a vital factor in geothermal, and also in nuclear generating facilities."

AVERT MELT-DOWNS

Possell explained that the Three-Mile Island shutdown became a near meltdown disaster because the cooling water pumps could not continue the water circulation once the temperature rose too high.

"Even though they shut the nuclear reactor down," he said, "the cooling water was needed to eliminate the residual heat. But the pumps couldn't handle the high-temperature water and lost their prime."

Possell added that he is working quietly at this time with nuclear authorities to provide his pump technology to the A-plant design "arsenal."

However, with geothermal coupled to Possell's bladeless technology, the radiation danger inherent in nuclear plants need no longer be an issue. Geothermal is cleaner, cheaper and abundant.

"The godfather of geothermal, John Carlo Facca, has pounded his fist for years trying to tell utilities that the energy within the Earth is virtually free," Possell added.

Essentially, the actual energy bubbling up is free, and the costs need not be astronomical to harness it.

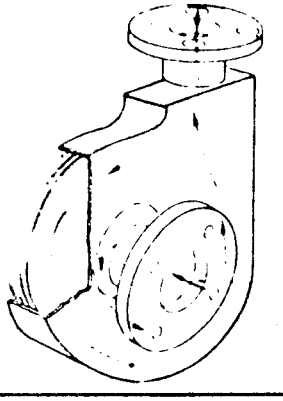
"Tests with our bladeless turbines indicated up to 60 percent more shaft horsepower can be achieved by using a mixture of hot water and steam, coming up out of the ground. This is really amazing to many engineers, because bladed steam turbines cannot tolerate water at all.

"We can drive the dynamos directly with total effluent geothermal without the expense of cleaning up the steam, and we do it more efficiently."

And, he could have added, bladeless pumps and turbines may be built for about half the capital cost of the others.

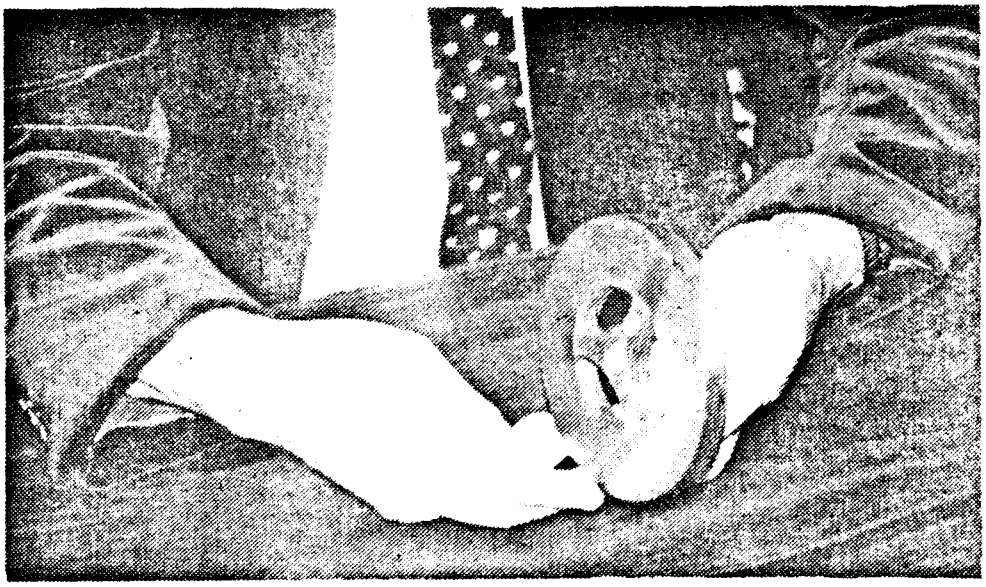
Possell has explained all this to various firms, utilities and government agencies for more than 15 years. His first patents were issued in 1966 and have been continuously updated ever since. All his perseverance has been needed as he encountered negligence and apparent complicity and conflict of interest at high levels.

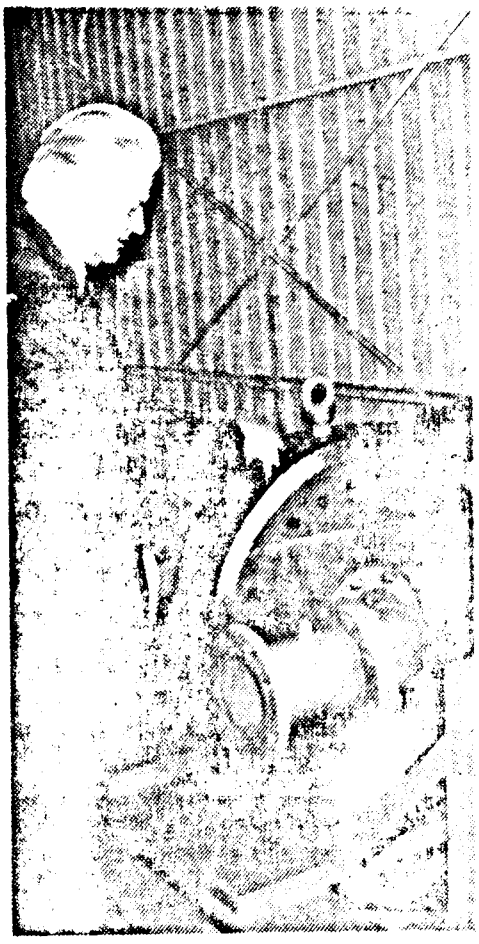
"I've been patient, and I'm not trying to bring the established houses down. The system isn't threatened by my technology, just streamlined," he said.



Possell Bladeless Process Pumps use rotor discs instead of impellers or blades. As pumped fluid enters at the center of the discs, boundary-layer drag on both sides of the discs imparts energy, and the fluid moves in a spiral path, discharging into a diffuser outside the case. There is little or no impact on the discs themselves. . .thus virtually no erosion. Absence of internal static pressure (the pump works only on the pumped material's *velocity*) puts minimal pressure against the seals, insuring longevity.

** A British thermal unit, or BTU, is the quantity of heat required to raise the temperature of one pound of water by one degree Fahrenheit, at or near 39.2 degrees F.





Here's How It Works

Engineers with excellent credentials can look at the "bladeless turbine" and disbelieve. So it's no easy task explaining exactly how and why it works.

The turbine was first conceived by Nikola Tesla in 1909. In 1966, C.R. Possell patented his innovations on the concept of bladeless turbines and pumps.

The idea of a fan or turbine working without blades is so foreign to standard turbine thinking it's no wonder Possell was forced to persevere through more than 20 years of entrenched dogmatism telling him it "can't be done," while the device was already functional.

The accompanying photographs will help the explanation of just how the turbines and pumps work.

The photo above shows a prototype of the interior rotor for a Tesla-Possell turbine. It appears to be merely a wide disk. But it is actually about 20 very thin disks, held a few hundredths of an inch apart from one another. The purpose of this design is to utilize the "boundary layer turbulence," or "drag," that always causes air to cling to moving metal.

"The shear plane between the boundary layer of air molecules and the metal is measured in thousandths of an inch—0.002, for example," Possell explained.

"In all turbines, including our bladeless models, a nozzle or series of nozzles directs the heated fluid or steam directly at the blades," he added.

But there are no blades.

To understand what the invisible blades are all about, it's best to first understand standard turbines.

"The initial push against the blade in a standard turbine is called the impulse, and on the opposite side of the blade there is a suction pulling at the same time, called the reaction. A bladed turbine is literally trading temperature and pressure for velocity."

Standard turbines are known for high rpm and power.

"The bladeless turbine obtains the same energy tradeoff, with amazing efficiency, by accepting the nozzle-directed heat and pressure at the edge of the disks, then having the boundary layer turbulence spin the disks and transfer the energy inward and downward, in a helical configuration, until it exhausts out the center."

In the photo above, Possell's index finger is poked into the exhaust port.

However, we need to explain his explanation.

A "helical configuration" is a helix or spiral. The energy potential inherent in the spiraling motion is exemplified by the vortex of a tornado when it touches down.

In the left photo, Possell is shown standing by one of his geothermal test turbines. The piping from the center of the large disk housing is the exhaust. If this were a bladeless pump, the process would be reversed—the exhaust pipe would be the inlet port.

Materials to be pumped are literally drawn up from the center of the disks and spiraled out the top edges.

"The thin boundary layer does an amazing amount of work," Possell explained.

American

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555 BIRCH COURT, FAX (715) 825-5911

November 17, 1988

Luther Rangely
P. O. Box 1574
Grass Valley, CA 95945

Dear Mr. Rangeley,

Thank you for your letter of November 8, 1988. My company designs and builds turbines based on Tesla's Boundary Layer Drag principle. However we have our own patents on "Bladeless Pumps and Turbines" since Tesla did no development work on his original idea which had very low efficiency. Since, we only build to order, I cannot give you any general information except that our turbine are approximately 1/2 the cost of our competitors and generally have higher perofrmance due to the unique configuration of our design.

I am enclosing photos of several units we have built for other people to show their simplicity and small size.

Thank you for your interest in our equipment.

I don't know anything about Tech Developments air turbines or their price. If you have a solid requirement we would be happy to bid on your specification.

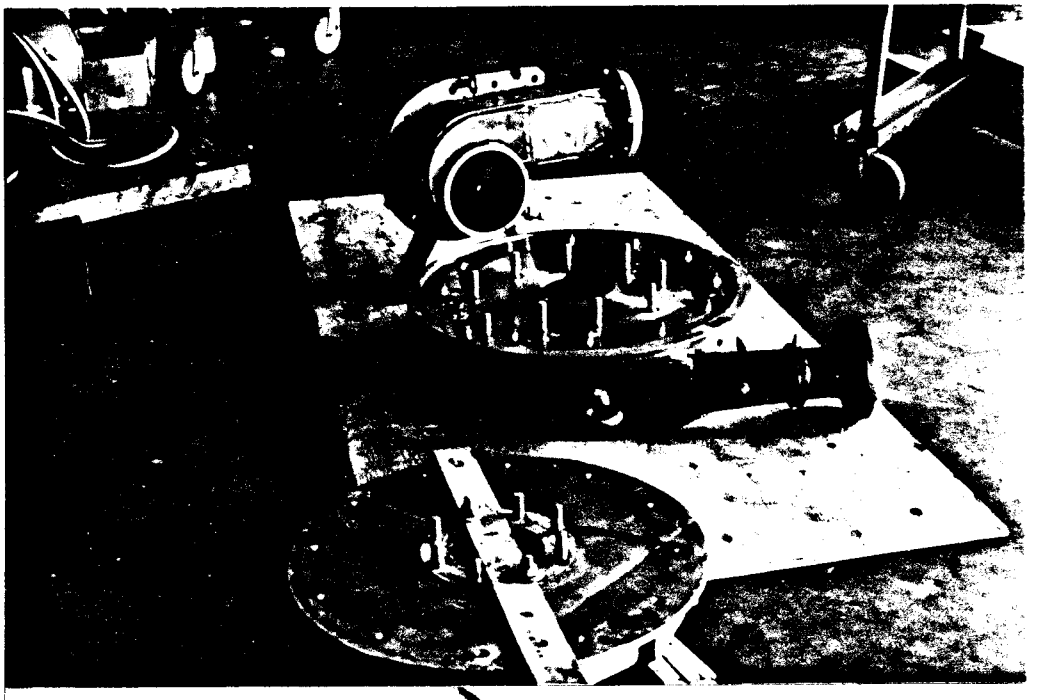
Very truly yours,

C. R. Possell

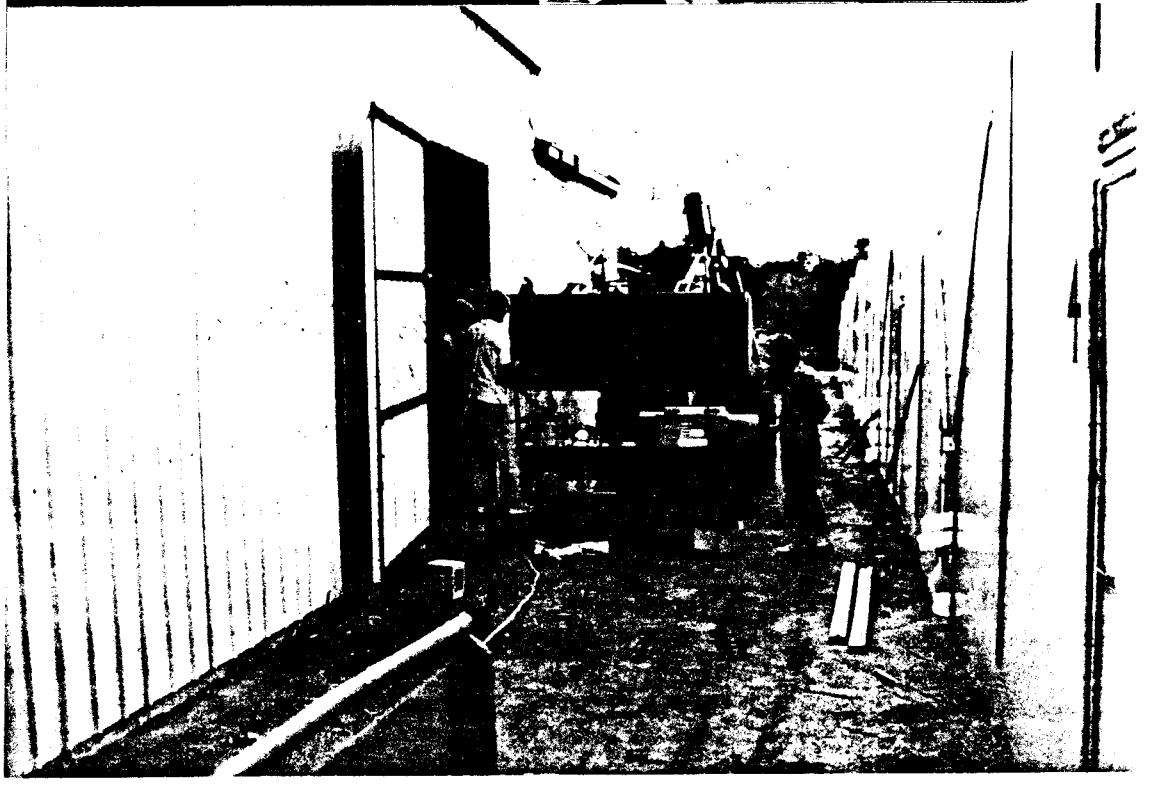
C. R. Possell
Chief Engineer
A.D.M.

American
Development & Manufacturing Inc.
555 Birch Court
Colton, CA 92324

C.R. "Jake" Possell
President/Chief Engineer (714) 825-5882



250 KW Bladeless
Steam & Water
Turbine



100 KW Bladeless
Steam Co-generating
Set

Boundary-Layer Breakthrough: The Bladeless Tesla Turbine (as developed by C.R. "Jake" Possell by Jeffery A Hayes. 1990: High Energy Enterprises Inc Box 5636, Security CO 80931, p. 26-43, 96-101.

BOUNDARY-LAYER BREAKTHROUGH

presented before the 1986 Tesla Symposium!

by C.R. "Jake" Possell

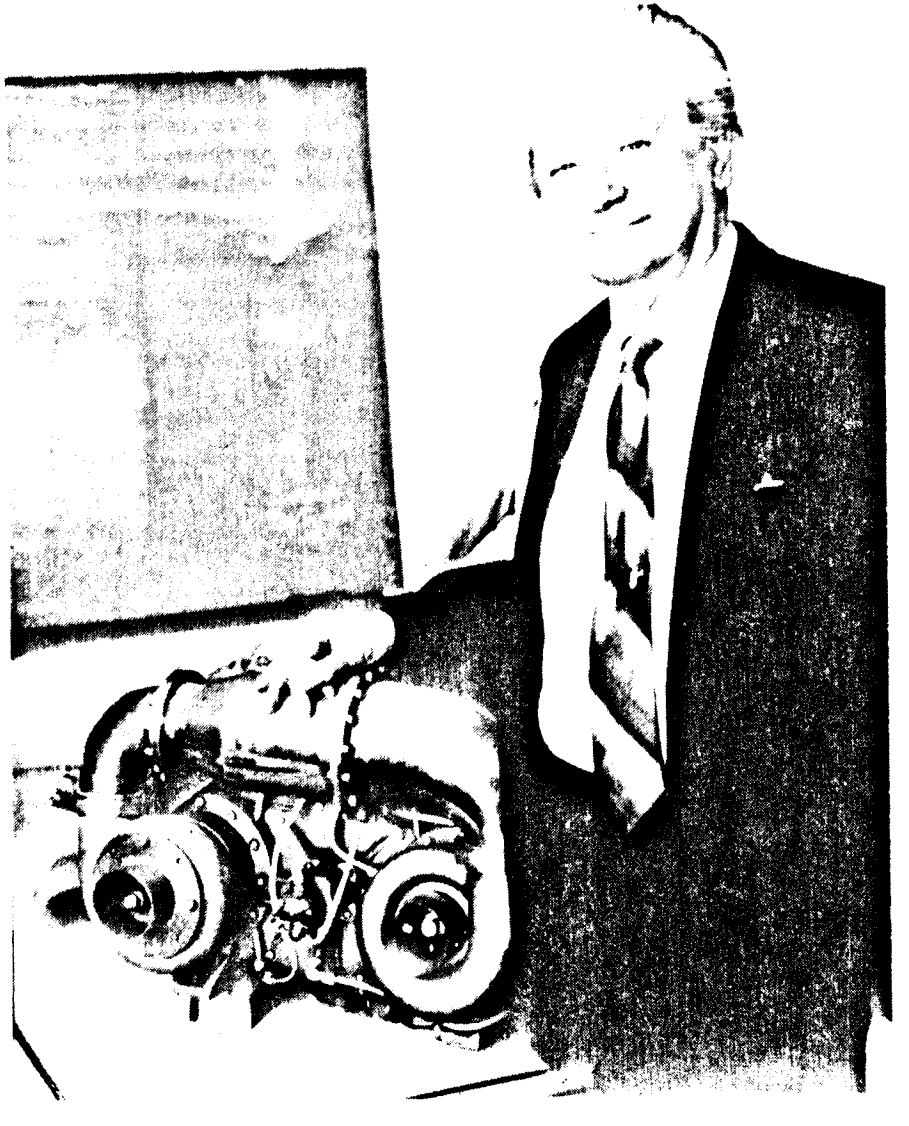
Ladies and gentlemen, members of the Tesla Society and our Guests. I am a mechanical engineer by trade. I have a private company, that has been involved for the last twenty years, in the development of a different type of technology. This technology came from the consummate genius Nikola Tesla. Tesla, the man that he was, made a number of excursions into other areas other than electrical engineering. Unfortunately, Tesla being twenty five to thirty years ahead of his time, metallurgy was not what it is today.

The ideas that Tesla purported at that time languished and were literally buried under the table for twenty to thirty years. About twenty years ago I stumbled across some of Tesla's work. Like a lot of people I had never heard of Nikola Tesla. Of course, everyone in this society is well aware of his capabilities. This excursion that he did was into an area of turbines and pumps. It is best alluded to in a new technology that we are all a little more familiar with--aerodynamics.

The Boundary Layer Effect

When an airplane flys through the air, at very high speed, there is a thin layer of air that sticks to the wing, the fuselage and all parts of the airplane. This layer of air goes the same speed as the airplane. There is then a sheer action or sheer plane between that boundary layer and the surrounding quiescent air around the aircraft. At that sheer plane is where the drag is formed that holds the aircraft back. Aerodynamicists tell us that if we could, wave a magic wand over an aircraft and, eliminate or minimize boundary layer drag the aircraft could fly 40% faster or further with the same amount of horse power. In aerodynamics, boundary layer drag is a totally unwanted precept.

Tesla was able to turn that precept around 180 degrees. Tesla perceived that boundary layer drag could be used to do something useful. Generally speaking, when we can take an unwanted precept like this and turn it around, we can make a very significant change or advance in a technology. This is exactly what happened in boundary layer drag technology. What



we have done basically, or what Tesla purported to do, is to make a turbine out of a series of flat discs. These disks are stamped out of metal. Stamping, you have to remember, is the cheapest form of metal fabrication that there is. If the automobile people had their way, the entire car would be constructed out of metal stampings. Stamping is the least expensive way to make parts. We stamp these flat disks out of flat metal and stack them up. As you can see, the rotor is constructed of thin disks of flat metal. There are appropriate spacers between the rotor disks. The rotor is then mounted on a shaft.

The Tesla Turbine Pump

This device is perfectly reversible. If power is put into the shaft and rotate the disk, we have a pump. If a fluid is brought in at very high velocity, it will create a boundary layer on either side of these disks, we have a turbine. The fluid proceeds, in an inward helix, until it comes out at the center holes which are the exhaust in the turbine. If we do just the opposite again and put power into the shaft, and rotate it, then these inlets become the inlets for the pump. The fluid then goes in an up outward helix path, until it comes off of the periphery of the outside of the disk, at a very high velocity. If we look at Bernoulli's equation, total pressure is equal to velocity squared plus P sub s or static pressure ($P_{TOTAL} = V^2 + P_s^2$). We can see from that simple equation that if we convert velocity squared it has to go into P sub s or static pressure. When the fluid comes off the periphery of the pump, at very high velocity, all of the total pressure of the energy in the fluid consists of velocity. That has some interesting precepts, that I will explain in a moment, in pump technology.

That high velocity then is taken outside the case of the pump into a device that is called a diffuser. The purpose of a diffuser is expressed through surfaces that gradually expand. The fluid is slowed down, velocity decreases and static pressure increases. That is the function of a pump, to increase the static pressure of a liquid. The unique thing about this particular piece of apparatus is that there are no lifting surfaces as there are in conventional pumps. If we look at the centrifugal pump, the vane pump, the piston pump, they all have a lifting surface that moves the fluid along. Therein lies the basic difficulty of using that type of equipment.

This is where the Tesla approach or the boundary layer approach exhibits all of it's advantages. The boundary layer on each side of each of these disks pulls the liquid by the boundary layer drag and accelerates it. Then centrifugal force moves it to the outside of the device. The liquid is then taken out on the periphery. In a turbine we do it just the opposite. We bring a fluid in that has high temperature, high pressure and we put it through a series of nozzles. The purpose of a nozzle is just the opposite of a diffuser. Here we are going to take pressure and temperature and change it into high velocity. We bring the fluid in tangential to the outside of the turbine. We bring it in at very high velocity. It forms a boundary layer on either side of the disk of the turbine. The pressure ratio is pushing it toward the center of the turbine. It forms a helical path down into the center of the device, and exits in the center, after it has transmitted all of its energy to the disks through the boundary layer drag.

Analysis tells us, this type of energy process of transfer has the potential of being much more efficient than by pushing something, as you do in a conventional bladed turbine through impulse or reaction. Of course, because there are no lifting surfaces, there is no way for the fluid or the material going through the turbine to impinge on anything. These are the limiting factors in a conventional pump.

There is a phenomena that occurs in a conventional pump called *cavitation.* Cavitation occurs where you have a liquid that has bubbles of gas in the liquid. These bubbles can be from outside gas that is mixed into the liquid or they can actually be formed by the liquid boiling inside the device as it

passes through. There are many liquids that have very high vapor pressure and they boil at very low temperatures. When we reduce the static pressure on a liquid it boils at a lower temperature than it normally would. If we take water for instance and carry it up on top of Mt. Whitney or Pikes Peak we will find that water does not boil at 212°F it boils at some lower temperature. So if we happen to be putting water through a conventional pump, at say, 165°F, because of the lowering of the static pressure on the suction side of the pump the water will begin to boil inside the pump. These bubbles of vapor of that particular liquid will be formed, inside, and will have to pass through the pump. What happens is that a lifting surface comes along and hits the bubble and collapses the bubble. This causes the opposite of an explosion--an implosion. At the same time that this implosion is occurring, the liquid rushes in the hole where the bubble was. This inrush digs away at the lifting surface of the pump. If we operate a pump under extreme cavitating conditions for any length of time, the rotor of that pump will look like lace-work curtains. In fact, I've seen pumps where there was a shaft and a little nubbin left on the end of the shaft and the rest of the rotor had completely disappeared due to cavitation. That's pretty serious trouble for a pump.

In addition to that, there are other phenomenon that happen downstream of the pump that could be even as bad or worse than the cavitation effects on the rotor of the pump. These are large, huge pressure fluctuations that occur downstream of the pump because of the cavitation that is occurring within the pump. These large pressure fluctuations can split a line in half. It can tear a heat exchanger apart. It can do all kinds of serious damage in a petrochemical plant or a chemical plant where cavitation is occurring in a pump. If we are trying to pump a liquid, not water, say something like isopentane that boils at a very low temperature, it is almost impossible to pump that material without having some cavitation occur within the pump.

The Tesla turbine or boundary layer pump has no lifting surfaces. So by the very definition of cavitation it cannot occur in this pump. That means that we can pump liquids and gases together. In fact, as it turns out, we can pump three phase flow. Three phase flow consists of a liquid, a gas, and a solid.

Why can we pump solids when other pump companies are very wary of solids? Again, it is this impingement, the lifting surface comes around and hits the solids, say a grain of sand. It impinges on that lifting surface and it abrades or digs away at that lifting surface. So if we were pumping three phase flow with a conventional pump, we could actually have both cavitation and impingement of solids occurring at the same time. If that happens, of course, that pump is not long for this world. It will be destroyed in a very short period of time. And it's all dependent upon on how fast this occurs.

If we increase the speed of the pump, this phenomenon will all occur under faster conditions and the pump will be destroyed, in a much quicker manner. If we slow it down, the effects will take longer over a longer period of time to act on the pump. That is another bad thing for conventional pumps. In order to get reasonable life out of a conventional pump, we have to slow the pump down. This prevents, the process I'm talking about, from occurring at too rapid a rate and causing the pump to be damaged in too short a period. Out in the real world, where people are using pumps every day, you would be surprised how many people are using pumps under this condition. They say, well, there is no other way to pump fluid or what we are trying to pump. Water is a pure Newtonian fluid and at 60°F it meets the standard conditions. Unfortunately, none of my customers pump clean, cold, pure water. They want to pump something nasty like sulfuric acid, ground glass in water or dirty oil that's loaded with sand and other chemicals that can attack metals. The problems out in the real world of pumping compound, unfortunately, with the present type of pumps that are used in this world--the lifting surface devices.

We have a unique problem. We have a small company. We're literally like a child in front of the candy store. We have so many things to do and so many things to accomplish we can't do it all. We have to pick the most important things that we see out there where this technology can do the most good for industry. We have done a whole variety of things. I'll be as brief as possible and tell you some of the things we have done.

We can pick pump three phase flow. We can also pump any two of those phases together without damaging the boundary layer equipment. In other words we can pump a gas and a solid. That is very unique to pumping. I think we are the only company in the world that can say we can pump a solid and a gas without any damage to the rotor. The boundary layer protects the rotor and the inside of the pump. The solids that go through the pump actually never touch the surfaces of the inside of the pump. In other words, they go in between the boundary layer from one disk and in between the boundary layer of the next disk adjacent to it.

Being an engineer as I am, I have told a lot of people about these facts, of course, but there is always a gnawing little word in the back of your head that says you know that to be true, but what if it really doesn't happen that way? So to satisfy ourselves, during one of our tests, we were pumping sea water off a beach with a very large pump, we were planning a long duration pumping time, so our shop superintendent took a grease pencil, and he put a little mark on the outside of the rotor of one of our pumps before we put it together, then we assembled the pump. We pumped this sea water which is nasty stuff because it has so much salt in it. It also has lots of sand, it has sea shells, sea weed and all kinds of other things in it. All these things pass through the pump. Two weeks later after having run this pump twenty four hours a day we took the pump apart. Eureka, right on the side of the rotor was the little mark from the grease pencil. We could take our thumb and wipe that grease pencil mark off. The boundary layer protection does work. It is a real part of this type of pumping.

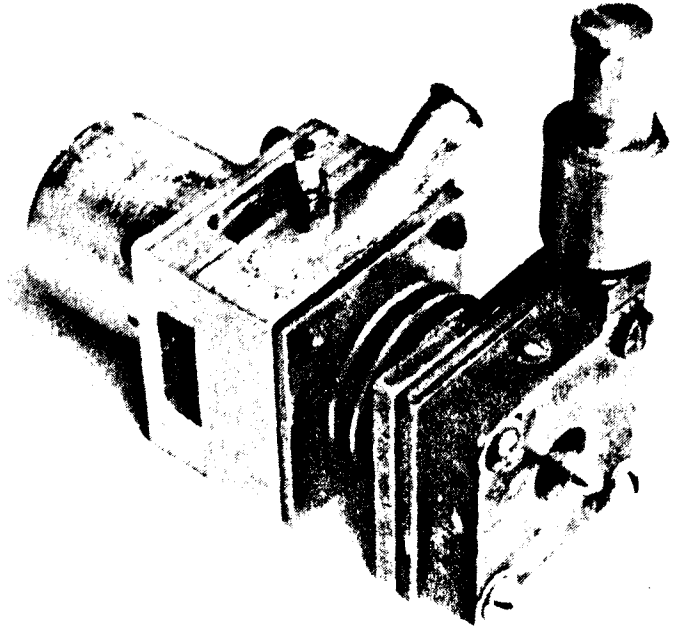
The Tesla Turbine Generator

Before we started on this particular technology I had worked for a number of companies that were in the gas turbine business. When I first stumbled over the works of Tesla I saw this as a cost breakthrough in the field of gas turbines. I said to myself, aha, with today's metallurgy, with the techniques of today's improved bearings and seals and all the rest, I believe we could do what Tesla set out to do so many years ago. Of course, we went to the military naturally.

The military has classically been looking for what we call an expendable engine. That is an engine that lasts for a very short period of time. Normally if we look at conventional technology, say a General Electric, a Westinghouse, a Toshiba or a Rolls Royce gas turbine were looking at a piece of equipment that is so capital intensive or so expensive to manufacture that if you bought one of those devices and brought it back to your plant and said to your accountant: How are we going to amortize this piece of equipment? He would say: We better amortize it over the next ten years because that is the only reasonable way to spread this tremendous capital cost over a number of years. Bear in mind now what the military is looking for is an engine that doesn't last for ten years but lasts for ten minutes. So it's a little more difficult to amortize the cost of that device over ten minutes than it is over ten years.

I brought a little device with me that I hope you can see from where you are sitting. We built this for the US Navy during the Korean War. I can't tell you what it's for because that is still classified but I can tell you what it does and you can draw your own conclusion. This device is called an expendable gas turbine electrical generating plant. In other words , I'm holding in my hand a complete electrical generating plant. It consists of a two stage compressor. It takes air and compresses it and mixes a fuel with it. In this case, it's propane. That propane/oxygen mixture then goes into a burner can, which is a tiny piece of jewelry on this little device and it burns in there. The burner can holds the flame front so that the flame will not blow out. That hot gas then comes down to a little tiny bladeless turbine, which is only one inch in diameter.

This device is launched at thirty thousand feet altitude. It comes up to speed in a fraction of a second, in millionths of a second. This is done by a charge of gun powder. On the end of this little device, here this little silver thing that's on this end of the turbine, is a two pole alternator or generator, as we know, and it generates electricity. Its all on the same shaft with the turbine.



Hand held 180 Watt turbine. Note: Spark plug is mounted on compressor section for storage. For operation it would be mounted on burner-can (vertical section).

At thirty thousand feet altitude it comes up to speed and it produces one hundred watts of electrical energy. It takes the place of a battery which is quite large--a foot square. Of course a battery does not produce much electricity when its at -65°F which we have up at that altitude. We have to put around the outside of that battery, some means of heating it. That means that we have to put more energy in to make the battery work. Here all we have is this tiny little device that makes 100 watts of electrical power.

Ten minutes later, after it is launched, it is back at sea level and it is producing about 180 watts of electrical energy. Then it is destroyed. Its total life is only ten minutes. That does not mean that this device can't run more than ten minutes. It means that in its mission, for what it was designed to do, it has a ten minute life. That was a very interesting engineering exercise for our company. Unfortunately, after that occurred the Korean War was over. Our company determined that it is very difficult for a small, undercapitalized company to interface with the military. We decided to put this on the back burner, which we did.

That was kind of hard for us to do, as you can appreciate, because we were so enthralled about building larger ones and what we could do with the boundary layer concept as far as generating electricity. We had to put that on the back burner. That is more amenable to a small company that's undercapitalized. The military does not understand that you have to make payroll on Friday afternoon. They say we'll pay you next week or next month or something like that. It's not too important to them but to the people that are running the company it is a matter of life and death.

We decided to put turbines on the back burner and go into the development of boundary layer pumps. That turned out to be a good decision for our company because a pump is about one tenth the complexity of a gas turbine. With developing these pumps that could do all of these unique things, our company has prospered over the years. We are now in a position where we feel we can go back into the turbine field and possibly interface with some larger companies and the military and so forth and do the things that we should have done twenty years ago in the turbine field.

Problems Inherent with Gas Turbines

Now there are many types of turbines. There are water turbines, there are steam turbines and there are gas turbines. We're looking at the whole wide field of turbines. As I showed you this turbine was built during the Korean war which is what? Thirty-eight years ago-something of that nature. Over the years, we have been able to produce a number of patents on this technology. The first patent, that we took out, cite Tesla as prior art. Of course, we always tell everybody that this whole idea of boundary layer drag was first purported by Nikola Tesla the genius who invented alternating current.

There are some very interesting and exciting things that are occurring in this field. Its kind of refreshing to me to come to a meeting like this and hear a doctor who can get so excited about dividing the speed of light into a factor. I can get just as excited about boundary layer turbines and what they can do for the world up ahead. I would like to give you a little insight into what we're working on and what we feel can happen. In a steam turbine there are series of blades that look like a twisted airplane wing. They are attached to a big rotor in the center of the turbine. The steam comes in and impinges on the wing or the blade of the turbine and it pushes it. That's called impulse. The back side of the turbine is curved like a wing. The fluid goes around there and forms what is called reaction which drags the blade around. As I told you, in the boundary layer turbine we have no lifting surfaces, we have merely a system of flat disks.

In a turbine the crux of the whole problem, that people are looking at, is the effects of temperature and stress on that blade. You can appreciate that as we raise pressure and temperature the cycle efficiencies of steam cycles, gas cycles and so forth, all become more efficient. Everyone is striving, monumentally, to increase turbine inlet temperature and pressures. This will increase the efficiencies and we can get more electricity out of a ton of coal or a gallon of oil, what ever it might be, that fossil fuel that we're using to make the electricity.

People sometimes have a mistaken idea of a particular technology. For instance, back a few years ago in California, we had a whole bunch of people that were reinventing James Watt's steam engine. They were reinventing the steam turbine. Unfortunately, they were not smart enough to go to the public library, and pick a book up there, and read about what the efficiencies of such devices are. A steam turbine and a steam piston engine, and so forth, unfortunately, are very neat arrangements of mechanisms but they are inherently very inefficient. The most complicated steam locomotive that was ever built, triple expanding, double compounded device, was only 14% thermally efficient. Unfortunately the diesel engine is a good deal more efficient than that it's about 36% to 38% efficient. That's why we don't see anymore steam engines running out on the railroads steam track. We have

a much better way of making motive power with a diesel engine at a much higher thermal efficiency.

There are some rules of thumb that I go by as an engineer. If I tell you some of these rules of thumb you will be able to understand what I'm talking about much better. In a gas turbine, lets take for instance, the limiting temperature today, if we burn fossil fuel, at stoichiometric combination, which is the optimum mixture of fuel and air, we can generate temperatures well over 3,000°F. We don't dare put 3,000°F on those blades in that turbine. If we did the blades would stretch and hit the outside case. It would be an incipient failure of that device. What we do is take tremendous quantities of cold air from the outside and we mix it with that hot gas and cool it down to some temperature that we dare put into that device.

You don't have to be an aero dynamacist or a thermo dynamacist to know that's not good. If you raise something to a temperature you have a certain amount of energy in it. If you put cold air in with it you're reducing the energy content by quite a substantial amount. Today the limit of such gas turbines is about 2,000°F. The most advanced military engine, in the world that I know of, runs at 2,050°F. Westinghouse, General Electric, Toshiba, Rolls Royce--everybody that is working in the gas turbine field--is putting most of their effort into some way of being able to increase this temperature that goes in to the turbine. Every year they raise it eleven degrees or fourteen degrees or what ever it happens to be. That's through a combination of metallurgy, unique mechanical design and maybe some new materials. We're able to raise it eleven degrees. Then they publish a big report about as thick as a big telephone book. It says, look what we've done this year we've raised the temperature eleven degrees.

There is a rule of thumb that is most interesting here, that I'd like to interject, if we can raise turbine inlet temperature 350°F we double the horse power output of that engine. In other words, if we had a General Electric engine standing there, and we could wave a magic wand over it, and raise the temperature 350°F it would have 80,000 pounds of thrust instead of 40,000 pounds. I'm not the only one that knows that. Everybody else in the jet engine business is well aware of it, let me tell you. They are up against a stone wall, literally, to raising turbine inlet temperature.

Along comes the boundary layer turbine. We can see that there are no blades here. At the root of the blade, we have a phenomenon, a stress phenomenon, called combined bending. That is the worst stress level. That will drive a stress engineer right up the wall. Combined bending is the worst kind of stress that you can put on any device.

To accentuate that, in a conventional turbine, the back end of the blade goes out to a sharp edge. That's Charpy effect or the notch effect. That is where the bending effect is greatest at that sharp edge. That is where the blade fails. It starts to crack at that point. If we put too high of a temperature on, it's goodby blade. It flys off and that's the end of the turbine.

If you are ever getting on an airplane you might look at the engine pod, as you're getting on the airplane. There is a red line that is painted around the outside of the engine pod. The purpose of that red line is to tell the mechanics, who are working around that engine or aircraft, not to stand in line with that red line. That is where the blades are going to come out if they ever choose to fatigue and come off. They will come out of that engine like a rifle bullet. They will go into the next county. Let me tell you something, every engine mechanic watches that red line. If you watch, they will step very quickly when they get to the red line. They don't want to be in line with that area of possible danger.

In the boundary layer turbine we have no blades. We have no combined bending. It's gone. There is no such thing in this turbine. All we have is two forces. One of them is centrifugal force. This is due to the weight of the flat disk itself as it spins. There is a centrifugal force developed within the disk. That tends to move that disk, break it and make it move out. Centrifugal force is a very small percentage of what a combined bending stress level would be in a conventional blade. The other stress is a small force due to the torque that is generated in the shaft for transmission out to the disk.

I would like to tell you about a little thing that we did for our own amusement and to prove to ourselves the tremendous advantage of bladeless turbines. We physically set the turbine up as a compressor or a pump and attached it to a variable frequency motor which allowed us to control the speed. After we finished our test, we decided to find out ourselves, what would happen if we overdrove this disk up to some point where the stress level, the centrifugal force and the sheer forces would tear that disk apart.

The mass of a flat disk is a small percentage of the mass of a blade in a conventional turbine. That's why that conventional blade goes into the next county. Its turning at a very high velocity and has a relatively high mass. It will continue to travel in the same direction its flung out at. In the Tesla turbine the mass of the disk is much lower than that of a blade.

We drove the Tesla Turbine to increasing speeds, five thousand revolutions per minutes at a time. Somewhere up around 85,000 rpm one of these disks reached its point of no return and it broke. We had all been standing around cringing waiting for a big explosion. We were saying, raise it another five thousand. We were kind of worried about whether it was going to blow the whole laboratory up or what was going to happen. All of a sudden one of the instrument men who had earphones on said, "Sir, there has been a little blip in the rpm. I believe something has happened to the turbine." We didn't hear any noise. Nothing that we could detect had occurred. We decided to slow the turbine down and take a look at it. We let it coast down and it took quite some time for it to coast back to zero r.p.m.

To our great surprise and astonishment, when we opened the case of the turbine, the disk on the outside of the turbine, which is slightly thicker than the other disks, had exploded and disappeared. We thought that it would be somewhere just downstream. Maybe inside of our flow meter or in our control valves something of that nature. We started to unscrew the pipe. We looked for these elusive particles. We didn't know what to expect. Finally we went all the way over to the wall and took the last piece of pipe up and shook it. There were no particles in it. We said: "Now where could those particles have gone? Aha, they're out in the parking lot."

Sure enough, we went outside (we were in the second floor of a large concrete building) down in the parking lot and with a pair of tweezers we picked up these little pieces of the disk. They were about one quarter inch square. They had managed to go through the turbine, through the control valves, through the flow meters and all the instruments, out the pipe and into the parking lot without us even knowing it. We think we proved our point beyond the shadow of doubt, that this turbine has little danger from exploding parts if you over drive it. I'll explain why that is important in a minute.

The other thing is that a turbine has to be coupled to something. If we're going to put a turbine into say a vehicle. Lets look at that for a moment because that is an exciting part of what we're talking about. A turbine has maximum torque at zero rpm. Now just think about that for a minute. What does that mean as far as acceleration is concerned. When we let the clutch out in our automobile, we have to rev the motor up to about two thousand rpm. before we let the clutch out. If we don't the motor dies and we have to start it again. At zero rpm a conventional internal combustion engine has zero torque. Not so good for accelleration. We have to speed that engine up before we let the clutch out so that we have some torque available to accelerate the vehicle. The turbine is exactly the opposite. It has maximum torque at zero rpm. When we let the clutch out we better be ready. Our head is going to get snapped back and we're going to take off and probably spin the wheels.

Back in 1967 there was a gas turbine that was put into a Indianapolis racing car by a gentleman that we all know who is on T.V. That bladed type turbine had so much torque, they told the driver; "Be careful, don't push on the throttle to hard or you'll twist the half shafts off in the back of the race car." He could pull out along side another car and pass them at will, with the turbine. You might well say; "Why don't we have turbines in Indy cars today and other types of vehicles?" There are two reasons. One is that USAC the governing body of that racing sanction outlawed gas turbines. All the other owners said; "We can't afford to put a gas turbine in our race car and this thing can blow our doors off. We don't want gas turbines in racing cars." Collectively USAC outlawed the gas turbine as a power plant in race cars. Not because it wasn't good or better or anything else, it's because it was too good.

Today our ultimate objective is to get boundary layer gas turbines back into vehicles. Maybe first in an Indianapolis race car then into a truck and then possibly come up with one that will go into a car. Why is that so exciting? Let me explain to you. We touched a little bit on that rule of thumb about 350°F. Why is that so important? I told you that we have a lower stress level on the flat disks. I told you about our unique experience. Why is that so important?

The Ultra High Temperature Gas Turbine

It means that we can raise turbine inlet temperature above that temperature our competition can put on ordinary bladed turbines. I can tell you this, today we have run gas turbines with a turbine inlet temperature of 2,200°F- this is 150°F higher than any other turbine has been run. About ten years ago an English company came up with a new material. Normally metal, as we heat it, gets weaker and weaker and weaker. Up in the temperatures we're talking about, from 2,000°F on up, temperature really effects metal very adversely, metal gets very weak. At 2,500°F the best high temperature metal that we know of has a tensile strength of about 3,000 psi, which is like wilted lettuce. That is not too good to make turbine parts from. We are continually looking for new materials.

This English company came up with a new material which I call aerospace ceramic. Its a very high purity ceramic made out of silicon nitride. Silicon is the seventh most prevalent element on the face of the earth. Beaches are largely all silicon. Silicon is not only a very interesting material it is also very plentiful and therefore very inexpensive. What happens with this silicon nitride to our utter astonishment, don't ask me why this happens because I don't know, for some reason silicon nitride gets stronger instead of weaker as the temperature is increased. Silicon nitride gets stronger as it's heated. We said; "All right, lets take it on up and see what happens to it." Lets take that magical temperature 2,500°F. The strength of silicon nitride at 2,500°F is 80,000 psi instead of 3,000 psi. We could see the tremendous advantage. Why don't we make the internal parts of a jet engine out of hot pressed silicone nitride? People are endeavoring to do that. You can well appreciate the fact this material is a very high purity material. It has to be put in a mold. It has to be put under tremendous pressure. At the same time it has to be heated to temperature in the range of 3,000°F. The equipment needed to make hot pressed silicon nitride is not cheap. Its a new process called HIP-Hot Isostatic Pressing.

There are a number of companies doing this. You probably have seen an advertisement in a magazine that says HIP. What in the world is that? Now you know. HIP is taking materials like hot pressed silicone nitride and putting them under tremendous pressure and temperature and after a long period of sintering it will develop these unusual properties that are so advantageous in a gas turbine. I call that turbine the Ultra High Temperature Gas Turbine. Fortunately, the strength of hot pressed silicone nitride goes up to somewhere close to 2,700°F before it takes a nose dive as the other materials do at a much lower temperature. What I can tell you is that we can make these flat disks with relatively low cost simple tooling. We can make these disks out of hot pressed silicon nitride. We feel that in one swell jump we can go from 2,200°F to 2,700°F.

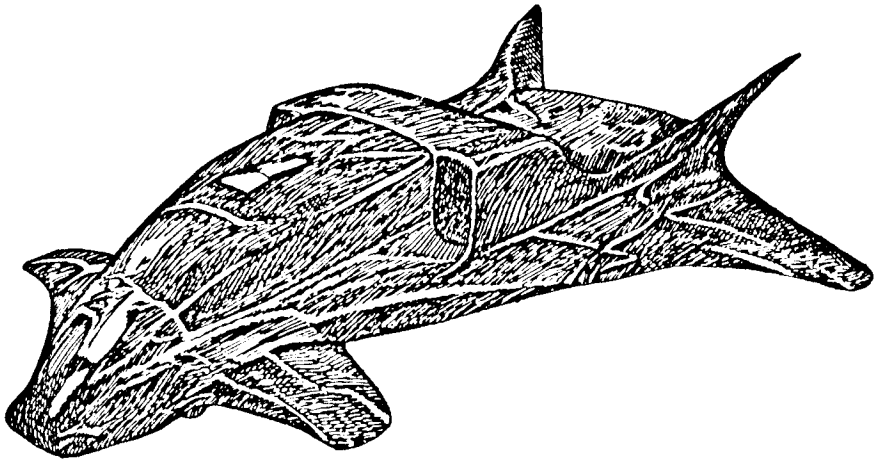
Lo and behold, We're going to have an engine that has three times the horse power of a conventional engine that we know today. Let me tell you, that's going to make one hell of an impact upon industry. It will first be, of course, very attractive to the military because of the tremendous additional power that we can achieve with this type of engine. Ultimately, because we can go to these high temperatures we also enjoy an increase in thermal efficiency of the device. We read alot about the airlines carping about the fact that their profits are going out the window due the high cost of fuel to run an aircraft. So if you can come in with any kind of a means, device, material, whatever to lower the cost of fuel per passenger mile, they will grab you with a hook and pull you in. That is a tremendous possibility in the ultra high temperature turbine.

The Phalanx

I'd like to tell you about a very interesting project that I'm working on. It's not classified. Although you may think it might be. Its been revealed to the world. It can no longer ever have a blackout put on it by the military. It is a new type of aircraft. It is what we call a true VTOL. That's a vertical take off and landing aircraft, ala the Flying Saucer. Why can we build this vertical Flying Saucer at this date when we couldn't do it before. The reason is power to weight ratio or thrust of an engine that's available to do this. This new ultra high temperature Tesla bladeless turbine is going to make this possible. We have already built a prototype of this engine. We have run it at 2,200°F, as I told you. We have plans of, in the very near future, running the engine at 2,700°F. When we do, we will achieve this power to weight ratio that is phenomenal in that industry. It will make possible aircraft of a whole new dimension.

The aircraft is similar to an aircraft you might of heard about, the Harrier built by theBritish. The Harrier was built by a company called Halker Sidley. It was built thirty eight years ago. Everybody thinks the Harrier is a brand new aircraft. It is not. It's an old airplane, but it is a unique aircraft even today. The military and others are just becoming aware of the Harrier's potential and what it can do. The Harrier has a conventional, bladed type engine called the Pegasus. The Pegasus, when it was first put into the prototype Harrier, had so little thrust, that in order to get the aircraft to take off from the ground, the fuel tanks had to be emptied. They had to remove all the wrenches and everything that wasn't bolted down in order to get it to lift off. It did fly under those conditions. It is an STOL --slow take off and landing. It has to move forward, in order to generate a small amount of lift, to help the poor Pegasus engine lift this aircraft off the deck. Today this is done on the bow of a aircraft carrier. The aircraft carrier goes fifteen knots and generates it's own wind. This is the fifteen knots the Harrier needs to takeoff from the deck of the aircraft carrier.

This new aircraft is not going to have to fly forward at all. You'll be able to park this in your garage and back it out in the alley and take off without an airfield. Just think what that is going to mean alone. We can eliminate all the vulnerability of an airfield. In other words, all an enemy



The Phalanx (see page 99)

has to do is come in and put a couple of bombs in right in the middle of our runway and it's goodbye airport until somebody can get out there and fill those holes up to be ready to take off again. This new aircraft can be parked under some trees. It can be pulled out and take off. It can do everything a helicopter can do. Why is that so important? All of our military strategy is predicated on the helicopter, ala the Vietnam war. We lost some 27,000 helicopters in the Vietnam war. That was from some guy standing with a rifle on his hip, plink. Put one rifle bullet through that rotor, that's spinning up there, and down comes that helicopter. This new aircraft is not vulnerable to that single shot.

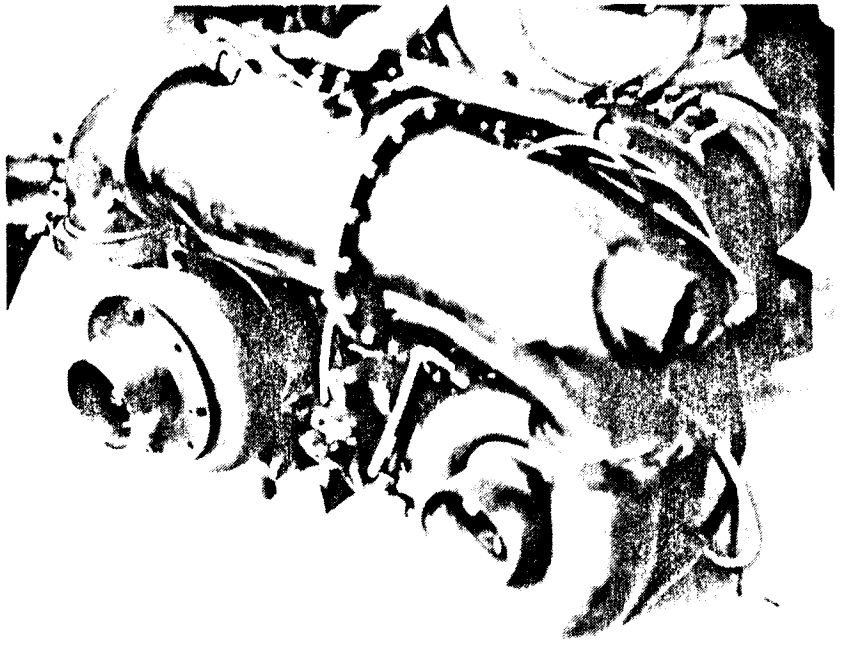
In fact, with two of the ultra high temperature turbines in this aircraft, it will rise off the ground, fly to any altitude vertically, then all of that thrust can be put to the rear of the aircraft with some variable nozzles that are part of the support system of that aircraft. That aircraft can fly through the air at Mach 2.5. Gentlemen, that is faster than the F-16 and the F-18! Not only do we have a device that out modes the helicopter it can also fly faster than our conventional military aircraft that we have today.

That gives you some idea of what the Tesla turbine is going to do to aviation. Naturally the military is very interested in this. Of course, there are other people who are very interested in it. I was at a meeting the other day where the president of Federal Express looked at this. He said: "Boy, would I like those to deliver packages". We were goading the military. We said you guys better get going with this. If you don't, later on when there is another war that starts, you'll have to go and take the Federal Express aircraft and paint over the signs and put the military signs on the Federal Express aircraft.

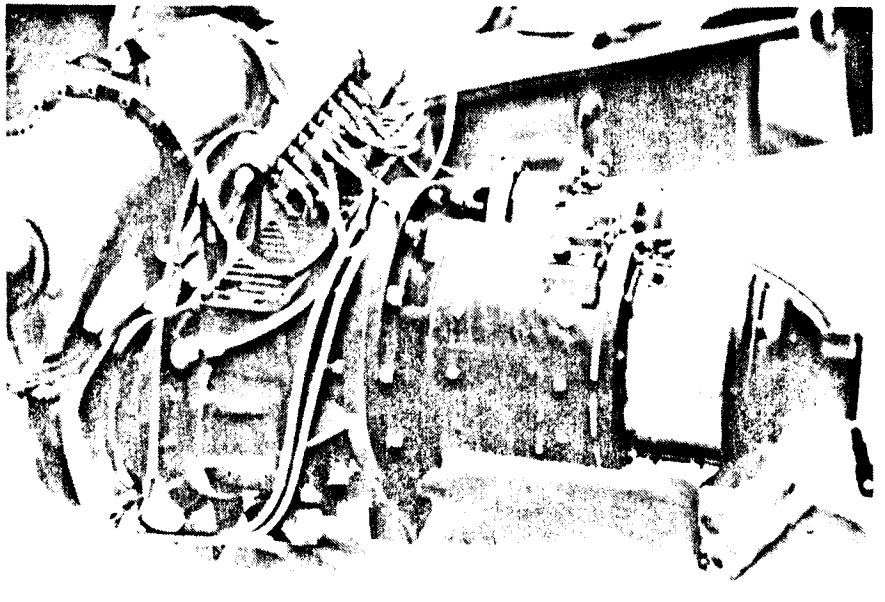
If any of you are interested I have some photographs of that aircraft. There are five of those aircraft in existence today. They have been built over the last two years. By the end of this year they will be in the air. Believe me, when the first one rises up over the hangars in Long Beach there is going to be another H.G. Wells episode. Everybody's going to call in and say; "Hey, I just saw a flying saucer and it was right here in Long Beach!" It's not a flying saucer, ladies and gentlemen, it is the new Phalanx VTOL aircraft that is being produced there. Don't become worried that there are some aliens from outer space approaching your area.

The other thing I want to say, one more word, about the use of these turbines in vehicles. It is literally possible with the high temperature turbine--and you'll have to take my word for it at this point because I can't show you anything (I wasn't able to bring anything with me)-- it will be possible very shortly to have a gas turbine that is so small that you can put it in the glove compartment of an automobile. It will produce enough horse power to make that car perform like none that you've ever been in before. It will out accelerate. It will be quiet. It will do a number of things that present day internal combustion engines cannot. It can burn any kind of a fuel. You can put oil just as it comes out of the ground into this engine. You can put powdered coal into it. You can burn saw dust in it, if you choose. You can use any fuel. You can put Shenaleeze Black Label into it if you like. It will run on that fuel--however it is a terrible waste of Shenaleeze Black Label.

With that in mind, I thank you all for being such a good audience.

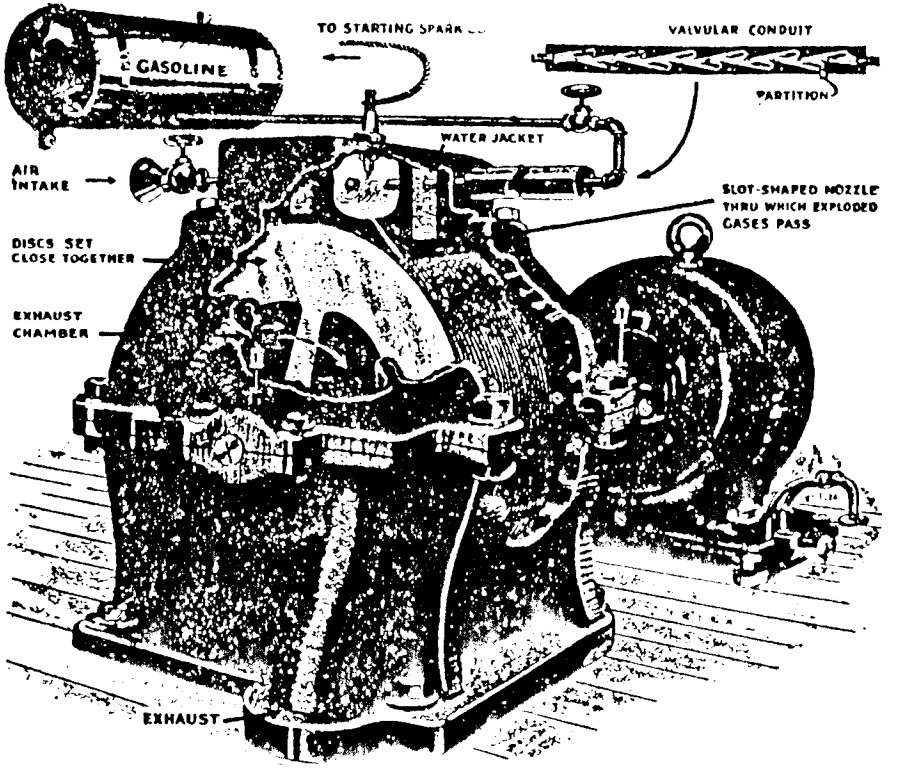


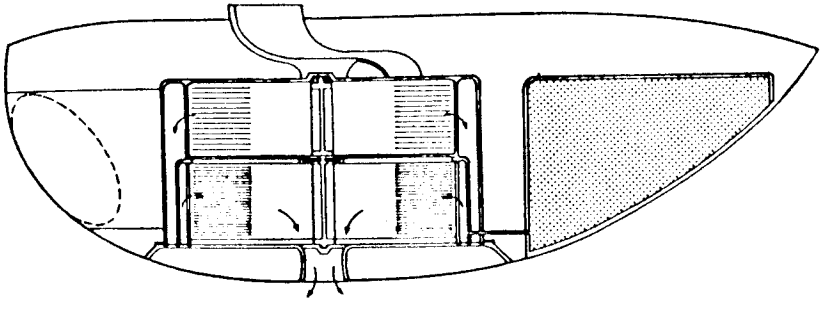
Bladeless Turbine on Left - Bladeless Compressor on Right



High-Speed Gearbox Attached to 30,000 Watt Generator

Tesla's Original Engine Generator

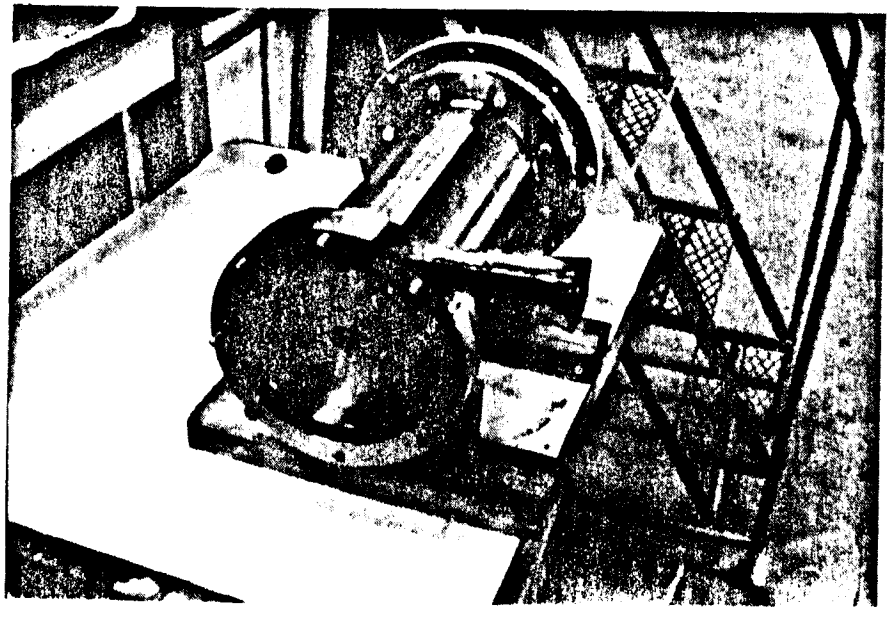




Tesla Hover Craft Design Using Bladeless Turbine

Bladeless Jet Engines

This is an R&D turbo jet engine. It was designed to produce 600 pounds of thrust. It is a simple cycle having a bladeless turbine on a shaft which is also connected to a bladeless compressor. The air from the compressor goes out externally to a chamber which contains the burner can, fuel nozzle and igniter. This unit runs at 35,000 to 38,000 RPM. This unit can be mounted in a test stand so that the fuel flow, pressure, temperature, RPM, and thrust can be monitored. The bearings on the turbine side are water cooled externally to prevent overheating. This bladeless turbine is of metallic construction. The metal is a very high temperature nickel alloy called incolloy. It can operate at temperatures of 2,000 degrees F or 2,200 degrees F for short periods. Ultimately this turbine will be fabricated from hot pressed silicon nitride, a space age ceramic, which will enable the turbine to be run at approximately 2,600 degrees F uncooled. This turbine was designed to show that as the turbine inlet temperature goes up the power output of the turbine increases. For every 350 degrees F increase in inlet temperature the output horsepower of the turbine doubles. The next step in the development of this engine is the redesign and use of materials for the lowest possible weight and highest possible thrust. We are ready to do this as soon as funding becomes available. The next generation of engines should be capable of over 1000 lbs of thrust.



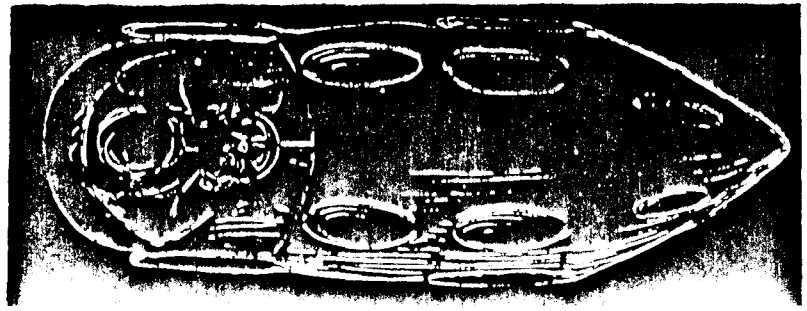
600 Pound Thrust Bladeless Jet Engine

The Phalanx

This is a photo of a revolutionary new aircraft being developed by All American Aircraft in Long Beach California. This aircraft is called the Phalanx. The Phalanx is a lifting surface body that rises vertically. It is a true Vertical Take off and Landing (VTOL) device. The one pictured is a four place device. It has a power system that is very similar to the English Harrier system that is now being used by the Marines in the United States who purchased two squadrons of these aircraft. The Harrier, however, is not a true VTOL device. They are a STOL (slow take off and landing) they need either a ramp or an induced wind to take off or land. It requires additional lift surface. The Phalanx device has enough power to take off and land vertically without any forward motion or any head-wind. This device can be flown with the most effective conventional bladed fan jet type engines. However, the intention is to ultimately power this device with the bladeless high temperature turbine.

This is a science that is very important and is coming into its own very shortly. It makes use of the rule that if you can raise turbine inlet temperature 350 degrees F, twice the horsepower is developed from the engine. It is a very significant statement to say the power will be doubled or tripled for a given size and weight of an aircraft engine. Today bladed turbines are limited to a turbine inlet temperature of around 2,000 degrees F. Yet when we burn fossil fuels, that are common today, we get temperatures up as high as 3,400 to 3,500 degrees F. We don't dare put those high temperatures on the blades of a bladed turbine. It would be too hot, resulting in the blades coming apart and a catastrophic demise of that particular turbine. Another disadvantage of the conventional bladed turbine aircraft engine is the tremendous noise created by this engine . This is one of biggest negative aspects of these engines. The noise is an unwanted parasitic energy that does not occur with the bladeless boundary-layer turbine.

So the bladeless turbine offers us a tremendous advantage. The bladed turbine operates under the most difficult stress level imaginable which is called combined bending at the root of each blade. The impulse or reaction that happens on the blade causes this very severe stress of combined bending to be applied to the blade. If the stress level is exceeded the blades will come apart resulting in the destruction of the turbine. The boundary layer turbine has a tremendous advantage in this case. The stress level is entirely different on the bladeless turbine than it is on the bladed turbine. There is no combined bending in the bladeless turbine. Therefore we can operate the disks, which take the place of blades, at much higher temperatures. The bladeless turbine can survive operating under conditions that a bladed turbine cannot. It is possible today to build a bladeless turbine that can operate at three time the horsepower of present state of the art bladed turbines. It has been suggested by many orthodox engineers that the blades can be cooled in conventional engines allowing them to operate under higher temperatures than is currently possible. Unfortunately the amount of energy saved by not cooling the blades is exactly equal to the energy that is required to blow the cooling air into the bladed turbine. We can forget about cooling the blades because the net power increase is zero. Any one that would spend money and time trying to cool blades just does not understand this very severe limitation. The bladeless turbine is going to run at these higher temperatures. We call this new turbine the Super High Temperature Bladeless Turbine. There are several other things that can be done besides raising tit (turbine inlet temperature). The overall thrust and efficiency can also be increased by raising the compression ratio of the compressor. That is true of almost any engine including the internal combustion engine of an automobile. If you raise the compression ratio the efficiency and power of the engine goes up significantly. In this case it goes up very rapidly. If we were to raise the compression ratio in a normal turbine from simple cycle (around five to one compression ratio to twelve to one or as high as twenty five to one) and raise turbine inlet temperature at the same time, we have a very unusually powerful, light-weight engine that will accomplish some of the missions that are waiting to be done in the future. One of the most important applications will be an aircraft that will fly at very high altitude and will be capable, with its thrust and power, of flying out into space without being launched by a rocket. It will literally fly itself out into space. That is a project that is now being contemplated. In the future a space shuttle will not require expensive and dangerous rocket engines to take it beyond the pull of gravity. The aircraft of the future will be able to fly in a proper orbit to leave the earth's gravity field without vertical ascent. It will cost much less to launch a space vehicle than it does today and with much less danger. The space plane will land under full power rather than gliding back to earth, further enhancing safety, allowing controlled reentry to any location.



A Tesla Sketch of a Turbine Powered Vehicle

Tesla Pump Patents by Possell & Gurth

Clarence R. Possell (srchd 66-84):		class:
3719836	pressure sensor	307
3636289	pressure sensitive electric switch	200
3848775	valve for liquid dispenser	222
4050636	water-driven Tesla-turbinelike garbage disposal	241
4043036	shaver, Tesla-turbinelike	30
4347032	slurry pump	415
4347033	concrete pump	415

3244255	jet engine noise suppressor	4-5-66	181
3279170	gas turbine power plant	10-18-66	60
4232992	geothermal turbine	11-11-80	415

misc. Possell patents: 4056209, child-proof bottle; 4186554, cl. 60; 4403911, cl. 415.

Max Gurth (srchd 70-91; should be six more?):

4335994	Meth. & App. for pumping large solid articles	6-22-82	415
4514139	M & A for pumping fragile aticles	4-30-85	415
4768920	Method for pumping fragile and other articles in liquid medium	9-6-88	415
4773819	Rotary disc slurry pump	9-27-88	415
4940385	Rotary disc pump	7-10-90	415

(Gurth supposedly claims to have 11 patents)

(also srchd Discflo 70-91)

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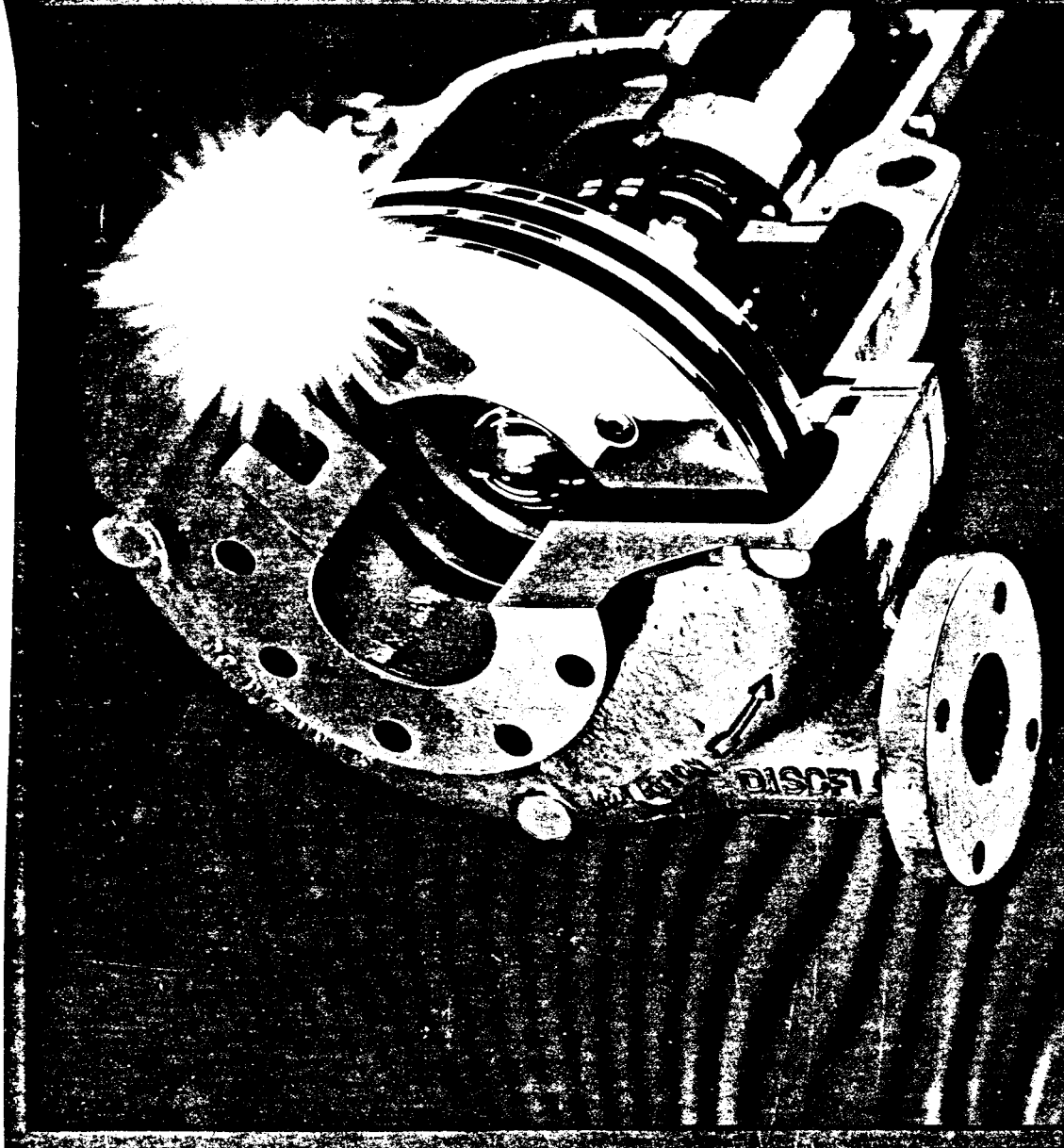

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DISCFLO...

A Quantum Leap in Technology

by **Steven R. Elswick**
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Extraordinary Science

&

JW McGinnis
President
International Tesla Society

A dramatic revolution in the pumping industry is taking place as rotary disc pumps (which most Tesla enthusiasts refer to as the Tesla Turbine Pump) take the marketplace by storm. Up until now, conventional technology consisted of positive displacement pumps and dynamic centrifugal pumps. Now a radical breakthrough in pump technology has resulted in a non-impinging, product friendly pump that is the solution for previously unsolvable pumping problems.

While many of the basic concepts and principles of operation on which the disc pump is based were developed around the turn-of-the-century by Nikola Tesla, it took intensive research and a series of bold unprecedented experiments by Max Gurth, founder and president of Discflo Corporation, to turn those concepts and principles into reality—to create the technological breakthrough that is the Discflo Pump.

Principles of Operation

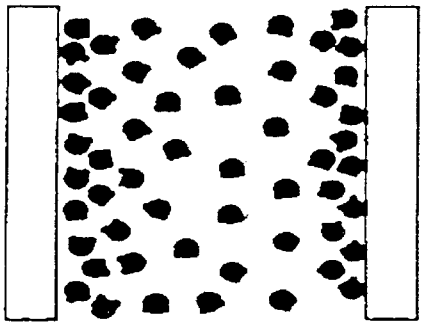
Unlike normal pumps that act in opposition to Nature, the Discflo Pump operates on three of Nature's own dynamic forces—boundary layer, viscous drag and laminar flow.

In **US Patent 1,061,142**, is Nikola Tesla's description of the boundary layer:

> *It is well known that a fluid possesses among others, two salient properties: adhesion and viscosity. Owing to these a body propelled through such a medium encounters a peculiar impediment known as "lateral" or "skin resistance", which is two-fold; one arising from the shock of the fluid against the asperities of the solid substance, the other from internal forces opposing molecular separation. As an inevitable consequence, a certain amount of the fluid is dragged along by the moving body. Conversely, if the body be placed in a fluid in motion, it is impelled in the direction of movement.*

Michael Faraday, in his famous lecture ***The Forces of Matter,*** illustrated the internal forces that tend to hold matter together and described it as *cohesion.* It is through cohesion that the energy is transmitted from one layer of matter to another. A measure of cohesiveness in fluids is *viscosity.* For example, oil tends to stay in one mass better than water does. The more viscous a material is, the greater is its ability to suspend material placed in it by resisting the material's movement through the fluid.

Many of us have witnessed the Prell Shampoo commercial in the 1960s where the sphere is dropped in the bottle to show the "richness" of the shampoo. The ball fell through the shampoo slowly demonstrating the high viscosity of the shampoo. It is easy to imagine this fluid carrying lots of these little balls when it moves. This ability to suspend material is *viscous drag.*



The initial boundary layer attracts and "drags" along additional layers of molecules!

Due to the manner in which movement is generated within the fluid, close observation reveals that the flow is actually layered, with each layer flowing at a different speed but in the same direction as an adjacent layer. This is known as *laminar flow*. By definition, laminar flow is a smooth, gentle type of flow in which there are no abrupt interruptions or changes in direction.

This is possible in the bladeless turbine pump (or as some prefer—disc rotor pump) because there are no blades to block the flow as in centrifugal and other bladed pumps. In bladed pumps, the flow of the fluid is interrupted when the fluid hits the blades in the pump, thus eliminating any semblance of layers within the flowing fluid. The interrupted flow is termed *turbulent flow*.

Primarily due to the way scientific thought and theory was based on intense observation of natural phenomenon, scientists during the Victorian age were referred to as *Natural Philosophers*. So it would be obvious to the scientists and researchers of that age to note that the most natural form of movement in Nature is a *vortex*.

A vortex is the most powerful and efficient manner of transforming the direction of energy flow, for this reason it is Nature's method of choice in changing the direction of unobstructed forces. This is readily observed in tornadoes, whirlpools and hurricanes.

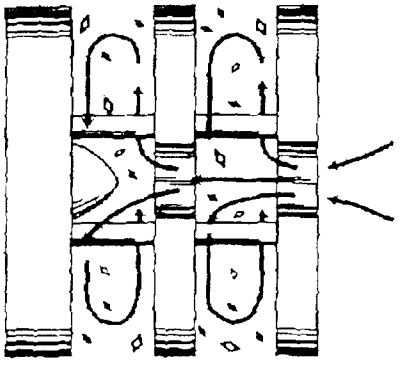
Although Tesla refers to the importance of a spiraling flow of fluid through the pump, Victor Schauberger is perhaps the most significant expert on vortices and vortexian flow. In the book, Living Water, by Olof Alexandersson, Schauberger explains the different forms of the vortex:

The form of movement which creates, develops and purifies and grows is the hyperbolic spiral which externally is centripetal and internally moves towards the centre. We find it everywhere in Nature where growth or movement is taking place, in the spiralling of nebulae in space, in the movement of our planetary system, in the natural flow of water, blood and sap.

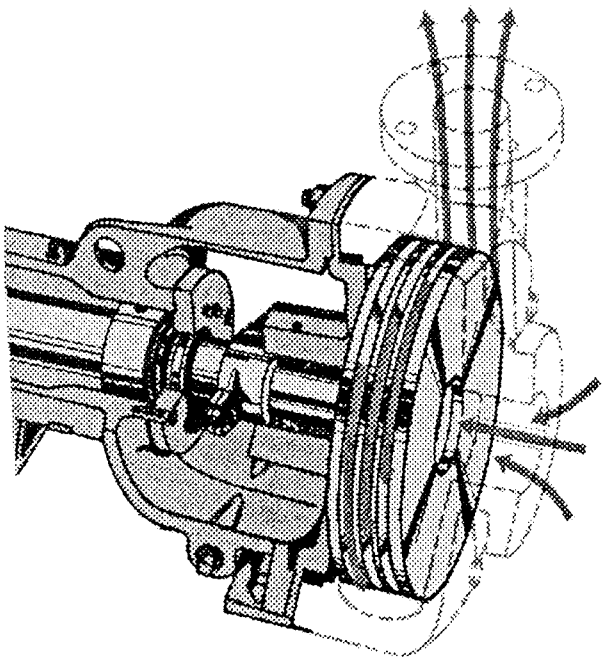
On the other hand, the destructive or dissolving form of movement is centrifugal in Nature—it forces the moving medium from the centre outwards towards the periphery in straight lines. The particles of the medium appear to be forced out from the centre. The medium is first weakened, then it dissolves and breaks up. Nature uses this action to disintegrate complexes that have lost their vivacity or have died. From the broken down fragments, new co-ordinated forms, new identities can be created as a result of this concentrating form of movement.

The centripetal, hyperbolic spiral movement is symptomatic of falling temperature, contraction, concentration. The centrifugal movement, on the other hand is synonymous with rising temperature, heat extension, expansion, explosion. In Nature, there is a continuous switch from one movement to the other; but if development is to occur, then the movement of growth must be predominant.

The bladeless turbine pump (or as some prefer—disc rotor pump) embodies both types of movement. It is through the outward spiraling that energy is imparted from the disc to fluid. In this mode it is an effective pump. Conversely, in the inward spiraling movement energy is imparted from the fluid to the disc making it into an efficient generator—this is covered in much greater detail in the Tesla Turbine patents. Thus Nature's balance is evident in this apparently simple and geometrically elegant device.



The fluid is drawn through the central orifice of the pump and confined between the parallel discs of the rotary discpac.



Discflo Pumps utilize the powerful forces of Nature--boundary layer, viscous drag and laminar flow.

The Vortex Controversy

There seems to be a lot of confusion between the terms *turbulent flow* and *vortexian flow*. This is partially due to a misnomer prevalent within the pumping industry since the introduction of so-called *vortex pumps*. The "vortex" pumps profess to utilize the power of a vortex, but in the past an improper understanding of fluid dynamics resulted in poor system design that damaged products. As a result, many engineers in the pumping industry cringe when they hear the word "vortex." ***The rotary disc pump is NOT a vortex pump!***

The *Reynolds number* is indicative of the relative importance of inertial and viscous effects in the fluid's motion. It is based on the velocity of the fluid and its viscosity, therefore it is different for every fluid and application. At low Reynolds numbers, viscous effects dominate and the flow is laminar. At high Reynolds numbers, the inertial effects lead to turbulent flow characterized by eddy currents within the mainstream of the fluid.

A true vortex is not a turbulent flow—the Reynolds numbers do not determine whether there is a vortex or not. Rather, hydrodynamic analysis of fluid flow attributed to Helmholtz clearly indicate that in order for a true vortex to occur, laminar flow must be present. A true vortex has many filaments (or layers) which are the locii of the centers of circulation (a spiral). These filaments must be continuous and the strength of the vortex is constant along filament. Obviously, if the flow is disrupted by turbulence it is not constant.

Vortexes are natural phenomenon that occur whenever there are changes in temperature or pressure. It describes the **direction** of fluid flow... not the manner of flow. Unfortunately, this misunderstanding often prevents the proper study of vortexian motion in fluid dynamics.

Overview of Pump Operation

The pump itself has a fairly straightforward mode of operation. The fluid is drawn through the central orifice and confined between the parallel discs of the rotary discpac. The layers that adheres to the rotating discs, generate within the fluids themselves, pressure gradients by which the entire fluid mass itself begins to rotate. As the fluid gains energy it moves outward to the edge of the disc—and beyond. The actual path of the fluid through the pump thus becomes a spiral path—in other words, a vortex.

Boundary layer occurs when the molecules of the fluid attach or "lock onto" the surfaces of the rotating discs. The force of this rotating boundary layer is then transmitted to adjacent layers of fluid molecules—generating pressure gradients and a dynamic force field within and between the molecules. It is here that the principle of viscous drag comes into play—for the energy transmitted to the boundary layer is then transmitted to other layers of the fluid's molecules until the entire mass of fluid is rotating carrying other materials immersed (or suspended) in the fluid with it.

Thus, the initial boundary layer attracts and 'drags' along with it additional layers of molecules setting in motion numerous parallel streambands of physically interacting molecules. As the kinetic energy from the discs is imparted into the fluid, the fluid moves in a spiralling unobstructed outward flow.

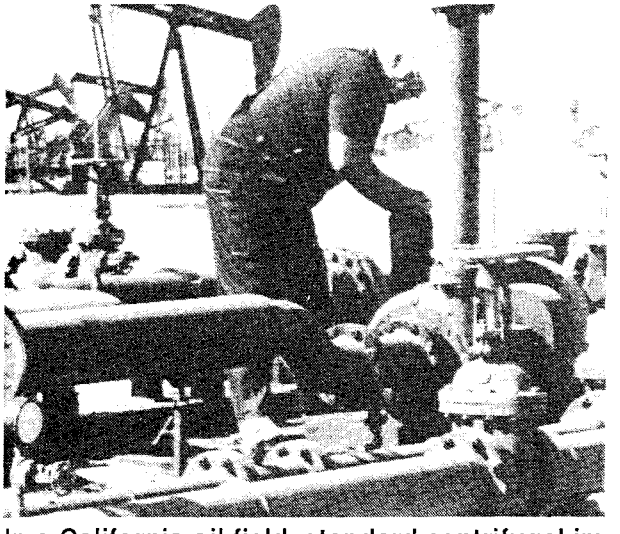
This combination of boundary layer, viscous drag, and vortexian motion thus generates a powerful dynamic force field which propels the fluid naturally, producing a smooth hydraulic flow profile and a non-pulsating operation.

Moreover, not only does the boundary layer transmit kinetic energy from the moving surface into the fluid, but it acts as a "molecular buffer" between the disc surface and the fluid contents. This protects both the product from the full impact of the disc and the disc from the full impact of the product—thus eliminating excessive product damage and pump wear.

Characteristics of the Discflo Pump

A genuine innovation in the pumping industry, the Discflo Pump combines the best features of both the dynamic centrifugal pump and the positive displacement pump. The result is a simplicity and a versatility unmatched in the pumping industry.

The Discflo Pump is designed to handle a wide variety of difficult fluids more efficiently and effective than ever before thought possible... and not constantly break down in the process.



In a California oil field, standard centrifugal impeller pumps which were pumping diatomaceous earth, sand, oil and water (60% solids) required a complete rebuilding every 15-30 days due to high wear. For the same operation, the Discflo Pump has run continuously with no pump-related breakdowns! Similar reports are coming in from those who are pumping lime slurries, steel mill wastes, coke slurry and/or highly abrasive slurries.

In conventional pumps, if abrasive substances are being pumped, there will be a tendency for the pumped material to abrade, or dig into, the lifting surface of the pump--this phenomena is known as *impingement.* After, an extended period of time pumping abrasive materials, the pump's performance will be degraded to the point it no longer works.

Alternatively, conventional pumps face the problem of *cavitation.* Cavitation occurs where liquids with bubbles of gas entrapped in it is pumped. (Often, bubbles are formed in pumps themselves as a pressure interface is created between the high pressure and low pressure areas by the pumping action.) When a lifting surface hits the bubbles of gas, the bubble collapses (or implodes). As the liquid rushes in to fill the area left by the collapsing bubble, the inrush digs away at the lifting surfaces. In extreme cases of cavitation, pumps can be left looking like lace curtains.

The secret of the Discflo Pumps is the non-impinging disc rotor that does away with lifting surfaces! The absence of lifting surfaces in the Discflo Pump virtually eliminates the cavitation and impingement problems encountered by conventional pumps. An additional benefit of the bladeless rotor is the ability of the pump to "swim" through blockages without damage!

In contrast to both the impeller driven centrifugal pump and the positive displacement pump—both of which impact and push fluids through the system—the Discflo Pump generates powerful dynamic forces which pull fluids through the system. The results are dramatic! Its reliability and durability on difficult applications is unsurpassed by any known process pump on the market today.

Throughout the Industrial World, Discflo Pumps are showing superior performance in handling the following difficult fluids:

Severely Abrasive

For highly abrasive fluids the "impact/push" principle is a direct cause of the high wear rates experienced by other types of pumps. In the disc pump the same bound-

ary layer that generates the dynamic force field that pulls the fluid through, also creates a "molecular buffer" between the fluid and the disc surface. Because the boundary layer adheres to the disc surface and rotates at the same speed, abrasive particles are kept from impacting and grinding the rotor—significantly reducing rotor wear and the high costs of down time and repair. In addition, there is virtually no performance loss as the result of rotor wear.

Highly Viscous

Viscous liquids (100 cps) have a very adverse effect on conventional centrifugal pumps—making them ineffective devices for use with viscous fluids. The Discflo Pump, which requires no close tolerances, is designed to handle stubborn viscosity problems. The key here is that the discpac can be engineered according to viscosity, density and solids. When you add this versatile design feature to the powerful boundary layer/viscous drag forces generated by the innovative disc technology, there is little wonder why the Discflo Pump is so effective on highly viscous fluids. In fact, with Discflo, the higher the viscosity, the more efficient the performance!

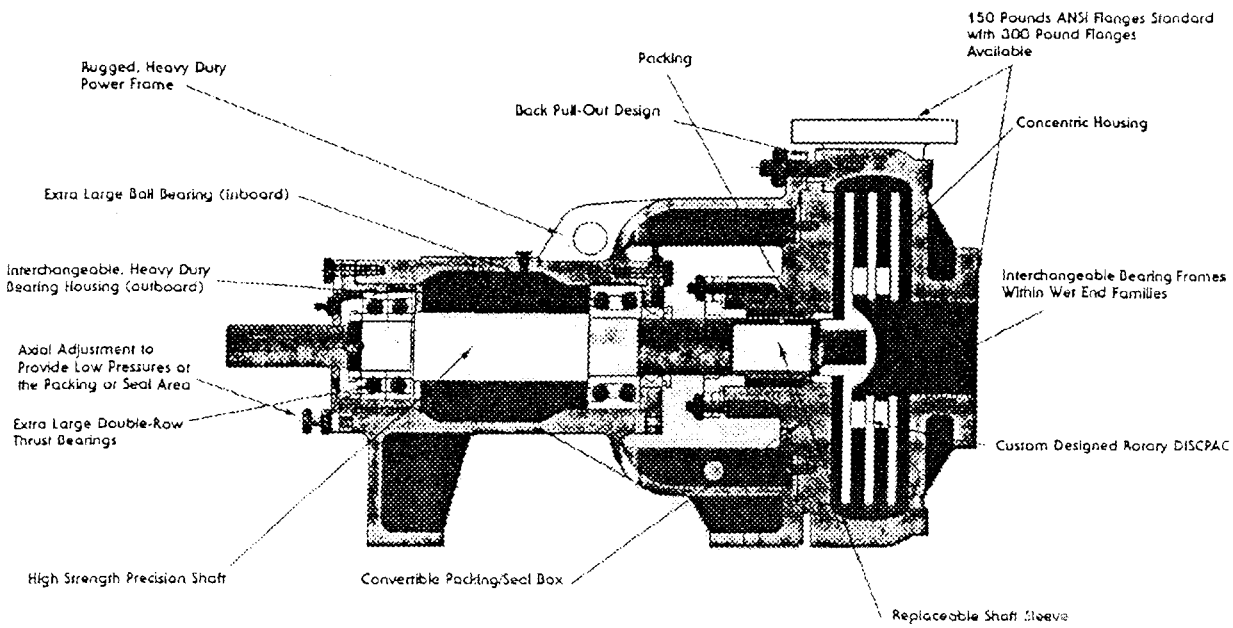
Pluggage Prone

Wastewater treatment, paper stock and meat packers are just some industries which have notorious problems with pump pluggage. Stringy, bulky materials, such as rags, hair, entrails, plastic bands, etc., all clog impellers (even recessed impellers) and progressive cavities on a regular basis. Discflo's revolutionary open design discpac runs virtually clog-free—handling with ease high solids content fluids (over 80% by volume) and large soft spherical and stringy solids. The reason it runs virtually clog free is because the Discflo Pump is a non-impingement device—with no close tolerances-which allows the particulates to stay suspended in the fluid throughout the system.

Delicates and shear sensitives

Delicates and shear sensitive fluids receive sympathetic treatment by Discflo pumps. Because there are no vanes or paddles, there is no impact on delicate or shear sensitive particles.

At a major corn chip plant, the Discflo Pump has virtually reduced product loss to zero. The savings are in the millions of dollars. The disc pump has almost totally


150 Pounds ANSI Flanges Standard with 300 Pound Flanges Available
Packing
Rugged, Heavy Duty Power Frame
Back Pull-Out Design
Concentric Housing
Extra Large Ball Bearing (inboard)
Interchangeable, Heavy Duty Bearing Housing (outboard)
Interchangeable Bearing Frames Within Wet End Families
Axial Adjustment to Provide Low Pressures at the Packing or Seal Area
Extra Large Double-Row Thrust Bearings
Custom Designed Rotary DISCPAC
High Strength Precision Shaft
Convertible Packing/Seal Box
Replaceable Shaft Sleeve

Key Features of the Extraordinary Discflo Pump

eliminated maintenance. Where previously a good deal of the product was macerated, now less than .2% is lost.

Air/Gas Entrained Fluids

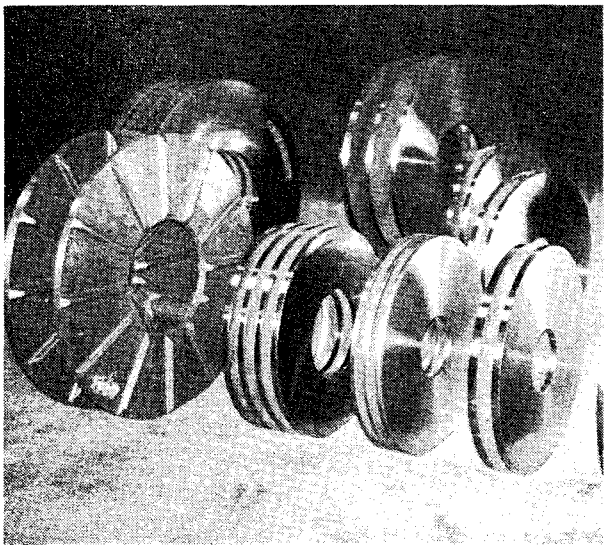
Because the Discflo Pump does not require close tolerances and mechanical impellers, it can handle fluids entrained with air and gas without destruction to the pump and system due to cavitation or by running dry. However, careful attention must be given to pumps using mechanical seals that they be lubricated at all times.

There are numerous installations where Discflo pumps are being used which are processing as much as 70%+ air and gas entrained fluids. At one installation in the mining industry, a company is processing air-entrained frothy mineral slurry that just could not be handled by standard pumping equipment... and it is doing so without costly repairs and down time.

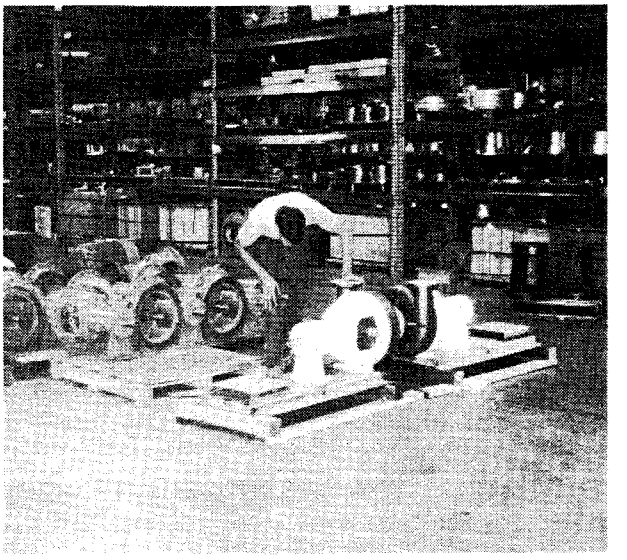
Features of the Discflo Pump

The simplistic design of the pump lends itself to a number of features which make it superior to conventional pumps. These features combine to make the Discflo Pump the most cost effective pump on the market today.

The reason again lies in its revolutionary disc technology. The boundary layer laminar flow



The revolutionary discpacs are available in various styles and sizes.



Each pump undergoes intense scrutiny and testing—*before* shipping.

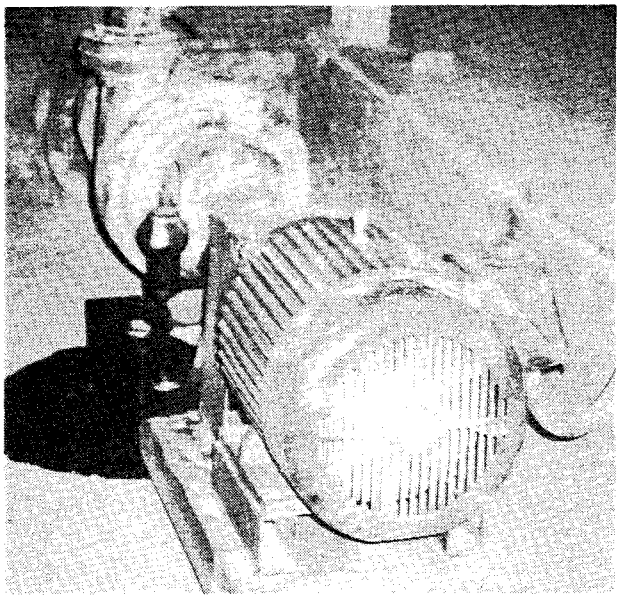
created by the rotary discpac generates not only a dynamic force field which propels the fluids, it also creates a "molecular buffer" between the surface of the disc and the contents of the fluids being pumped. This protects the disc from the fluid (resulting in minimum pump wear). The savings in downtime and repair costs have been 60% and more annually.

The revolutionary discpac enables the design engineers to rapidly resolve the various pumping problems their customers encounter--everything from shear sensitives and frothy liquids to heavy viscous and abrasive slurries. With the discpac technology they can configure a pump that will handle the job properly and quickly.

Truly quality-oriented, Discflo will not even discuss building a particular pump until they know the fluid being pumped and head and system requirements.

In addition to the standard plain discpac, Discflo also manufactures a High-Head Discpac which incorporates a number of straight radial ribs on inside surfaces of the discs. For special applications, the High-Head Discpac provides significantly higher flow rates and pump efficiencies.

But the long life of the Discflo Pump results from more than its disc technology. The pump contains a heavy duty shaft which is combined with



Discflo Pumps are designed tough... To last in tough environments!

close to zero axial and radial loading to give less than 0.002 inch shaft deflection under full load, full speed pumping. This translates to extra long life for seal, shafts, shaft sleeves and bearings. When lubricated according to manufacturer's specifications, the bearing are designed to run over 80,000 hours before 10% of the bearings would theoretically begin to fail.

The Discflo Pump is a well-designed pump with ease of maintenance in mind. The back pull-out design of the pump allows easy removal and service of the bearing frame and rotating assembly without disturbing piping connections. The rugged, heavy-duty power frame provides maximum mechanical reliability, performance and pump life. The convertible packing/seal box was designed to accommodate virtually any packing arrangement size or style, and all single or double mechanical seals in US and metric sizes–no mean feat in itself!

Many of the components used throughout the pump are designed for interchangeability, which allows for minimum inventory costs and maximum standardization of parts. The heavy-duty, rugged, high-quality parts used throughout helps provide a degree of reliability unmatched by other manufacturers. All of these features combine to give an incredible savings in downtime and repair costs.

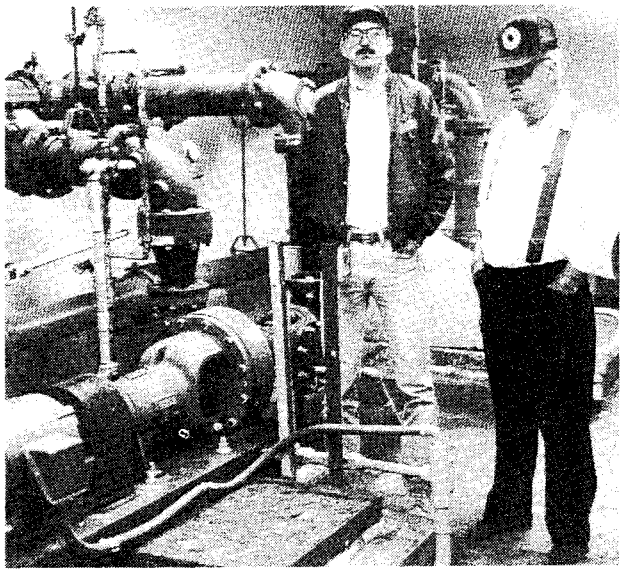
Low Maintenance=High Savings

A supplier of paint intermediates in Knoxville, TN. Their products are shear sensitive. For years their major problem was the cleaning of the spent solids treatment basin. Prior to their conversion to Discflo pumps, their monthly cleanup took four men two days... with two of the men assigned to permanent standby just to keep pumps running. Since installing a Discflo Pump, downtime has dropped from two days to three hours and replacement costs have dropped from $3,000 to zero dollars annually. The total cost savings as a result of using the Discflo Pump is more than $57,000 annually.

Not only increased savings... but increased productivity... results when companies use the little-to-no-maintenance Discflo Pump. At a major laundry detergent company in California for instance, the Discflo Pump increased production by 22% because there were no more "routine maintenance checks."

The company manufactures dry laundry detergent.. .involves spray drying of hot (180 degree F) corrosive, extremely abrasive, high solids (75-80%), high viscosity (4000-6000 Cp), and heavy (1.3) specific gravity slurry.

This combination of "bad actors" spelled high maintenance and early death for the industry



Before Discflo Pumps were installed, we had our hands in the sludge. After Discflo pumps were installed, we have our hands in our pockets.

standard positive displacement gear pump the company used for years. As the gear pump "wore in" pressure fluctuations, consistent but random downtime led to a multitude of mechanical, production and quality problems downstream.

The company installed a Discflo Pump, and has since then been pumping any combination of high density , high viscosity, abrasive, aerated liquids with almost zero maintenance and zero pulsation. From their original $750 per hour downtime costs to the now zero downtime, they have retrieved the cost of the pump in less than a month. at the same time, their zero maintenance has resulted in a production increase of 22%.

Case Histories of Discflo Users

Today, many industries must process aggressive and hard to handle fluids. The answer for an increasing number of companies is the disc pump. Discflo Pumps have a reputation of simply outperforming all other types of turbo-machines when it comes to handling difficult fluids and they do so by increasing long term efficiency and productivity and reducing down-time and the high cost of maintenance and product loss. The following examples from Discflo, testify to the wide, successful use of their pumps by their customers.

Wastewater

For three years the operations manager at a wastewater plant in Texas has used a Discflo Pump to backup all of his service pumps...from incoming sewage through activated sludge, grit, scum, and digested sludge. After three years, in all of these services, there have been zero failures with Discflo.

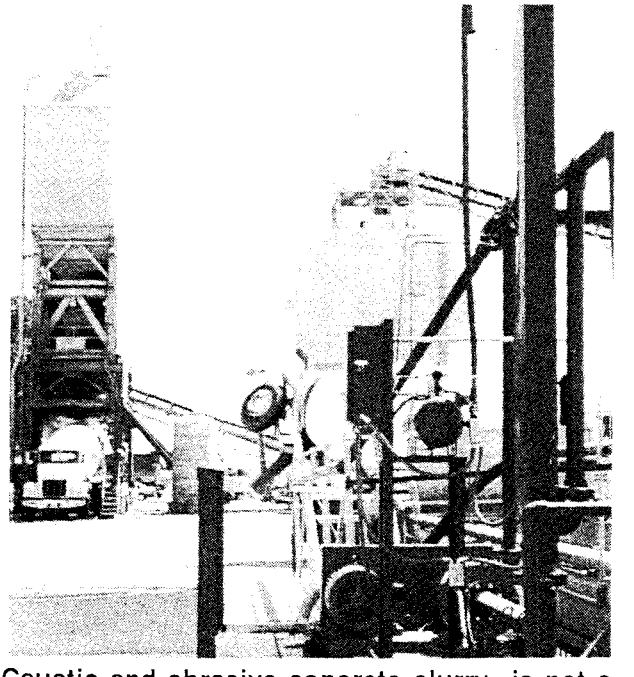
At a Midwest facility, digested sludge with 18-27% solids eroded every pump tried... until they installed a Discflo. For the last several years the Discflo has operated without failure, pluggage or excessive maintenance expense. In addition, with Discflo Pumps, the plant has been able to eliminate the use of dilution water to pump these heavy solids.

The first reaction at a Minneapolis waste water plant was, "No way are we going to install that funny looking pump that the front office ordered. It can't work, we'll just have to take it out again." When Discflo's field engineer showed up and helped install the pump they were amazed. When the pump handled their sludge and grit, they were impressed. Currently the plant has several Discflo Pumps...and they are satisfied with the return on investment received.

Chemical/Petrochemical

For over ten years at a major petro-chemical plant in Texas, the pumps used to transfer filter cake slurry to disposal containers experienced frequent failures and excessive repair costs. The slurry was routed to disposal containers using progressive cavity type positive displacement pumps. Since it was impossible to strain out entrapped material from the thick slurry, solid foreign matter would gouge or tear the rotors. Shutdowns were required once or twice a week, at an average of $650.00 for parts and labor. A 2 week run was considered good, and there were up to 5 hours of lost production. Disposal costs for the sludge were also high.

To resolve these problems the plant's engineers installed a Discflo Disc Pump. After 4 years of run time it performed so well, they ordered two more disc pumps. After a year and a half, the pumps were operating without any problems. Maintenance costs are essen-



Caustic and abrasive concrete slurry is not a problem with Discflo Pumps.

tially nil...resulting in a savings estimated at $65,000 per year. In that time there were no mechanical failures or shutdowns to replace worn parts, and no lost production.

A major Bakersfield, California oil field operator has found that Discflo pumps deliver more than a three for one advantage. Prior to discflo this company used three large progressive cavity pumps to transfer wellhead, oil, sand and water slurry. They commissioned an 8 month extensive test of Discflo versus progressive cavity pumps. Their findings showed that for the same service: the progressive cavity pumps cost over $63,000 in parts plus labor, plus production shutdowns. In 4 months of testing the progressive cavity pump failures lead to 3 oil field shutdowns. Conversely, Discflo had zero breakdowns, zero maintenance costs and zero production down time.

"Our impossible service eats pumps alive. Not only does the high gas volume stall our pumps, the abrasives wear them out at an astronomical rate....one repair every two weeks per pump. "We've tried every pump we know of to no avail." That was before Discflo. One Discflo Pump replaced two of their "best" constant maintenance pumps and for the last 2 years Discflo has sold no spare parts to this Canadian oil refiner.

Food and Beverage

For years a fruit juice company had suffered through continuous failures of their progressive cavity pumps. Their boneyard had piles of stainless steel pump skeletons. The incoming mash of high viscosity grapes, juice, skins, seeds, air and stems was just too much trouble for the progressive cavity pumps to handle. Their plant engineer found it hard to believe when from day one, a Discflo pump replaced two progressive cavity pumps and exceeded the juice company's requirements while requiring zero maintenance.

A sugar company has turned Discflo into a competitive advantage. After installing a Discflo pump, maintenance costs were greatly reduced and they greatly benefited from Discflo's no shear pumping technology. Now 99% of the pumped sugar meets or exceeds premium grade quality standards.

One of the world's largest snack food suppliers used to have trouble transferring cooked corn for processing into corn and tortilla chips. It was normal to have 12-17% product breakage for lobe type pumps and air pressure transfer. After one year of research and in-plant tests Discflo pumps gave incredible results... less than 0.2 percent breakage, resulting in a very significant 92% savings in product degradation.

Cement

Hawaii, California and Detroit cement plants all had the same problem. How do you get a pump to pump a cement truck washout for recycling? The pumps these plants were using constantly wore out while trying to lift sand, rocks and cement up from a pit to the recycle tanks. Discflo Pumps were installed and have been operating for the past decade without any pump-related failures.

"How can we pump recycled cement slurry fast and without pump downtime? This was a problem that went unsolved until we bought a Discflo Pump." says the manager of a transit mix company in California.

The slurry is often thick and is always abrasive. This always spelled death for all the normal technology pumps we tried. More than 2 years ago we purchased a Discflo Pump that uses a revolutionary boundary layer/ viscous drag technology instead of the normal push principle. Since that time the Discflo has successfully pumped whatever we feed it. No matter how thick or thin, the Discflo runs and runs and runs. In over 2 years in this service we have not needed to replace any parts. It's an amazing pump and we're sold on it!"

Printing

A printing machinery manufacturer in Pennsylvania made a breakthrough when they switched pumps. "The diaphragm ink pumps required constant maintenance. Discflo handled that without any problem." The real surprise to the production manager was the quality enhancement. "We used diaphragm pumps because of their supposed low shear. When we converted to Discflo the ink color brightened noticeably to the eye. No shear Discflo pumping made the difference."

Seeing is believing. A booth set up at a trade show lets the pumps virtually sell themselves. There will be a Discflo exhibit at the upcoming Tesla conference in July.

A major paper coating company told Discflo: "Your pumps have eliminated shear in our dilantant Calcine clay paper coating operation. This not only translates to higher quality paper, but also to significantly decreased costs. Discflo gives us a considerable competitive advantage!"

The Cutting Edge

Not content to rest on its laurels, Discflo is constantly seeking out new applications to apply this amazing technology to. In order to meet the challenges of tomorrow, the Discflo non-metallic pump was developed. In today's manufacturing sector, there is an explosive growth in the field of aggressive fluids..and a growing need for high safety assurance. The non-metallic pump is a proven success in resistance, performance and application. Many of the same features in the standard pump are carried over in the non-metallic pumps.

In addition to the horizontal pump, they have also developed a totally integrated range, such as self-priming pumps, vertical pumps, sump pumps and armored pumps for extreme conditions.

Battle of the Patents

In 1913 Nikola Tesla was awarded US Patent No. 1,061,142 for the Bladeless Pump. Lacking resources, Tesla was unable to commercialize or to do extensive testing on his bladeless pumps. The world had to wait half a century before some serious and unprecedented experiments led to the Discflo pump—experiments that proved the experts wrong and went far beyond Tesla's claims.

Today, Max's company, Discflo Corporation is the sole patent-protected manufacturer of the revolutionary rotary disc pump. Discflo pumps are covered and protected by United States and foreign patents. These patents cover optimum designs of the discs, castings and relationship ratios. They also cover the spacing of the discs in relation to the range of fluid requirements.

It took eleven years of filing documents, conferring with officials and countless trips to the Patent Office in Washington D.C. to convince the authorities that the use of boundary layer technology proposed in Max Gurth's patent claims described a new concept—differing from earlier patents dating back to the late part of the 19th century.

Throughout this decade of effort the so-called boundary layer technology experts claimed that the concept of increasing the space between the discs beyond 0.025-inches would simply cause

the discs to spin ineffectively. They surmised that if the spacing was opened up, energy would not be transmitted efficiently from the spinning discs to the fluid surrounding the discs. Research by Mr. Gurth established that this was not the case and, in fact, the spacing could be opened up to several inches (as much as 19" or more!) without causing the boundary layer principle to fail. Max Gurth proved them wrong and went on to win the initial patents. He has gone on beyond all of this to secure nine patents in the United States and abroad with additional claims pending.

It is a testimonial to the strength of Max's patent claims that there have been no successful challenges to his claims. Due to his diligence and attention to detail, Max was able to do the impossible and bring to full fruit the realization of Tesla's dream.

The Bottom Line

It was a pleasant surprise to us that the market and product was so well developed in the field of bladeless (rotary disc) pumps—especially when all of the information we had received was to the contrary.

Many of the testimonials used were verified independently by phone calls from the staff at *Extraordinary Science*. Our verification of the testimonials, an on-site visit to the manufacturing facility and the delivery of a rotary disc pump to the museum within two weeks gives us the confidence to report to our readers that these Discflo pumps are **real** and **available** commercially.__**SRE/JW**

For information on Discflo Pumps contact:

Extraordinary Technologies, Ltd
PO Box 5382
Colorado Springs, CO 80931
(719) 632-6779

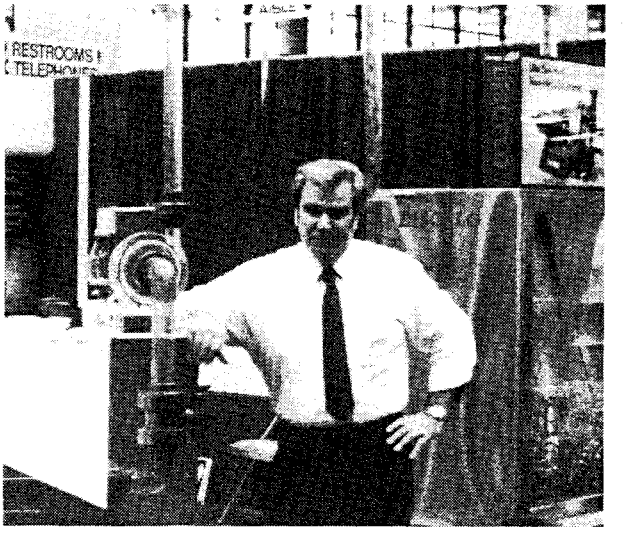
Register TODAY !!!

1992 International Tesla Symposium

(719) 475-0918 or FAX (719) 475-0582

Inventor Profiles:

MAX GURTH...
Bladeless Pump Pioneer



Max Gurth is the inventor of the revolutionary Discflo Pump, the commercially-viable advancement of the disc pump concept. Max has been in the design engineering and manufacturing business since his start in the equipment manufacturing business in 1965.

From 1970 to 1979, he owned and operated Brown Tool Engineering, Inc. which machined and assembled the Centaur and Mars turbines for Solar Turbines Inc. Brown Tool Engineering also manufactured aircraft and marine parts for such customers as Boeing, McDonnell Douglas and the US Undersea Lab. Brown Tool employed as many as 300 people.

Early in the 1970s, Max became aware of boundary-layer technology and his advancement of this concept led to his development of the commercial disc pump. Max was profoundly impressed with the market potential for his pump and in 1976 began to shift his efforts more and more toward the development and manufacture of the rotary disc pump. At this time, he founded Dicflo Corporation and became its President and CEO. For the past fourteen years, he has been engaged in the design, development and patenting of the Disc Pump and related technologies. Totally committed to Discflo—he has divested himself of all outside interests—Max has been the driving force behind the incedible success of the Discflo Pump.__SRE/JW

Unraveling the Web...

Debunking the Mythical Possell Bladeless Turbine!

by: The Plumbing Staff at *Extraordinary Science*

The Society's Position

The greatest impediment to the implementation of new technology is the appearance of "charlatans and scoundrels," in the field of interest. The broad fields encompassed by Tesla Technology are no different. It has been brought to the Society's attention that there are a number of charlatans operating under the umbrella "Tesla Technology" and implying that they are endorsed by the Society.

We wish to remind our members that the Society is merely a *platform* on which new and unusual ideas can be presented. These presentations do not represent an endorsement or proof that this is a "good" idea—and we caution our members to be prudent in any transaction involving money.

Through our magazine, Extraordinary Science, we introduce our membership to many new ideas and concepts. Unfortunately, new and little understood concepts form a target of opportunity for charlatans, to bilk millions out of unsuspecting investors, many who are willing to put their neck on the line for altruistic (yes there are still some that have these ideals) and great financial rewards. In an effort to give our members a more accurate picture of cutting edge technology we feel that it rests with us to inform our members when we uncover fraud, misrepresentation and misinformation in a Tesla-related field.

We are not intent upon becoming a *National Enquirer*, or a gossip mill. Information presented in our exposè articles will use legal documents, news articles and—in some cases—written testimonials and statements from parties actually involved.

In the rare case where you may have evidence of fraud or misinformation related to Tesla Technology or a related field, send it to us. We will check it out to the best of our abilities, and if it appears to have merit, we will publish our findings. Anonymity will be maintained except in the case of testimonials which will be used to help verify claims. Your concern could help save a fellow member from making a tragic mistake.

The Bladeless Turbine Myth Exposed

At the 1986 International Tesla Symposium, Jake Possell made his premiere presentation on the virtues of the Tesla Turbine and the work that he has accomplished on it. During this lecture, Jake introduced us to new concepts such as the Phalanx aircraft of which he stated there were five in existence and they were going to be in the air by the end of the year. He regaled us with anecdotes of tests on the turbine which proved how reliable it was... and he informed us about his invention...the bladeless pump.

In 1990, Jake came back for a repeat appearance at the urgings of Jeff Hayes, author of Boundary Layer Breakthrough. Jeff had learned about Jake's magnificent turbine from an audio tape of the 1986 Symposium. Impressed, Jeff personally transcribed the lecture and put it in the July/August/September issue of the Journal of Power and Resonance, the International Tesla Society's newsdigest at the time. Early in 1990, Jeff went to California to meet Jake and see this marvel for himself. After sending back glowing reports of Jake's work, Jeff Hayes returned to Colorado Springs to finish his book on the turbine.

At the 1990 symposium, once again we were mesmerized with stories about this wondrous turbine. This time there was an added twist... the Tesla Turbine was nearly ready for production. The catch... they needed orders for 50 turbines, and they only had orders for more than 25. Each order placed was to have a $25,000^{00}$ down payment. To accelerate the process, Jake's associates went to the 1990 Osh Kosh Air show to promote and sell the turbine.

Now, it is 1992. There are no Phalanxes in the air, and it is doubtful that Jake ever built a prototype turbine engine that ever worked! Instead, it is beginning to appear that we may have been duped.

After the symposium, we were receiving phone calls from members wanting to contact Jake Possell. The information was published in Extraordinary Science. Then we began receiving complaints that Jake never returned their phone calls and was never there to take them. In 1991, when Jeff returned from California, we queried him on that issue. Jeff indicated the probable problem was that Jake had sold his rights to the Ali Baba to a Japanese firm and retired from the turbine/pump business.

In March of 1992, we received a phone call from Max Gurth, inventor and holder of the Discflo patents. Max had learned about the Society through a legal

action he was pursuing with an Asian corporation whom had the misfortune to become associated with Jake Possell. The Asian corporation had built pumps to Jake's specifications, but they did not perform in the abrasive applications for which they were designed. The Asian company faces financial ruin and a bleak economic future because of their faith in Jake Possell.

Realizing the seriousness of the situation, the Society started investigating the activities of Jake Possell and associates. What began emerging was a mind-boggling sequence of events that seemed so incredible that it could have only come out of a novel. To simplify the process, a timeline of various transactions that Jake and/or his various corporations have been involved in has been outlined. We have only used written material to establish this timeline—the source is on each item.

We are still receiving material on many of the incidents that have been *alleged* in the timeline. One of the more critical is the geothermal station at Fallon (Nevada) Naval Station. A Senate investigation into that contract was to have taken place. One of the Senators from Colorado has been contacted for verification on the Senate hearings and a transcript (or summary report) of that Senate investigation. We will let you know his reply.

The Society is not making a determination of guilt or innocence. In fact, Jake should be presumed innocent until a court of law determines otherwise. We are presenting information from official documents, memos, and previously published material that we believe the readership of Extraordinary Science would be interested in having before proceeding on a business venture concerning bladeless pumps and turbines.__*TPS*

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Jake Possell Timeline

Jan 1973 - Letter (1/26/73) from the Law Offices of Howson & Howson to Roy Stiger of Sprout, Waldron & Company indicates:

Possell Patent #3,279,170 was reviewed by the law firm. The claims of the patent were limited to a rotary turbine compressor having the plurality of disc rotors and fuel burners as described in the patent. The law firm speculated that Possell may have tried to obtain claims to the air or liquid movement by such a device but may have been precluded by Patent #1,061,142 granted to Nikola Tesla in 1913. As of December 6, 1972, the record title to the patent was to Jake Possell.

Mar 1976 - Order Suspending Trade Issued (March 3):

Order #3014044 was issued suspending trade of General Ener-Tech for 10 days in the public interest and for the protection of investors.

Jan 1982 - An Industrial Revenue Bond Application was filed January 21, 1982:

Filed on behalf of General-Ener-Tech, bond was to be used for the construction of a pump manufacturing plant.

May 1983 - Claim published in *Wall Street Journal* (5/3/83):

General Ener-tech Inc. received an $18.1 million Navy contract for geothermal equipment. (This contract may have never been issued!)

July 1983 - Article about Jake Possell written by Doug McMillan in *Gazette-Journal* (7/17/83).

"Navy aims to tap Fallon's geothermal resources" touted the Naval contract which was allegedly awarded for geothermal energy. (In reality, this contract may have never been issued.)

Letter of Intent to file a stockholders lawsuit (7/21/83) alleged:

General-Ener-Tech entered into a secret agreement with Admiral Research and Development Corporation. This agreement effectively diluted stockholder equity and future profit. It was executed unlawfully without a stockholders' meeting nor any notice to stockholders.

In 1977, General Ener-tech officers—Possell and McAllister—entered into a transaction involving Western Engineering Corporation and during the course of this transaction the value of Western's machinery was fraudulently inflated through the use false and forged appraisals.

General Ener-Tech had not caused an accounting nor proper certified financial statements to be given to stockholders since 1979/80.

The officers of General Ener-Tech have not properly accounted to the shareholders for the proceeds of over 4 million shares of General Ener-Tech stock sold between the annual stockholders meeting of 1982 and 1983—the sales figure was admitted to by the officers at the April 1983 annual stockholders meeting.

General Ener-Tech field tested their product at Heber, California with the results being a failure. The results have never been conveyed to the buying public or stockholders in any way as required by laws governing public corporations for the protection of the stockholder and public.

Corporate officers Jake Possell and J.S. MacAllister conspired to commit Grand Theft against the stockholders of General Ener-Tech by causing some 600,000 shares to be reissued in Jake Possell's name when in fact Possell had given up the stock under certificate **#2070** in 1975 to Misters D. Del Grande and A. Doria. The said stock was being held as collateral for payment of monies owed to said Doria and Del Grande.

Possell issued a letter at that time instructing Del Grande to accept and transfer the collateralized stock as he could not pay the amounts then due. Said 600,000 shares of stock were transferred by said Del Grande and Doria into Hispanic names which Doria and Del Grande controlled with the full knowledge of Possell and MacAllister.

In the meantime Del Grande and Doria both died prior to 1979. In 1979 Jake Possell supposedly discovers four years later that his stock certificate **#2070** has been stolen. However, said Possell and MacAllister did cause a replacement stock certificate to be issued to Possell for the original 600,000 share amount.

In May 1981, the FBI, while investigating another matter, was suspicious of a large amount of General Ener-Tech stock offered for sale on the Salt Lake City Exchange. The FBI traced this stock back to the original certificate **#2070** and contacted MacAllister and Possell to inquire as to this certificate **#2070**. MacAllister and Possell used this opportunity to report the stock certificate **#2070** as stolen and made a theft report to the FBI in 1981... 6 years after the stock was actually transferred. The theft was reported to the FBI as being committed by the now conveniently dead Misters Del Grande and/or Doria. A stop transfer list supplied to brokers dated April 31, 1977 made no mention of this certificate **#2070** being stolen.

Aug 1983 - Letter (8/2/83) from Richard van Aken (consultant to General-Ener-Tech) to Tom Tratt (Economic Development Director, Placer County, California) indicates:

The consultant who applied for an Industrial Revenue Bond Application to build a manufacturing plant in Placer County which was filed January 21, 1982 rescinded and withdrew the application. After uncovering numerous detrimental documents, the consultant made numerous attempts to contact Jake Possell and other corporate officers for clarification, but was unable to make contact. His stated reason for withdrawing the application was in his own words:

It has been brought to my attention, and after the perusal of many documents, I am firmly convinced that this exercise has been nothing more than a successful endeavor by the above named company [General-Ener-Tech] to sustain over a period of time an open Bonding Application, which they could refer their customers to ... I do not wish to be an unwilling dupe in this type of action...

By keeping the bond application open, it would keep the company's stock price artificially inflated.

Letter (8/3/83) from Richard van Aken (consultant to General-Ener-Tech) to George Yanase (California Department of Corporations) indicates:

The consultant who applied for an Industrial Revenue Bond Application in Placer County, explained that he had withdrawn the applications due to improprieties and his belief that the pump manufacturing plant was never intended to be built.

Furthermore, Richard revealed that he had many conversations with Possell and others in upper management about the feasibility of Bahamian banking, and tax advantages of corporate operations outside the United States. These discussions, coupled with the announcement of their funding a Bahamian Company and a trip to England by Jake Possell, led Richard to conclude that the money was going to go offshore.

Memo (8/14/83) from Charles Schaub (stockholder/investigator) indicates:

A planned "Stockholder's Derivative Action" lawsuit outlined in the Letter of Intent on July 21, 1983 was NOT filed due to the following reasons:

Cost of Lawsuit;
Length of time for suit to reach courts for compliance;
California authorities accepted the matter for investigation and possible prosecution.

Memo (8/17/83) from Charles Schaub (stockholder/investigator) indicate:

The Industrial Revenue Bond application and approval for the General Ener-Tech Inc's proposed Rocklin plant was **withdrawn and rescinded.** Another industrial revenue bond filing in Oneida County, state of New York is also suspect.

There is strong evidence that a United States Senate investigating committee held hearings into the awarding of contract **N62474-82-C1035** for energy at Fallon Naval Air Station in Nevada to General Ener-Tech, Inc. It appears that General Ener-Tech officials may have lied and/or withheld information before this Senate Committee.

The United States Department of Energy provided funds for the General Ener-Tech bladeless turbine to be tested. The tests were conducted by Livermore Laboratory and Stanford University. The findings were that the bladeless turbine tested did not work. These were the same results as previously obtained from tests run at Heber, California by Chevron.

The suspension order #3014044 issued March 3, 1976, could not be reinstated because General Ener-Tech WAS NEVER AUTHORIZED TO ISSUE, TRADE, or SELL SECURITIES in California!!!!!

Oct 1983 - Richard van Aken (former General Ener-Tech consultant) files suit according to The Sacramento Bee (11/11/83):

Van Aken stated in the lawsuit that General Ener-Tech used him and his firm to create a false, misleading and unlawful market in the securities of the firm without any reasonable possibility of obtaining financing or bonds... Van Aken charged that General Ener-Tech refused to provide information or supplied false, untrue and misleading information to him and Placer County officials.

Nov 1983 - California Halts Trading in General Ener-Tech according to The Sacramento Bee (11/11/83):

On November 9, 1983, the California Department of Corporations suspended over-the-counter trading of the stock in California because the corporation had not filed current financial information.

Mar 1984 - Article about Possell written by Tom Valentine appears in Spotlight.

Tesla's Turbine Technology Now a Reality touted the Naval contract which was allegedly awarded for geothermal energy. (In reality, this contract may have never been issued.)

Jul 1986 - Jake Possell lectures at the 1986 International Tesla Symposium.

Jake claims tests prove safety and reliability of turbine. Super aircraft using the turbine are to be in flight by the end of the year.

Mar 1988 - Article featuring Possell among others written by Bill Lauren appears in Omni.

Rediscovering TESLA touted the Naval contract which was allegedly awarded for geothermal energy. (In reality, this contract may have never been issued.)

Letter (3/2/88) from Jake Possell to David Wilson (potential customer) claims:

That Jake prefers join ventures instead of licensees because he has several licensees whom he now considers infringers. Jake has offered Chrysler motors a bladeless turbine engine twice and they turned it down. Jake is using a 12VDC Panasonic pancake motor to drive an automotive water pump. Jake finally claims he is having a turbocharger made in China and shipped no-duty since the Chinese have Most Favorable Trade arrangements with the USA.

Sep 1989 - Article featuring Possell appears in the International Tesla Society's Journal of Power and Resonance (Jul/Aug/Sep 1989):

Technological Breakthroughs of the Tesla Turbine was transcribed by Jeffery Hayes from a tape of Possell's lecture at the 1986 International Tesla Symposium and published in the Society's newsdigest.

Feb 1990 - Jeff Hayes visits Jake Possell in California for the purpose of writing a definitive book on the Tesla Bladeless Turbine.

Apr 1990 - Jake Possell proclaimed the leader in the world's pump technology!

Jeff Hayes returns to Colorado Springs and states that Jake Possell leads the world in pump technology! He finalizes his book, Boundary Layer Breakthrough—the definitive book on boundary layer technology.

Jul 1990 - Jake Possell lectures at the 1990 International Tesla Symposium.

Jake Possell speaks at the 1990 International Tesla Symposium Colorado Springs. Jeff Hayes receives Boundary Layer Breakthrough from the printer and begins to market them at the conference.

Aug 1990 - The Possell Bladeless Turbine Jet Engine is marketed at the Osh Kosh, Wisconsin Air Show.

Selling price was $50,000 with 50% down. Production is to start when 50 orders are received. Claims were made in Possell's literature pack that the compressors were developed 25 years ago and extensively tested. Further claims were made that they had sold over twenty-five turbines already.

Feb 1991 - Jeff Hayes resigns from Board of Directors of the International Tesla Society.

After the Board of Directors asserted its editorial oversight and required all Directors to attend the monthly board meetings to maintain their directorship, Jeff resigned from the Board.

Jul 1991 - Jake's Retirement????!

Jeff Hayes returns to Colorado Springs and claims that Jake has retired from the pump business after selling Ali Baba manufacturing rights to a Japanese firm.

Apr 1992 - The Unraveling of the Web!!!!

The International Tesla Society learned of Max Gurth—patent holder and manufacturer of the pumps. An on-site inspection by JW McGinnis of Max's manufacturing facility led to further investigation into Jake's activities. After listening to numerous complaints of Jake's customers—including the Asian businessman who lost of thousands of dollars—the Society decides to expose this possible fraud!

YOUR AD COULD HAVE APPEARED HERE

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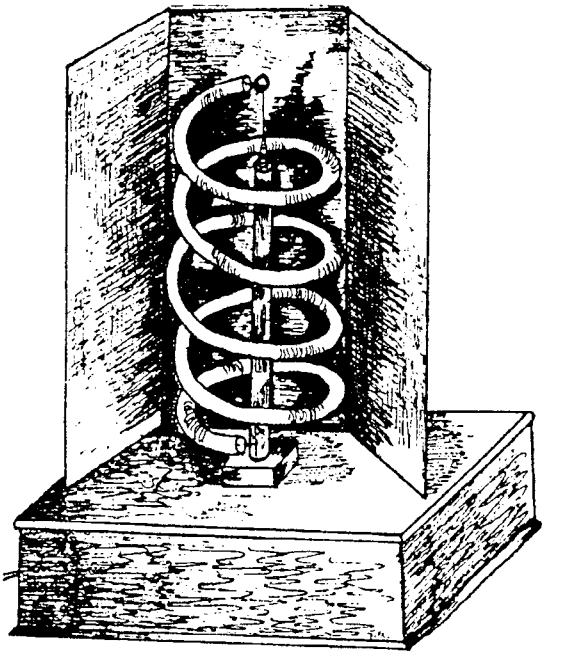

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Technology for the New Age

The Mythical Tesla Turbine... *Current Research and Results!!!*

Jerry LaBine -- Researcher

I will bring to the symposium two, perhaps three, Tesla turbines for demonstration purposes.

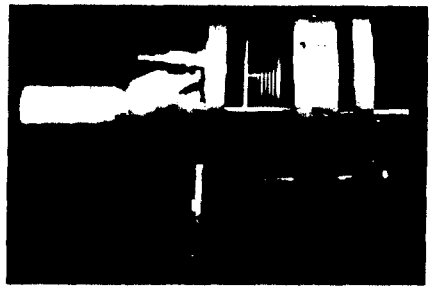
Engine 1:

The first engine I built, and even in it's crude form we will demonstrate the effects of viscosity against the disc. For this purpose, we will show the various disc that we tried in the turbine, which ones worked well, and a few which did not work as well, and explain why.

Engine 2:

On engine 2 we will show the change in placement of the injectors as it relates to *Engine 1*. We will demonstrate the difference in the blade configuration, and the various test we undertook with the blade placement, such as gap, thickness of the blades, exhaust ports, and rotor to end cap placement.

We discovered that the same viscosity that occurs between the blades also occurs against the walls on either side. To relieve this viscosity, Tesla designed special patterns in the end casings, and by placing a labyrinth half on the blades, and half on the casing



he was able to keep the back pressure from going directly to the shaft. By doing this it causes back pressure on both end caps forcing the flow to take the path of least resistance which is down through the blades. ***We know this because we tried it.***

To know if this effect actually functioned we built *Engine 2* with no rings inside, and then ran a continued test of 80 psi, it gave us a

net result of 20,000 rpm. The engine would not climb past that. Then we grooved both end caps in a specific manner indicated by Tesla in his patents. The rpms climbed to 30,000 rpm at the same pressure of 80 psi.

That is not to say that the engine was topped out, for under another controlled test—125 psi, it topped well over 65,000 rpm, at which point small imperfections in the milling process showed up dramatically. At this point you have to go back to the bearings.

We have found that in certain applications, such as *Engine 1*, that splitting the exhaust port, affording half of it to go through a hollow shaft which goes to the bearing compartment, will keep your bearings cool.

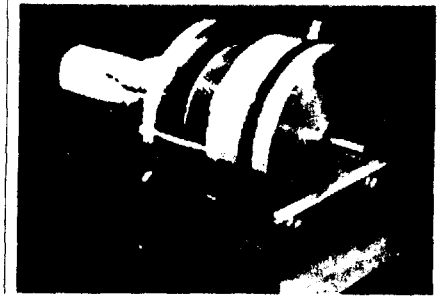
Conclusion:

We hope to demonstrate that there is more to the Tesla Turbine than Tesla gave us information on. The only way to perfect this engine is to do experiments, read between the lines, and try to see it from ***his*** mind's eye.

We will give a brief on how we came in contact with information on the Tesla turbines, and give an explanation of the first two tests that I performed to find out about viscosity, and it's effect on highly polished surfaces.

You can expect a basic overview of a functioning Tesla Turbine. Operational theory and construction tips will be liberally sprinkled throughout the demonstration and presentation.

The proper way to research Tesla is by experimentation. One of our goals in the future concerning the Tesla turbine is to thoroughly study the master--Nikola Tesla!!!



WANDERING STAR

"Independent Research Craft"

by Jerry LaBine & Doris Sewell
83-480 Avenue 45 #16
Indio, CA 92201

Introduction

The Wandering Star is a high flotation, alternative energy, experimental river craft. It has taken 8 years of design and over 3 years of construction to reach 90% completion. The propulsion system is electric over hydraulic, and the prime movers are high speed hydraulic jet pumps. No combustion fuels are used. We are designing variations of Tesla technology for our experimental river craft.

The onboard systems on the Wandering Star will be tested in the very near future. We have chosen the Mississippi River for our testing site. A few of the systems to be researched are:

- *Photovoltaics*
- *Air Conditioning*
- *Electrical Storage*
- *Solar Ovens*
- *Compressed Air*
- *Tesla Technology.*
- *Hydro generators*
- *Solar Hot Water Systems*
- *Hydraulics*
- *Four-Way Rudder System*
- *Water Purification*

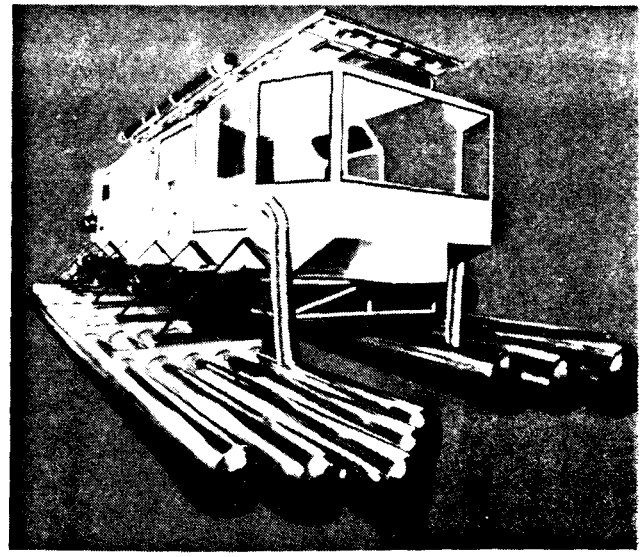
Design and Construction

Jerry and I are individuals, and are not involved with any organizations or groups. In August 1990, we were looking for a retirement vehicle. A motor home costs way too much, and the gasoline to power it would eat us up. We discussed this problem for several weeks, along with alternate energy theories, possible ways to achieve an ultimate power source, the universe, politics, etc.

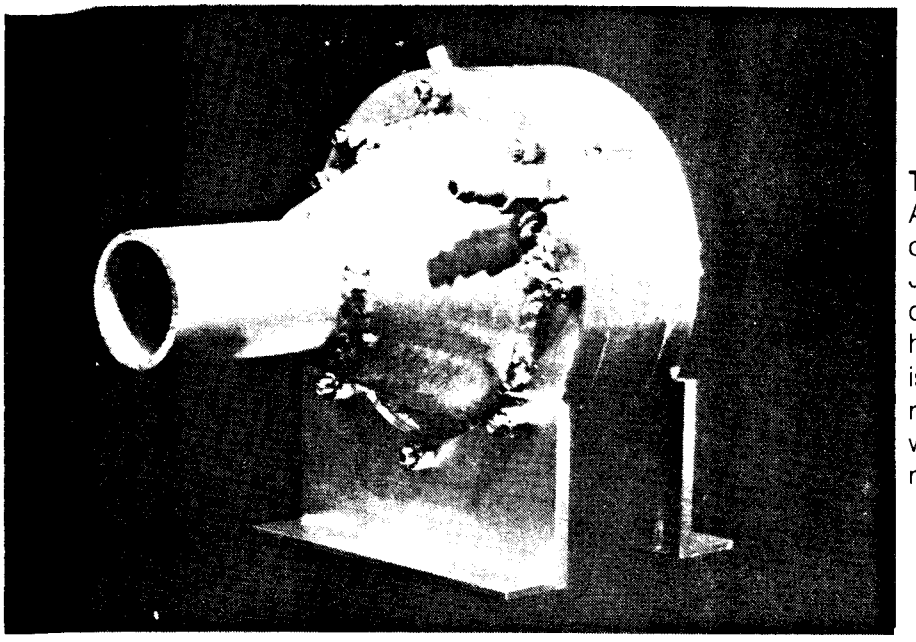
Eight years prior to meeting me, Jerry had begun refining plans for an electric river craft. When he mentioned these plans, we changed it to meet both of our needs. We came up with the name ***Wandering Star***. The *Wandering Star Research Craft* came into being when we discussed all of the theories we wanted to research.

In October of 1990 we rented a shop in order to start construction. Jerry had already made a model out of balsa wood. We started on the frame.

The frame was designed like an aircraft frame. It is composed of 1" thin wall square tubing, and gusseted with attention paid to the hard points. (*Hard points* is the terminology used in the aircraft industry where one would use heavier than normal gussets with either a hole or a bolt coming out of it for the purpose of attaching objects to the frame such as struts, boxes, etc.) Special welding techniques were incorporated into the frame to reduce any vibration and stress points. The frame by itself weighs 300 lbs. With only two strips of carpet keeping the frame 1/4-inch off the floor, it took both Jerry and myself jumping simultaneously and hitting firm onto the frame to get it to tap the floor. We gave the frame an acid bath, and painted it with epoxy.



A Dream of a Lifetime! The ***Wandering Star*** is a high flotation, alternative energy, experimental river craft which has been under construction for the past three years.



The Mythical Bladeless Turbine. After hearing rumors on both sides of the Tesla Turbine controversy, Jerry set out to build and test his own. After reviewing the test data, he has determined that the turbine is worth pursuing. He is now getting ready to build a steam turbine. We will keep you appraised of the results!

Testing our Tesla Turbine

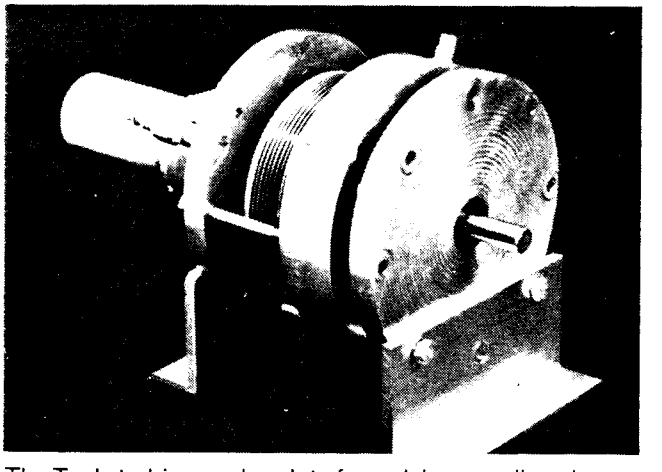
At this point, Tesla power is under investigation, with experimentation going on this very minute. After reading many books, and articles pertaining to Tesla's boundary layer turbine we decided to build one of the motors. Our first crude motor which you see here, reveals such outstanding possibilities that we decided that we would conduct in-depth research of our own. At this point we can honestly say that Tesla's boundary layer motors and pumps will be used on board the *Wandering Star.*

The circumference of the motor is 6" in diameter. The inside chamber is 5" offset. End caps are 1/2" thick. The center chamber is 1-1/4" wide. The bolts are stainless steel and the bearings are aircraft sealed pack ball bearings. The shaft is 1/2" stainless steel. The rotor disk which is the main carrier is stainless steel and welded to the shaft. The secondary disk (of which there are 4) are supported by the carrier disk.

The carrier disk of stainless steel has 3 taps in it and is 4-3/4" in diameter and 1/8" thick. The secondary disk's are 4-3/4", in diameter with a 2" hole in the middle that is 1/8" thick, with 3 corresponding holes around the 2" diameter that corresponds to the taps on the carrier plate. When the rotor is assembled disks are stacked with a 1/8" spacer between. The outer injector ring has a 3/8" inlet, and four injectors with a 1/16" bore. Two gaskets, and two angle plates serve as the base.

When the motor was operated on regular faucet pressure it yielded 850 rpm at a flow of 5 gal. per minute. Not impressive—but it moved! We cleaned it up and ran an air test. Even though we were astounded by its performance, and had run a variety of tests, we found that our pressure reading at the regulator on the wall was showing 110 lbs.. By putting a gage at the motor we soon realized we were only getting 30 lbs. of air pressure at the motor. With this revelation we were stunned!

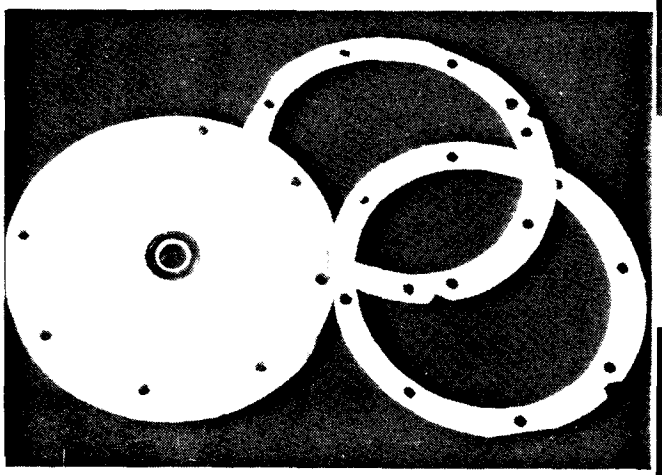
This crude-built quickie motor had been performing as high as 14,800 rpm, the torque was so much you could not hold it when air was applied!!! What will it do if the full 110 psi were applied. The results of that test will be disclosed at a later date, as the test is a near future event.

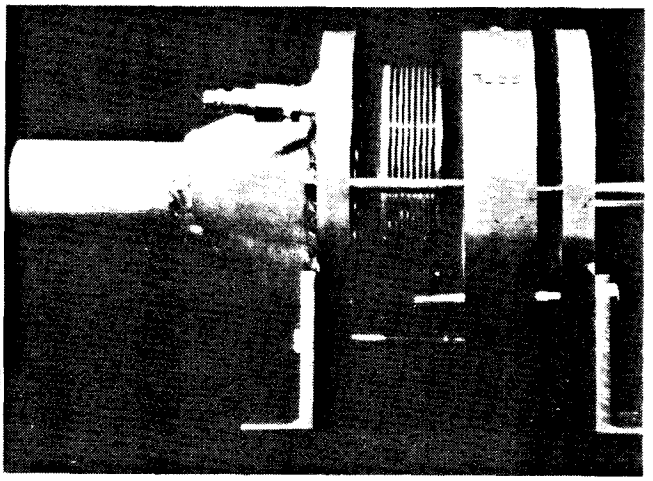


The Tesla turbine packs a lot of punch in a small package. Many since Tesla have built compressed air models...the real test is to make a steam or gas turbine!

The Tesla Turbine.... An Exploded View

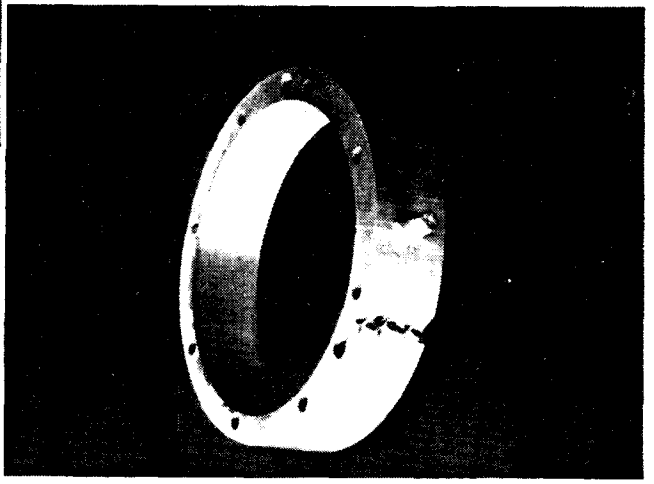
1) This prime mover with flat discs instead of bladed surfaces was crudely built with very little attention paid to tolerances. As a result, efficiencies are somewhat less than optimal. In this unit, 30psi of compressed air yielded 14,800 rpm. (right)

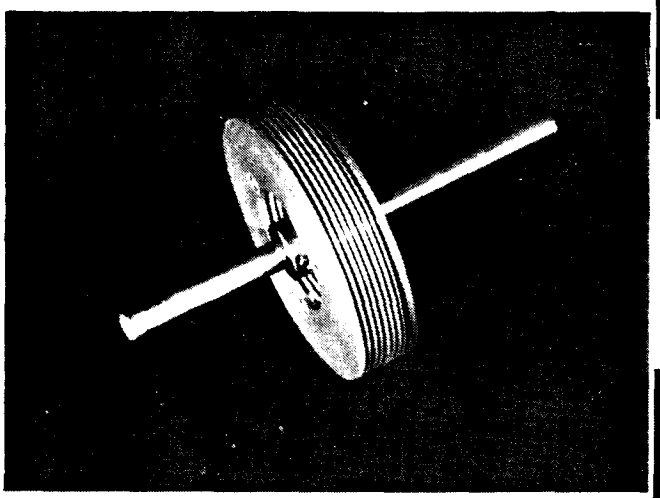




2) **The end plate and seals.** This prime mover with flat discs instead of bladed surfaces was crudely built with very little attention paid to tolerances. In this unit, 30psi of compressed air yielded 14,800 rpm. (left)

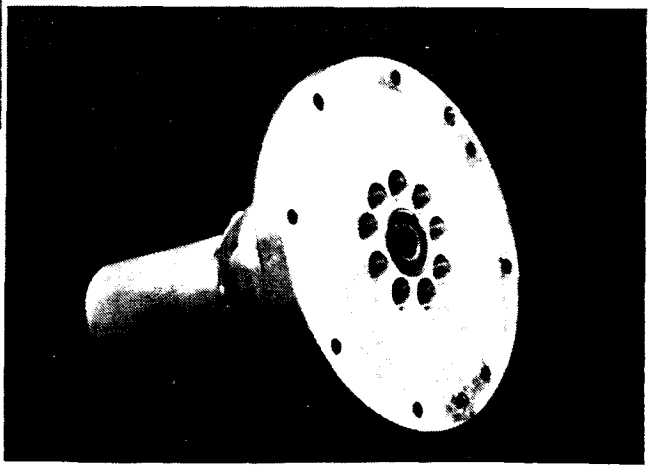
3) **Main Rotor Chamber.** At the top of the picture are the injector ports. Air (in future turbines, steam) is injected on the circumference of the rotors by means of these ports. The air/steam then spirals to the center of the chamber where it exits through the exhaust. (right)



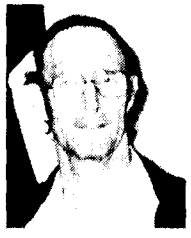


4) **Rotor Assembly.** Notice that there are 7 inside discs and 2 outside discs. (The outside discs are thicker. All but one disc is donut shaped. The support disc is welded to the shaft. The assembly is held together by three screws attached to the support disc. (left)

5) Exhaust ports. The Tesla turbine is exhausted through the center of the motor. According to Fritz Lowenstein (Tesla's assistant), the thermodynamic work being done by the steam on its free spiral path through the rotor is a feature unique to the Tesla turbine! (right)



SUPER STEAM—
Power for the 21st Century



by
Leonard Dorsett
PO Box 365
Grapevine, TX 76092

Introduction

With the rising cost of fuel in today's economy, and the ever-increasing pollution in our atmosphere, the time has come for drastic measures to be taken. For over 200 years man has utilized steam as a working force for many applications such as locomotion, heating food processing, cleaning, excavation, generation of electricity and so forth.

Now, in the 1990's, a new and bold energy system is ready for the ever-growing demands of the world. SST "Super-Steam Technology" is the answer to many of today's ever increasing energy needs.

What is SST? Simply put, SST is a system that produces steam at any pressure or temperature, *instantly*.

What is steam used for in today's world? Ironically, almost every aspect of everyday life is affected. In addition to the uses listed above, are the operation of large factories, cooling, water distillation, and much, much more.

With the SST system, steam can be produced instantly at any temperature or pressure without the need for water purification or huge conventional boiler systems. This differentiates SST technology as completely different from conventional boiler technology.

SST System Benefits

A single forty horsepower (40 HP) conventional boiler is approximately the size of a railroad tank car. The SST unit can be designed to operate from the size of a beverage can to any increased size as large as a house. An SST unit which is the size of a small shipping crate, can deliver the same BTU output as the aforementioned 40 HP conventional boiler.

In the SST system, pressure and temperature are independent of each other. The temperature achieved by conventional boilers is dependent on the pressure. The SST system can *instantly* produce steam from virtually any temperature, from hot water at 180F, to super-heated steam at over 3000F. It can operate at pressures from 1 pound(s) per square inch (PSI) to 3000PSI, or greater, with airflow volumes of 1 cubic foot(feet) per minute (CFM) to well over 2000CFM.

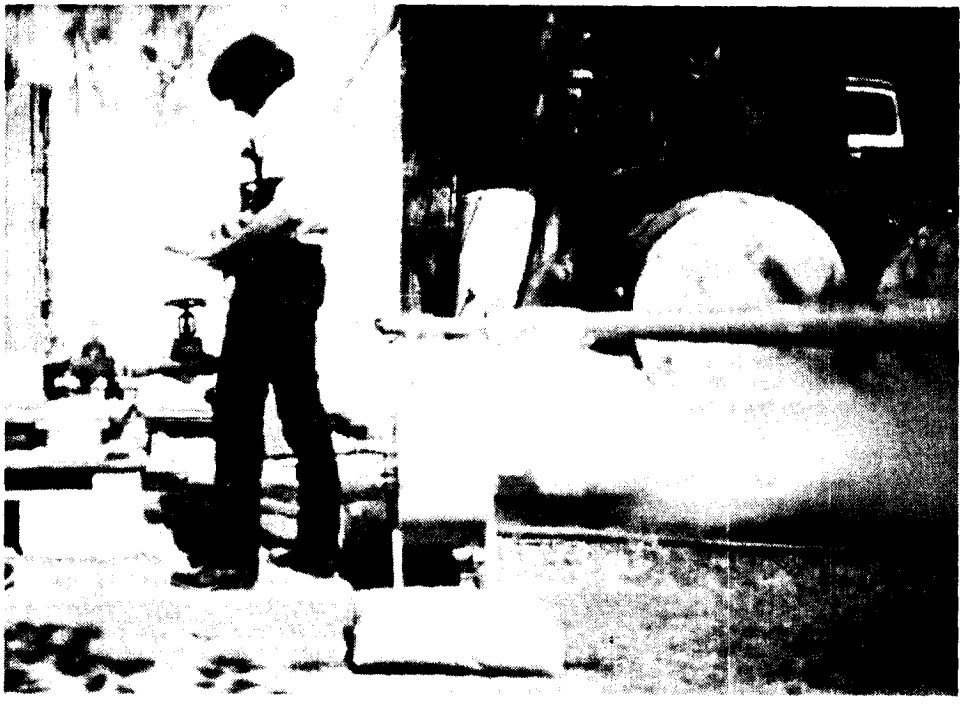


Figure 1. Norm Wootan takes some readings of the SST during a recent demonstration in Texas. One of the applications—the Tesla turbine—is shown in the foreground.

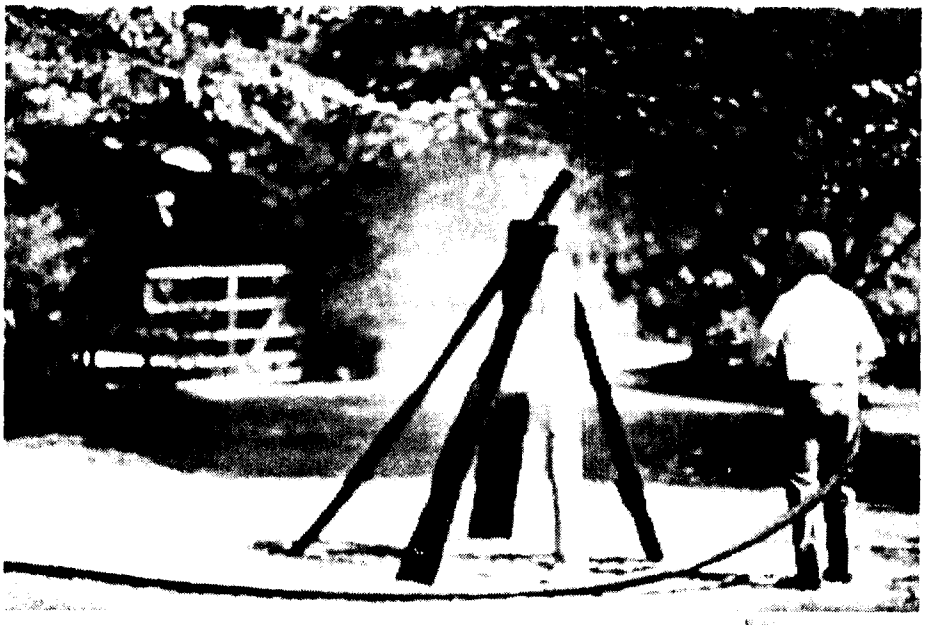
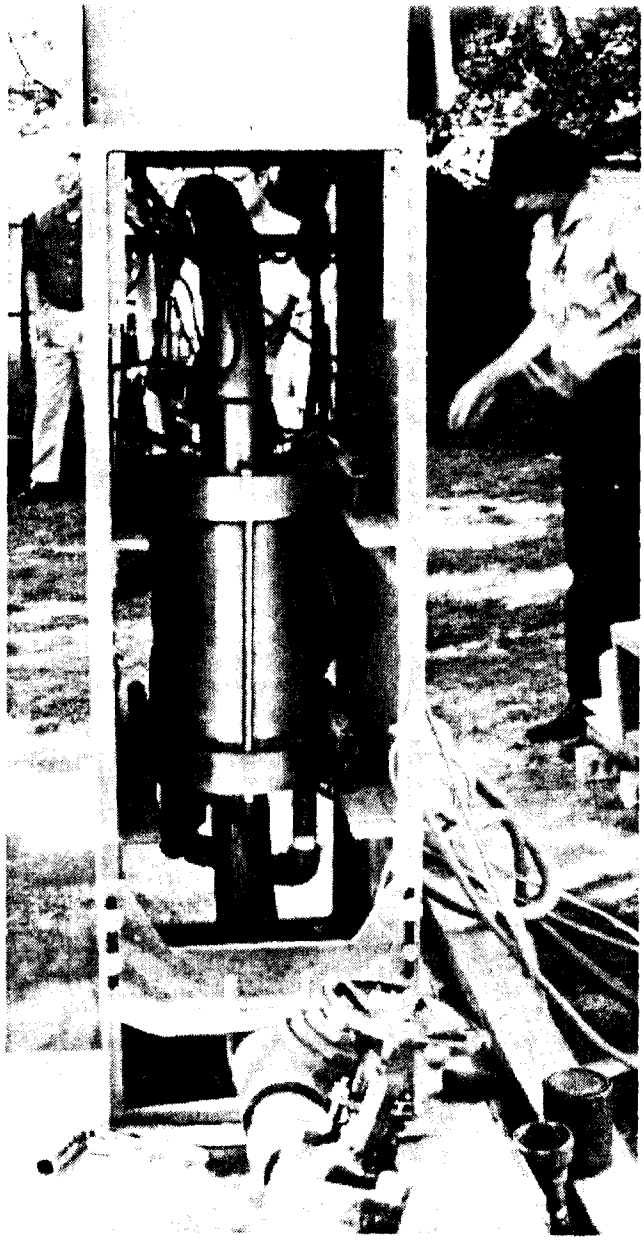


Figure 2. (left) Leonard Dorsett demonstrates the effectiveness of the SST in firefighting. With a miniscule amount of vapor, the SST can put out fires that can't be handled by regular firefighting techniques.

Figure 3. (below) Rear view of the SST. Although the SST appears to be a relatively simple device, it packs an awesome punch!



The SST system will operate on propane, methane, gasoline, kerosene, methanol, diesel, hydrogen, or any other combustible fuel, and it produces steam *instantly*. Virtually 100% of the fuel consumed is turned into usable energy, which means no pollution is exhausted into the atmosphere. It does not require hours of fuel consumption to reach operating temperature, as do conventional boilers. The SST system is over 90% efficient, whereas a conventional boiler is approximately 40% efficient. The energy savings alone has far-reaching economic implications.

The SST system does not require purified or treated water. It can take polluted water, salt water, or any other contaminated, liquefied material without doing any damage to the unit. Conventional boilers must have purified and treated water to operate. The SST system can instantly operate at pressures from 1 PSI to 3000 PSI with airflow volumes of 1 CFM to 2000 CFM.

The SST system has no moving parts to wear out. With its ease of use, space saving installation, and high efficiency, the SST system is capable of supplanting any existing steam application today!

Explanation of Operation

The heart of the SST is a specially designed open throat chamber with controlled, air, water and fuel interaction and high voltage ignition resulting in a direct fire process establishing thermal zones. This creates a stoichiometric condition resulting in a full

Figure 4. The SST version of the Tesla turbine has 24 stainless steel 18" discs. The turbine, driven by the SST BTU Converter with 350F saturated steam, operated at 1000 rpm on its initial test run.

spectrum of steam product. Selections may be from very dry to wet and supersaturated.

Desired applications determine the type of air resource to be used. A blower is ideal for high volume and low pressure. A compressor will effect a full range of options, especially supersonic steam, as used with turbines.

At the increased temperatures of steam... disassociation occurs. The hydrogen and oxygen reacts causing an exponential increase of energy resulting in super heated steam. It's interesting to note a similar superheat process was addressed by Tesla in the October 12, 1911 publication of the ***Engineering News Record***. I'm sure Tesla would have liked what is now birthing, as we may now move forward with certainty.

The Tesla Turbine

One of the most exciting applications is the famous Tesla turbine which Tesla first developed. Here is a unique situation of a bladeless turbine where advancement since its beginning in 1909, lacked only a continuous steam source of power, and a steam generator such as the SST BTU Converter—which has now come to the forefront after industry has stagnated at the level of pressure boilers.

Our Tesla-type turbine was designed and built in just two weeks for a May 21 demonstration. The housing is constructed of plate aluminum and sheet aluminum covers. The disc runners are made out of stainless steel and the shaft is built out of tool steel supported by two aircraft roller bearings. The design omits many features necessary for efficiency and power. This turbine, however as quick and dirty as it is, clearly demonstrates once again the Tesla principles for this technology! In some respects, the Tesla turbine seems to resemble the vortex refrigeration device.

The Vortex Tube

The vortex tube is an interesting device capable of providing both cooling and heating at the same time. Its source of energy is compressed air. It converts the compressed airflow into two streams of air, one hot and one cold. The amount of hot or cold air released from the outlets can be varied. For example, a unit with a 100 psi air supply at 70F. can be adjusted to cool half the air to -29F while heating the other half to 91F.

Looking at **Figure 5**, we can see that compressed air enters at A. It then goes into a number of small nozzles where it will lose some of its original high pressure. As it expands, it moves at near sonic speed (velocity).

Nozzles are arranged so that the air is injected tangentially (around the inner surface) to the circumference of the generation chamber. This makes the air swirl or spin like a cyclone. The control valve at the end of the hot air tube controls the flow of both the heated air and the cooled air. The position of the control valve determines how much air will leave the hot end and how much air will be forced out at the cold end.

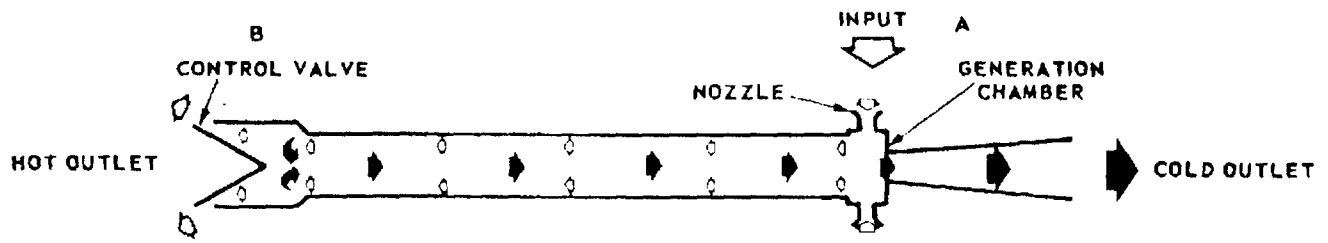


Figure 5. A schematic drawing of a vortex tube. A—Compressed air inlet. B—Control valve. (Vortec Corp)

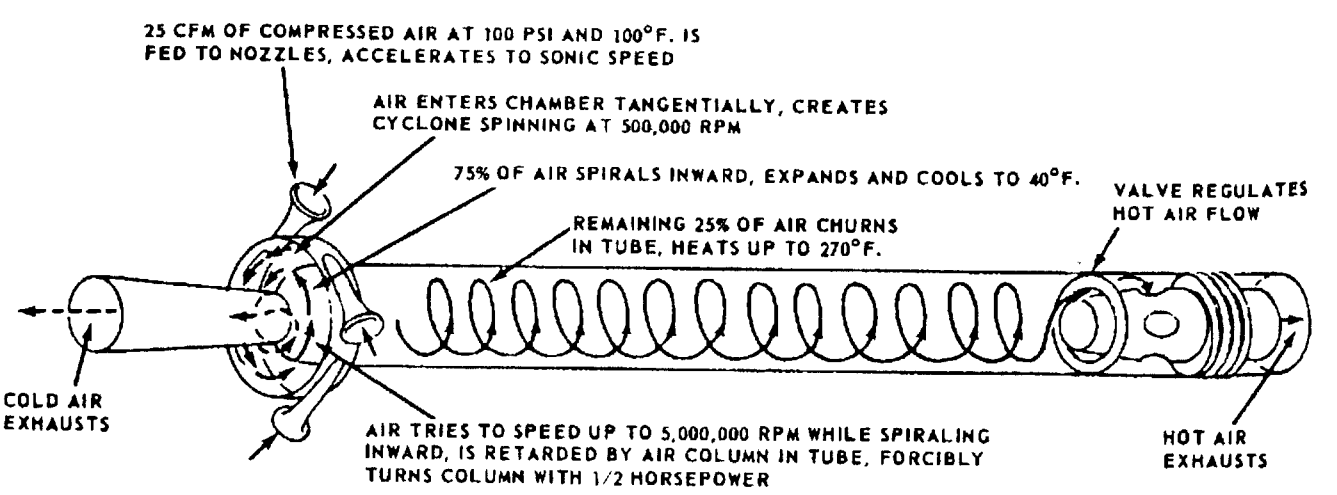

25 CFM OF COMPRESSED AIR AT 100 PSI AND 100°F. IS FED TO NOZZLES, ACCELERATES TO SONIC SPEED
AIR ENTERS CHAMBER TANGENTIALLY, CREATES CYCLONE SPINNING AT 500,000 RPM
75% OF AIR SPIRALS INWARD, EXPANDS AND COOLS TO 40°F.
VALVE REGULATES HOT AIR FLOW
REMAINING 25% OF AIR CHURNS IN TUBE, HEATS UP TO 270°F.
COLD AIR EXHAUSTS
AIR TRIES TO SPEED UP TO 5,000,000 RPM WHILE SPIRALING INWARD, IS RETARDED BY AIR COLUMN IN TUBE, FORCIBLY TURNS COLUMN WITH 1/2 HORSEPOWER
HOT AIR EXHAUSTS

Figure 6. Diagrammatic view of vortex tube. Note flow of air at 100 psi and 100 F into nozzles. Air is exhausted at temperature of 40F at cold end. Cold air is shown by the broken line and arrows. (Vortec Corp.)

There are two streams of spinning air going through the generation chamber. The outside layer moves toward the hot outlet and the inside layer moves towards the cold outlet. In this process, the air traveling through the center becomes very cold and leaves the vortex tube at the cold opening providing the required cooling.

Vortex tube cooling has many different applications. Since it must depend upon the supply of compressed air at fairly high pressure, it is often used where the air exhaust—either hot or cold—for the vortex tube can be used for other purposes also. It is particularly desirable in locations where both ventilation and cooling are needed. A common application is the cooling of miners' clothing in mines that are too warm for comfort. In most uses, the compressed air must be both clean and dry.

Many industries use the vortex tube for cooling tool bits and in similar applications where the use of liquid coolants would be undesirable. It is particularly useful in installations where the stream of cold air may be used for both cooling and removing chips or in installations where the exhaust air will provide good air for the operator to breathe.

Steam Jet Cooling

Another application at where the BTU converter may excel is in cooling. In the conventional state of the art, the steam jet system consists mainly of a venturi in the steam jet ejector, a condenser, evaporator or flash chamber, chilled water circulating pump, and condensate pump. The principle of operation is based on the fact that water under a high vacuum boils at a relatively low temperature. This causes evaporation to occur and reduces the temperature. However, since water is the refrigerant used in steam jet applications, only temperatures down to about 40F are possible.

The steam jet system is used primarily as a means of cooling water which will in turn be used for either comfort cooling or process temperature cooling. Thus, it is used in air conditioning, cold water gas absorption in chemical plants, beverage cooling in distilleries and many other applications where a steam power plant is needed for other uses. Its operation is normally limited to installations where there is an abundance of steam and condensing water at a low cost and where the desired temperatures are in the 40F to 50F range.

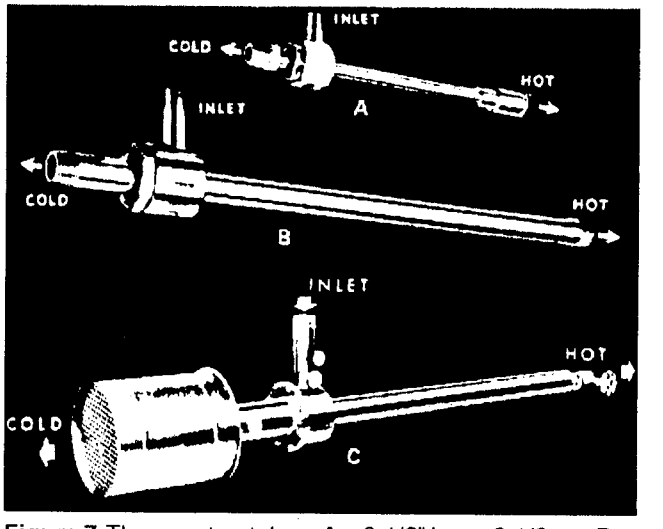


Figure 7. Three vortex tubes. **A**—6-1/2" long, 2-1/2 oz. **B**—10" long, 6 oz, **C**—Large capacity tube. Cold end is 10" dia. Inlet tube has dial thermometer and pressure gauge. Cold and hot ends have dial thermometers. (Vortec Corp.)

Figure 8. Leonard Dorsett explains the principles behind the SST and its numerous applications. He will be demonstrating much of this equipment at the upcoming ***1994 International Tesla Symposium.***

Since a relatively low steam pressure is required, steam jet cooling is often used in plants which use high-pressure steam for operating machines. The exhaust steam is right for steam jet refrigeration. With SST massive amounts of low cost steam can be supplied reducing the cost of refrigeration, or the need for existing high pressure steam facilities!

Other Applications of the SST System

The SST system can dramatically increase electricity production by eliminating costly conventional boilers and water purification. In addition, it will prove to be an ideal low-cost method for the heating and cooling of homes and businesses. Small portable units can be developed for home use, such as steam-cleaning, and insect and fire-ant removal, without the dangers of insecticides.

The SST can be used for low cost purification of polluted water or sewage. Salt water can be converted to drinking water both quickly and inexpensively. Furthermore, it is ideal for sterilization and nurturing of soil for agriculture where contamination has occurred.

SST will be used in many governmental applications as the fire-fighting applications are developed. In the northern portion of the country, SST will be used for snow and ice removal from roads, airport runways, resorts and businesses.

The SST system can be utilized with the upcoming sonic chemistry industry, which has far-reaching applications for previously-unknown and undiscovered chemicals and metals. And this is just the tip of the iceberg....

Conclusion

One aspect of technology that we are more concerned with these days is safety. This system can be shut down at any time of operation without any consequences. It can be immediately restarted to regain normal operations. The nitrogen/carbon dioxide components vapor product make for safe use around flameable materials. Since the BTU converter is not a pressure containment device it cannot rupture or explode—an increasingly frequent event with increased neglect for boiler maintenance.

Space does not allow us to go into further detail at this time. Many people from the Dallas area and some members of the Vangard Sciences Roundtable will be at the July 21-24 Tesla Symposium with me to demo this technology. Be there!__*LD*

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MERGING TESLA'S TURBINE AND PUMP TECHNOLOGIES

by:

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Much of Tesla's extraordinary career was devoted to the development of efficient energy conversion inventions. Tesla's turbine and pump patents clearly describe two energy conversion processes, the vortex and the liquid adhesion to a solid (skin effect). Both of these powerful energy transfer processes are found to occur frequently in nature. It was Tesla's keen observation of natural phenomena that led him to many of his great inventions and discoveries.

Both the turbine and the pump inventions he developed were based on continuous streams of gases or a liquid vortex. Just as resonance was the natural phenomena behind many of Tesla's early radio inventions, the vortex and liquid adhesion were two other important phenomenon that were the basis of his pump and turbine inventions. Tesla realized the same natural energy processes would work on solids, liquids, and gases. He also realized that different types of energy sources (electrical energy, magnetic energy, thermal energy) could be used to create rotational motion. By understanding natural phenomenon and applying his remarkable genius, Tesla envisioned and created ground breaking inventions.

Natural Energy Models

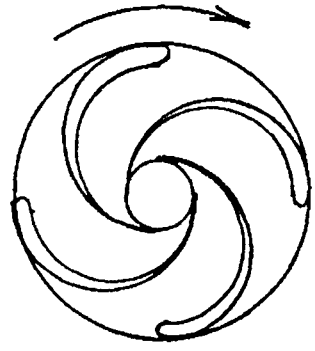
Tesla modeled many of his inventions by observing the universe. The universe contains galaxies with spiral shaped clusters of stars. These clusters of stars would spiral inward and collide by gravitational forces alone. The rotation of galaxies generate counter-balancing centripetal forces that keep the spiralling star patterns intact. Balance is maintained by natural opposing forces.

If you examine photographs of galaxies they have similar shapes. Each center is a circle or globe of filled stars and radiating from this center are attached arms that have the shape of a six. Some galaxies have two arms that are diametrically opposed. Other galaxies have six or more arms. Curiously, only two arms connect to the inner circle on most galaxies and the remaining arms are disconnected as trailing arms. The arms that connect to the inner circle of the galaxies intersect tangentially on opposite sides of the inner circle.

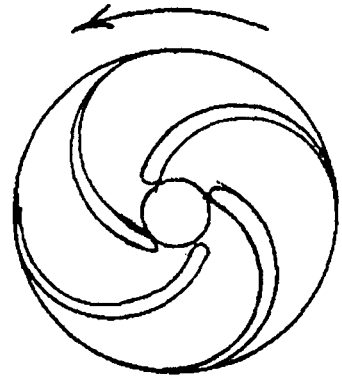
The important fact is that the galaxy's balance is achieved and maintained over time. Multiple arms are balanced in space according to mass and acceleration by an equal number of like stars or dark stars on the other side of the galaxy. A spiral shape is an important formation in nature and Tesla used this phenomenon to develop inventions based on vortices.

Galaxies are very efficient energy models. Since they exist in space and operate on gases, liquids and solids continuously, galaxy models have much to offer an inventor. A mechanical model of the galaxy is shown below.

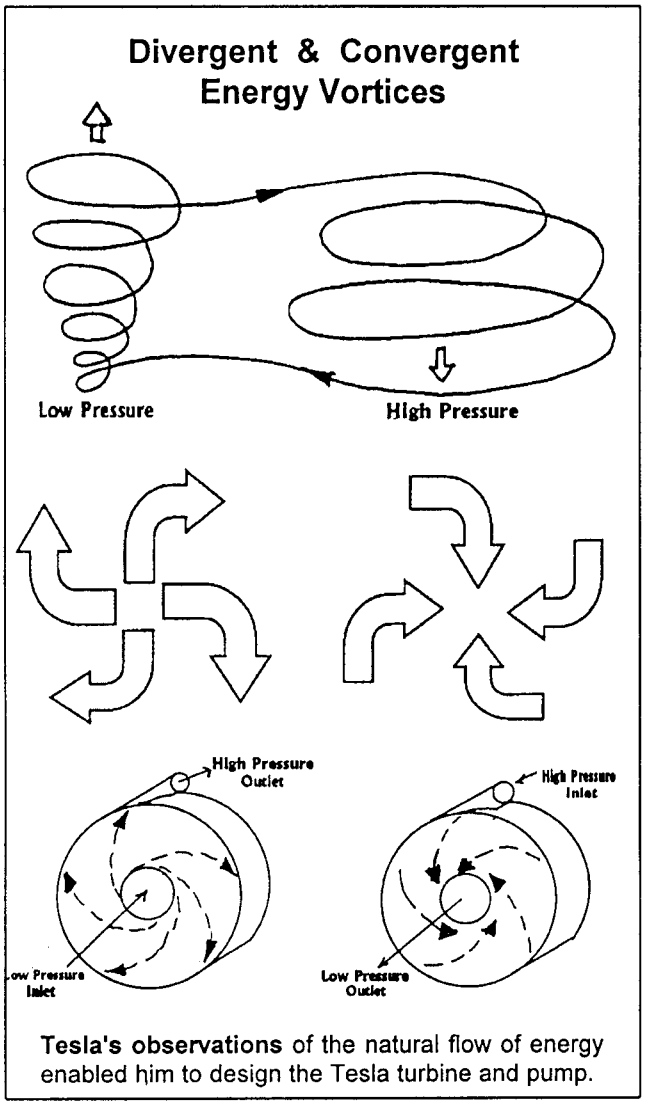
In nature every model of an energy source has a counterpart. Magnetics and electronics have *north*



Mechanical model of a galaxy. A disk with four streamers attached in a spiral configuration. This is equivalent to a compressor.



Mechanical model of divergent space. This consists of four blades and a disk. Gases flow from the center outward to the edge of the disk and beyond. Note the blades are tangential on the inner and outer edge of the doughnut shaped disk.



Tesla's observations of the natural flow of energy enabled him to design the Tesla turbine and pump.

and south poles or *positive and negative polarities* respectively. For the vortices there is the convergent (galaxy) model and the divergent model (empty space).

The Vortex
- Energy's Natural Path

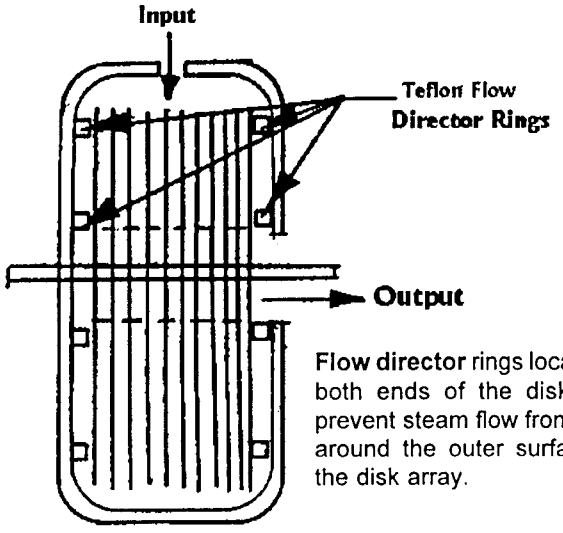
Tesla wrote of the principles of operation of his turbine in his patent notes. The vortex was described many times in the patent papers. A vortex is a naturally occurring phenomena found in space, the atmosphere, and everyday life. Galaxies, tornados, water spouts, and whirlpools of water draining in sinks are common occurring examples of the vortex. An atmospheric vortex is a rising column of warm air that spirals as it moves vertically. These spiralling air masses can cause clear air cyclones or create tornadoes embedded within storm clouds. Low and high pressure systems are an illustration of vortices in the atmosphere. Wind speeds in tornadoes can reach 600 miles per hour and extremely low atmospheric pressures develop in the funnel. Manmade structures can explode and be ripped apart by the powerful forces within a tornado's vortex. In nature there are two types of vortices, those that converge and those that diverge. Interestingly, Tesla used the convergent vortex concept in his turbine design and the divergent vortex concept in his pump design patents.

A convergent vortex is an inward spiralling mass of particles whether solid, liquid, or gas. The convergent turbine is usually driven by steam or compressed air. High pressure gas or steam is introduced at the outer edge of the turbine's disk array and moves in a spiral motion towards the cutout center of the disks. The pressure is significantly reduced due to the large opening in the center of the disks that lead to a large exhaust port.

Boundary Layer Effect
- A Powerful Natural Phenomenon

The rapid pressure drop from the input to the output causes a significant energy transfer to the rotating disks. The energy transfer process is also enhanced by the *boundary layer* or *skin* effect (water molecule attraction and adhesion to solid surfaces). During this adhesion process the moving steam condenses to a thin film of water that coats the disk and imparts further rotational energy to the disk. Tesla made mention of both these processes being key to the operation of the steam turbine. The compressed air version of the Tesla turbine works on the vortex principle alone. It would be an interesting experiment to see the power output difference between the same turbine run on steam versus compressed air at similar input pressures. This would isolate the boundary layer effect energy transfer process from the vortex energy transfer process.

In contrast, the divergent vortex is an outward spiralling mass of molecules or particles. Recently developed versions of Tesla's pump use the divergent vortex to handle concrete, oil, water, jellied substances, and powdered materials. If the disks were placed very close together in the pump it would move gases efficiently as well. In this mode, the pump could be a compressor or a vacuum pump.



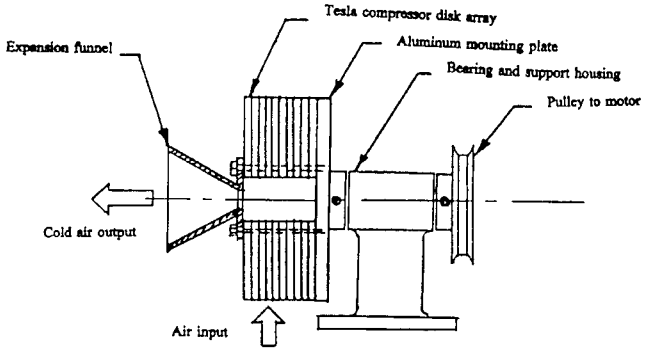
Flow director rings located on both ends of the disk array prevent steam flow from going around the outer surfaces of the disk array.

One of the interesting features of Tesla's pump is the smooth polished disk surfaces that have no impeller blades. The lack of impeller blades allow the flow of materials or fluids to be stopped without damage to the pump. No molecular damage occurs to the materials in the pump when the pump input is stopped as in conventional pumps. Again, the boundary layer effect occurring on the disks in the pump has an energy transfer effect on the material as it spirals outward to the edge of the disk. Low pressure exists near the input of the pump relative to the high pressure at the output of the pump.

A Practical Tesla Pump

To make a practical compressor or vacuum pump more than one disk is needed. The idea is to stack many disk and blades together in a cylindrical array for more effect. The figure above, shows how the array would be built up by laminating the disks and blades together.

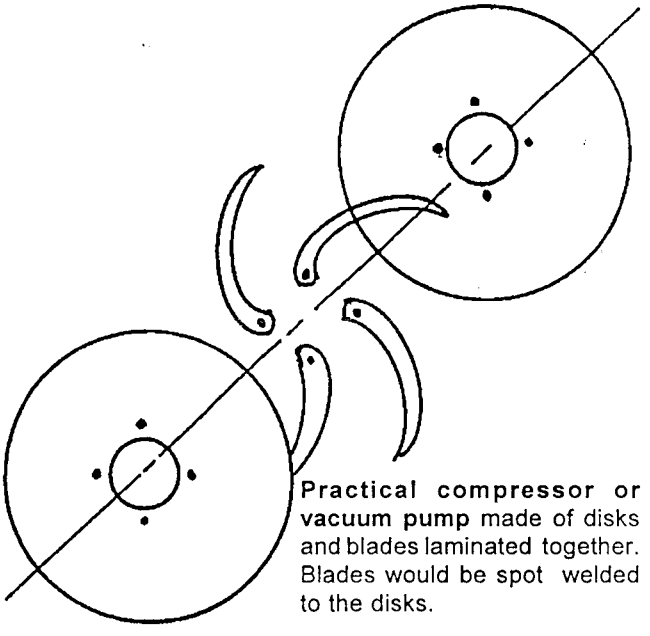
The compressor or vacuum pump will require ten or more disks and blade sets to make a useable cylinder.



The rotary compressor can be used as either an air conditioner or a refrigerator cooler.

This design is useful for rotary air compressors, vacuum cleaners, air conditioning blowers, vortex coolers, wind turbines, and vacuum generators. Spin the disk set on a magnetic bearing, power it with wind pressure, attach an alternator and you have a natural source of alternate power. By attaching a hose to the outlet you can use it for a powerful fan or suction pump. Connect an expansion funnel to the output and the compressor version will provide cooling for refrigeration or air conditioning without any freon required.

As you can see from the above applications, the galaxy vortex phenomenon can be put to good use here on earth. Natural energy sources have been around a long time and are going on around us

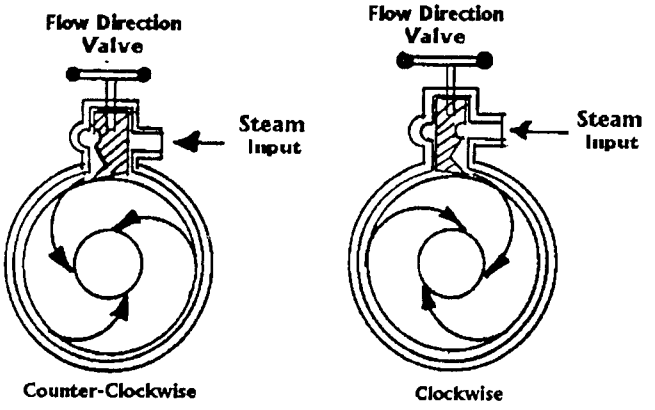


Practical compressor or vacuum pump made of disks and blades laminated together. Blades would be spot welded to the disks.

continuously without causing pollution and other side affects. We could all learn a lesson about good science by observing how nature does it. Note, the pump and turbine have similar construction but are the exact opposite in the way they work. Your comments and ideas on this topic or other natural phenomenon will be appreciated by the author.

The Tesla Turbine
- A Mechanical Design Masterpiece

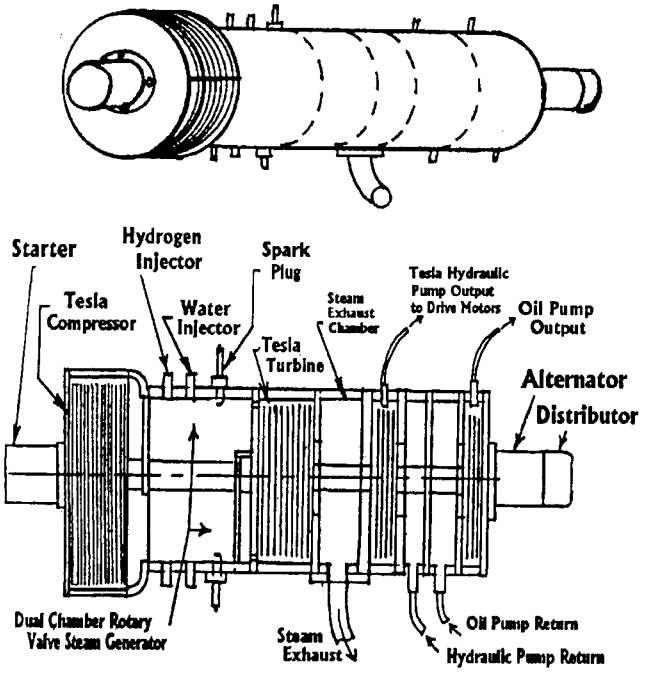
Tesla paid very close attention to mechanical details when he wanted to achieve maximum performance from his designs. One special feature of the Tesla turbine was a flow director set of rings that forced the steam to flow between the gaps in the disk array.



This reversible steam entry port enables Tesla turbine have either forward or reverse motion.

These flow director rings located on both ends of the disk array would prevent steam flow from going around the outer surfaces of the disk array on the right and left. These flow directors insure the pressure differential between input and output streams is felt mostly through the gaps in the disk array. These flow director rings improve the efficiency of the turbine.

Tesla's turbine has many important design features that should not be ignored by inventors looking for an efficient energy conversion system. Some versions of Tesla's turbine had a reversible flow entry port. As the entry port was rotated 180 degrees the steam input direction would also rotate 180 degrees. This reversible steam entry port would be a useful feature in car engine designs to enable forward or reverse motion. The spiral flow of the gases is also reversed in direction as you rotate the control valve 180 degrees.



The Hydraulic Tesla Engine envisioned by the author would utilize both Tesla turbines and Tesla pumps in a unique configuration.

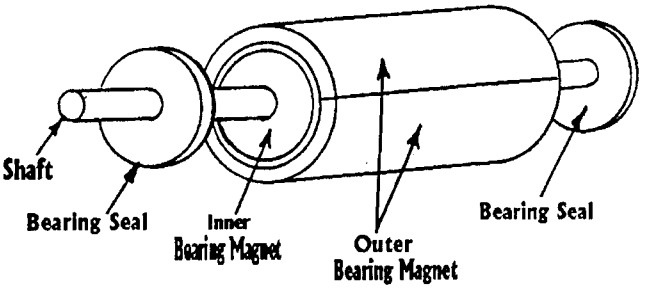
Tesla's patent papers indicate efficiency tests of the turbine were significantly better than the piston type of steam propulsion (the standard at that time). In comparison with the 30% efficient gasoline engines of today, a modern tesla turbine could well be 90% efficient if designed with modern technology and metals. Here are just a few of the Tesla turbine and pump advantages:

1) Highly efficient — high power to weight ratio.
2) Input can be abruptly shut off without destroying the unit.
3) Fixed installation or mobile applications.
4) Few moving parts — less things to fix.
5) Long operating life — if bearings are changed to magnetic bearings.
6) Low emissions. (If hydrogen is used the output is water vapor.)
7) Fewer and simpler machining operations are required to build than similar devices.
8) Direction is controllable by input flow direction change.

Tesla's bladeless turbine does have a few disadvantages. However, most of these problems could be overcome by design. Some of the Tesla turbine and pump disadvantages are as follows:

1) Small units require a high rpm to achieve useable torque.
2) Bearing life decreases with higher rpm.
3) Gyroscopic effect at high rpm.
4) When steam is used as the energy source, a condenser is usually required to recover water.
5) Belt, chain, or gear transmissions cannot tolerate high rpm or reversals.
6) Needs a quick starting mechanism to be practical.

The primary disadvantage of the Tesla turbine is the high rotational speed required to achieve useable horsepower. A one cubic-foot tesla turbine with 8.75-inch diameter disks could develop over a hundred horsepower at 50,000 rpm at 125 pounds



A magnetic bearing assembly between the turbine and pump would make the entire system wearout proof.

per square inch. A two-foot diameter disk turbine could develop 1200 hp at 2400 rpm. A car engine would probably require a 14-inch diameter disk array of 13 disks that would rotate about 6000 to 10000 rpm. This rpm could be reduced to 1000 to 2000 rpm by adding disks (for example 25 disks).

Centrifugal forces would have to be taken into account in a car turning a comer with this gyroscopic type of engine. Mounting the disks horizontally might solve this problem. The high rotational speed does create a problem of attaching the turbine shaft to a belt or chain or gear train. These belts or chains would be susceptible to disintegration at high speeds and abrupt reverse direction changes. The author's proposed solution solving the belt, chain, or gear problem might be to couple the Tesla turbine to the Tesla pump.

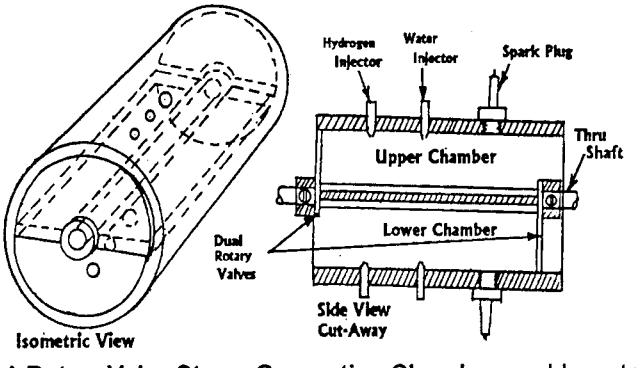
The Hydraulic Tesla Engine

Connecting the Tesla turbine on a common shaft with a Tesla pump allows the turbine to convert rotational energy to hydraulic power on the output of the Tesla pump. In an automobile application, external 50 horsepower Tesla hydraulic motors could be placed on the car's wheels and be driven by the pump output. Hydraulic servo valves could control the rate of flow to the hydraulic motors and change direction by flow reversal to the hydraulic motors.

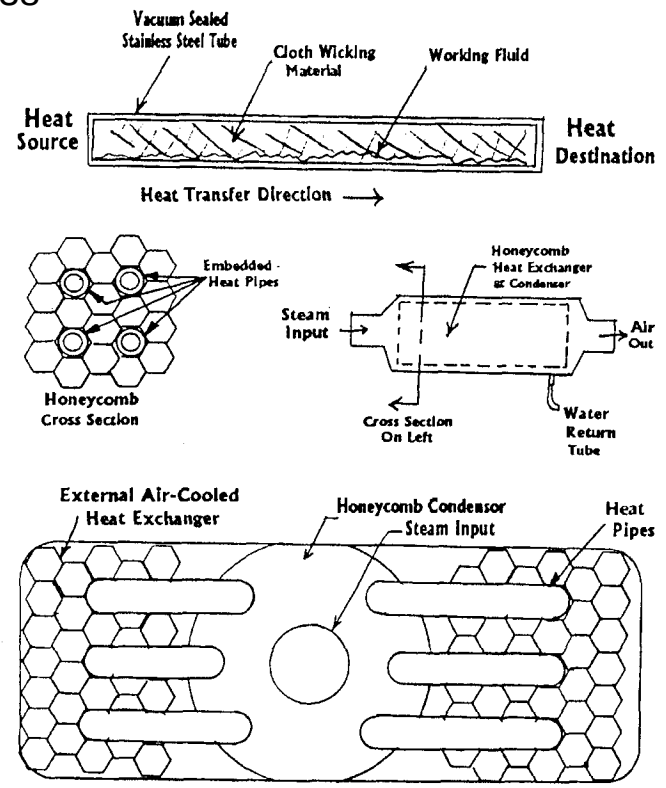
High speed Tesla turbines can also develop bearing problems. The Tesla steam turbine will run hot and the bearings will be stressed to the limit. Continuous oil flow over the bearings by an external oil pump and external oil cooling system may be required for long life. On one of the industrial versions of the Tesla turbines, the bearing housings were external to the turbine housing on external metal supports to keep the bearings cool and contain oil bathes for lubrication. An intriguing idea would be a magnetic bearing assembly between the turbine and pump to make the entire system almost wear out proof. The author is interested in information from any readers on the construction of magnetic bearings.

Another problem associated with development of the turbine is how to create instantaneous steam pressure and continuous steam pressure to activate the turbine. The author's proposed solution is to build a *rotary valve steam generation chamber*. The chamber would have two half circle rotary valves on both ends of a center divided cylinder. The divided cylinder would create two separate chambers with one chamber end covered and the other end uncovered by each rotary valve. A Tesla compressor would force intake air into the open chamber side while the closed chamber side receives a burst of hydrogen gas and a water spray from separate injectors. A spark plug on each chamber would ignite the fuel and water mixture and form instantaneous steam. The output rotary value would start opening just prior to ignition to allow the steam pressure to leave the chamber and enter the Tesla turbine.

Significant amounts of water are required to feed the turbine steam for continuous operation. A closed loop system of instantaneous steam generation and water recovery would be required to make the Tesla turbine a feasible automotive engine. A special type of condenser using convection cooling and heat pipe technology would be the modern way to reclaim the water effectively. A heat pipe is a 98% efficient heat transfer assembly consisting of closed metal tube made of copper with a vacuum in it and a working fluid, water, in it. A cloth wick



A Rotary Valve Steam Generation Chamber would create instantaneous steam pressure and continuous steam pressure to activate the turbine.



A Futuristic Water Recovery System would be needed to make the automotive Tesla turbine system feasible. The figure above highlights a number of features in the author's proposed system.

material is also enclosed in the tube to help distribute the small amount of water in the tube. The heat pipes would be coiled in the condenser and exit the condenser to extend into the aluminum convection coolers.

Conclusion

In conclusion, Tesla's turbine and pump technology could be combined into a tandem energy conversion system. This system could be expanded into an engine for transportation purposes by adding a steam generator, steam condenser, hydraulic motors, and hydraulic servo controls. Readers are encouraged to make comments, suggestions, or design improvements on this topic.

I hope this discussion of the Tesla pump and turbine will generate further interest and hopefully some invention on the part of the readers to help recreate modern versions of this technology. This technology is important in that it closely imitates natural energy conversion processes. Tesla was very much in tune with the idea that science and nature must be as compatible as possible to insure the long term coexistence of man and nature.__*DP*

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Several years ago, I purchased a booklet that dealt with some of Nikola Tesla's inventions. Among the discoveries that were addressed in that booklet was the Tesla turbine. I searched long and hard for material on this turbine, but only a few things were available. Since it wasn't an easy road, finding information, I thought there would be some interest in what I've accumulated. So, I've written it down.

Thank you for your inquiry on an invention that was buried, and remains buried, by economic and political forces. The original patents on the Tesla turbine were filed in 1909 and granted in 1913. Among the comments made on the turbine at its revelation are "greatest invention of the age", "the apotheosis of simplicity", "an ideal turbine", and so on. But before we go any further, let's understand a little history of the sciences.

Most of the principles of physics we work with today were uncovered many years ago. Even that abstract and fascinating area called quantum physics had its roots back in the early 1900's, and was well entrenched by the 1930's. The modern internal combustion engine is little different than the original patents in the late 1800's. All that's truly been done is to refine that engine. Modern turbines have been refined to a very high degree, but it must be remembered that the origins date back to the Greeks, thousands of years ago. The period of great invention truly was between 1700 and 1930. Virtually all physical rules were developed in this period, including the main theory of quantum physics. It was in the latter part of this period that Nikola Tesla made his contributions, among them the practical use of alternating current, without which we wouldn't have our modern electrical distibution systems. It was Tesla's discoveries of efficient generation and use of alternating current that made it all possible.

Tesla was not, as many have supposed, an electrical engineer. He was, by his own statement, a mechanical engineer sidetracked into electrical engineering by his own goal of building a flying machine. When, after years of thought, he devised and perfected his turbine, this was not a spur of the moment thing; and the first of the turbines wasn't very good. It was after a good deal of experimentation that the Tesla turbine actually grew up, into a very elegant, very efficient turbine of rather astounding abilities.

So...

How about a pump that will pump almost anything that will flow, including pulverized rock, without apparent damage to the pump?

How about an engine that has one moving part, and the only wear-prone piece used in it's construction is the bearing supporting the shaft?

This is the Tesla turbine - arguably the most elegant engine ever invented, and without question the most misunderstood. This turbine is, if proper guidelines are followed, the most efficient of the turbines (up to 95% proven efficiency), and incidentally, far exceeding in efficiency and power to weight ratio any piston engine ever produced anywhere.

This is also one of the most thoroughly suppressed inventions ever. Think of it - a simple, easily manufactured turbine with incredibly great low RPM torque, efficiency that more closely approaches perfection than any engine or pump yet developed, up to 95% efficiency in test situations, the ability to run on almost any fuel, including sunlight (similar to the Stirling engine), and with only the bearings to wear out! And how long will a properly designed and manufactured set of ball or roller bearings wear, if they are lubricated well and if there is little side load and no thrust load? A long, long time.

The Tesla turbine has been suppressed with good reason - it would put a lot of companies in economic peril, or so they believe. If Big Automobile Companies couldn't sell replacement engine parts (except ball or roller bearings), where would they be? Business would be cut by massive amounts! (It has apparently never occurred to them that other products could be sold, that the money saved by consumers would undoubtedly be used on other consumer products; and we consumers would be a lot better off!)

If Big Gasoline Companies sold less than one-quarter of the fuel they do now, what would happen to the huge profits they make? What would happen to the Energy Crunch? Somebody there might even be forced to think! And we can't really consider the good of the American public, now, can we?

If this turbine were used in all cars and trucks (yes, even the big trucks!), and fuel use dropped by 75%, where would the Arabs and their oil wells be? They would be in big trouble, while you cruised your 4000 pound big Ford at 65 on the Interstate, doing 60 or 70 mpg with the Tesla air conditioner on (no CFC's, just plain old air).

And the turbine would be able to run without a muffler, without a catalytic converter, without worrying about hot exhaust. The Tesla turbine utilizes almost all the heat from burning the fuel, and there are no blades to make noise as in a normal turbine.

The Tesla turbine can run from steam, too, even wet steam! And using compounding tactics, the steam can be completely converted back to water within the turbine itself! This isn't possible with a normal turbine. With a compounded Tesla turbine, no radiator is necessary, so

weight is cut way back, while horsepower is drastically increased (the latent heat of vaporization, the heat required to change the water to steam, is recovered by the turbine - about five times the energy as is required to raise the temperature of the water to the boiling point).

How about a water pump that's 90% efficient instead of 20-30% efficient (under ideal conditions, well built pumps can do 70%)? And a Tesla turbine configured for pumping water can ingest sand without damage! In fact, since there is no blade turbulence introduced into the fluid by vanes (lifting surfaces), the Tesla turbine can pump boiling water (something no other pump can do well, if it can at all) and still retain most of it's efficiency!

With minor design modifications, the Tesla turbine can be used as an air compressor, and a very efficient one at that. Or it can be used as an air motor.

This instruction manual explains how the Tesla turbine works, how it is that the Tesla turbine is so efficient, and most importantly, how you can build one yourself. Plans are included in the manual so a small turbine can be made for test purposes, to run on compressed air. There are also plans for a water pump, and some other devices that are easily constructed in an average garage with hand tools.

Also included are plans for a steam driven turbine. Both a simple turbine and a compounded version are presented.

There are plans for a water pump. The Tesla turbine can pump to higher heads than a normal pump, and is three or four times as efficient. Further, sand, grit, aquatic plants and so on, do not impair the operation of the Tesla turbine, nor does any detritus cause the pump to deteriorate. Pump speed is not limited, either: the faster the rotor turns, the more water is pumped.

There is theory included on construction of a solar powered Tesla turbine. Think about <u>that</u> a bit: if a normal solar cell is 10% efficient, and the Tesla turbine is 90-95% efficient, if the collector were properly designed, you could get six or seven times the capability of a solar cell. In southern areas, where about 370 watts of sunshine per square yard are available, you could actually generate most of a household's electrical needs with just a few square yards of reflective solar panels, which are much less expensive than solar cells, and the power generated would be standard 60 HZ AC.

The possibilities are almost unlimited. Quiet fans. Quiet airplane engines. Light, small automotive power plants. Electrical energy generation made easy and efficient. Pumps for nearly any fluid. Engines that run using nearly any fuel. Air conditioning. Anything using any kind of engine or motor can be powered by the Tesla turbine. Any fluid can be pumped with a Tesla turbine.

And the turbine is so easy to make, it's almost unbelievable! A few pieces of sheet metal, NO metal forming, NO blades, NO vanes, just a few pieces of punched sheet, some pipe, some shaft, a bearing or two, and a jet or diffuser of some kind (also easily made) will put the machine in business!

So there we have it. A truly remarkable device, capable of many different functions, easily made, and efficient.

And no, this turbine is not a joke. It does work, and how! It's terribly real. I'm a real skeptic, and I wasn't 100% convinced myself, till I tried it. After I realized it does work, I sat down and analyzed why it does, then went about building different types for different things. The turbine never failed me.

It's all contained in this well illustrated booklet. Hundreds, maybe thousands of different uses can be found for this device. There have been blowers (air movers pumping tens of thousands of cubic feet of air per minute), vacuum pumps, water pumps, pulverized stone pumps, pumps that move salt water without damage, and so on. There have been gas turbines, steam turbines, air engines and compressors, and even turbines that will run on pulverized coal (that would destroy a normal turbine in short order). I can't possibly come up with all the uses, but the more people that know about it, the better. At least with this device, we can take matters more into our own hands.

To take advantage of years of searching and experimentation, mail the order blank enclosed. The cost is $14.95, including shipping.

Your order will be shipped to you within forty-eight hours of receipt. Satisfaction guaranteed: return the booklet in salable condition within 30 days and your money will be refunded, no questions asked.

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